Renormalization for irrationally indifferent fixed points of holomorphic functions

Mitsuhiro Shishikura
Kyoto University

Frontiers in Complex Dynamics
Celebrating John Milnor's 80th birthday
BIRS, Banff, Canada
February 23, 2011

Frontiers in Complex Dynamics

use renormalization!

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include *boundaries* of Siegel disks

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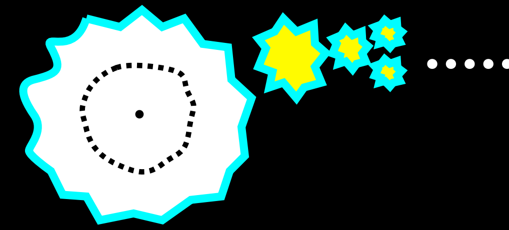

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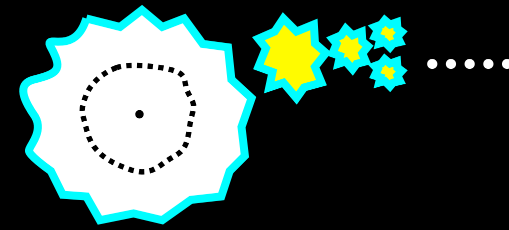

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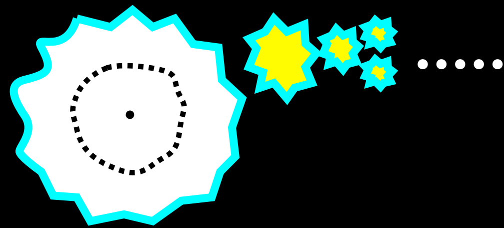

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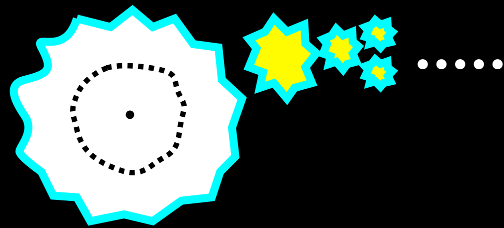

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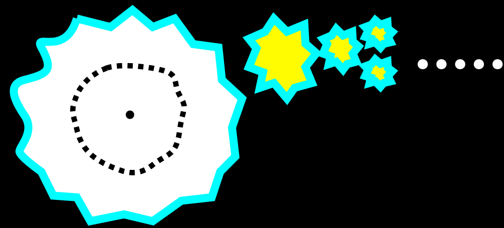

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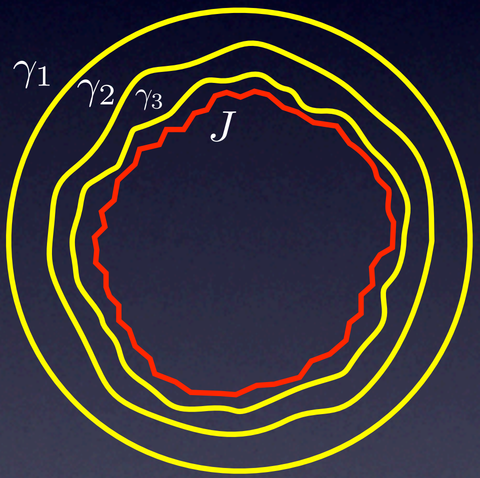

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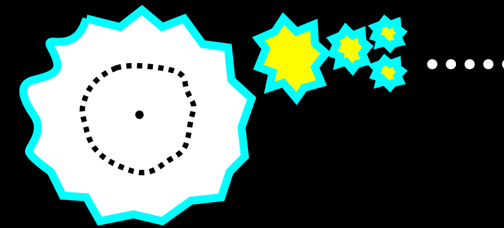
include *boundaries* of Siegel disks 

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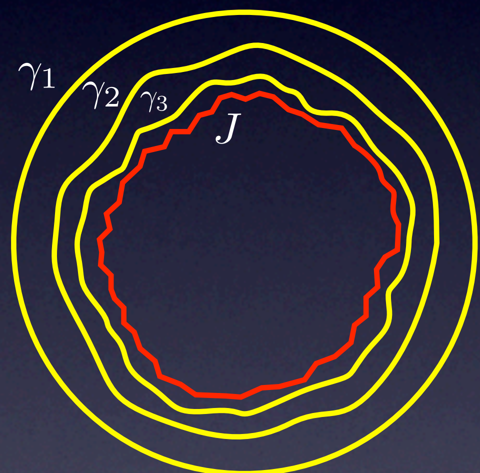

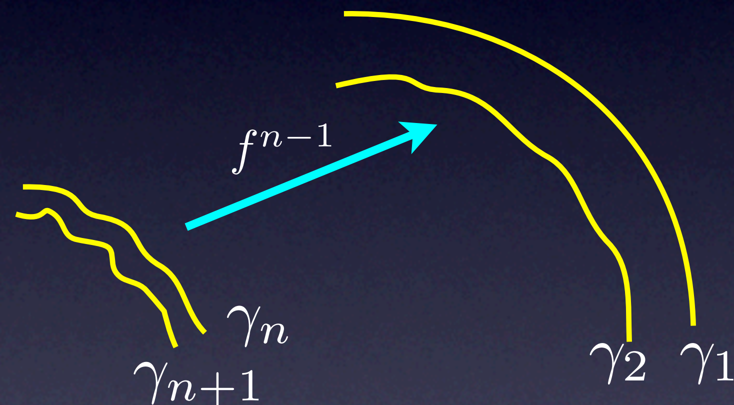

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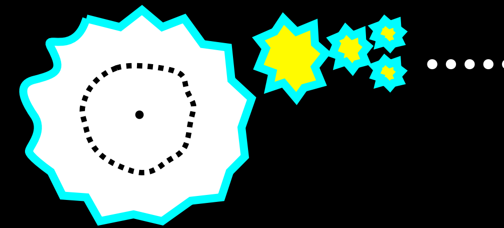

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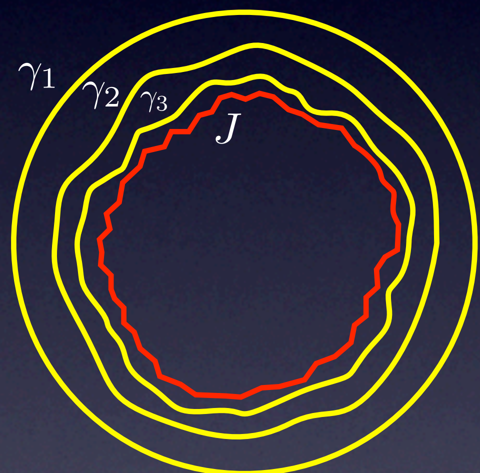

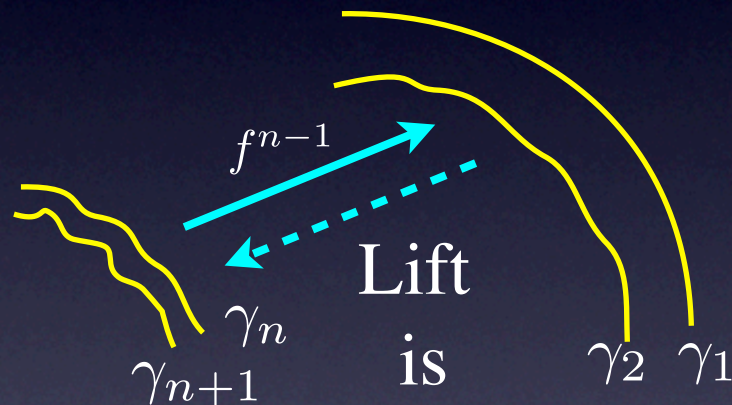

Lift is contraction

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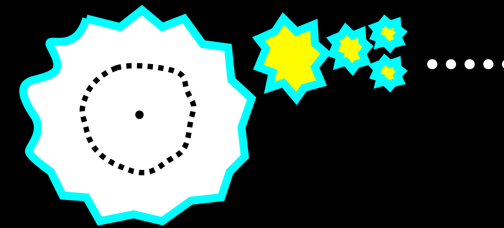

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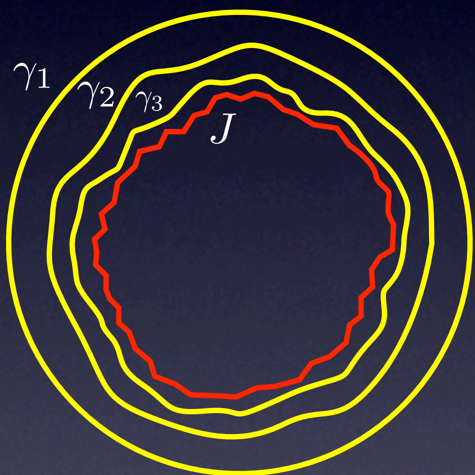

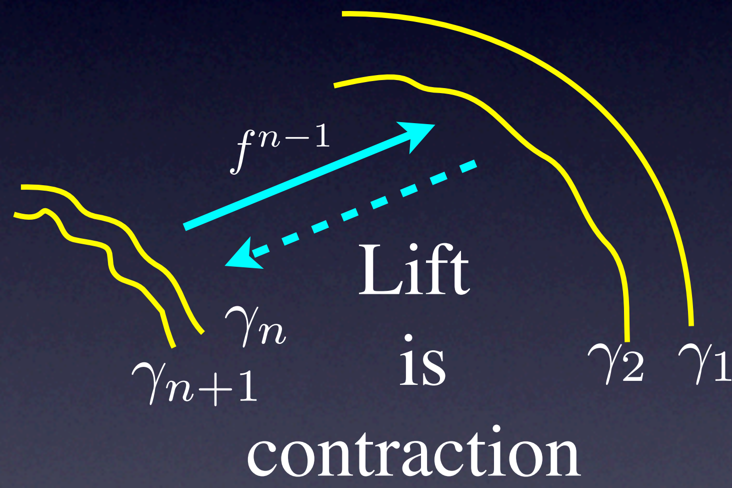

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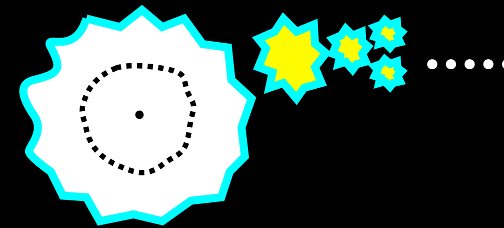

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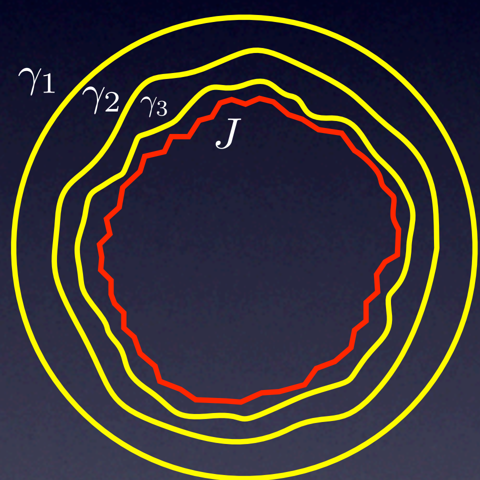

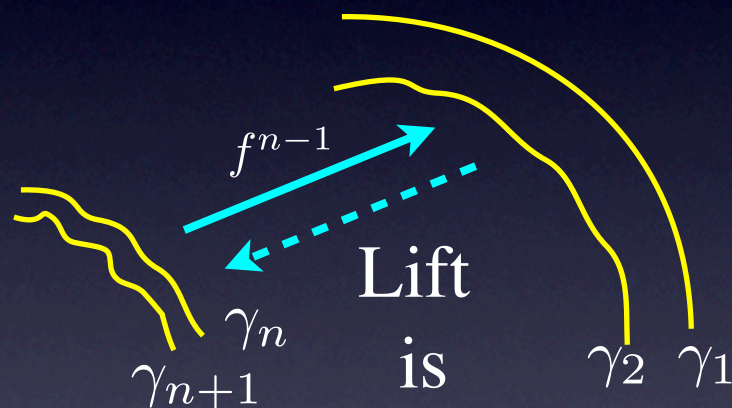

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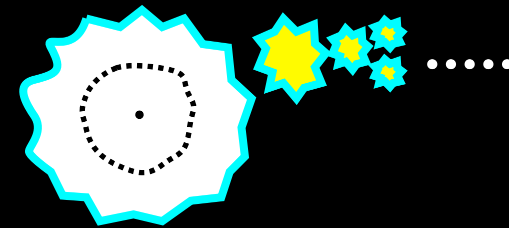

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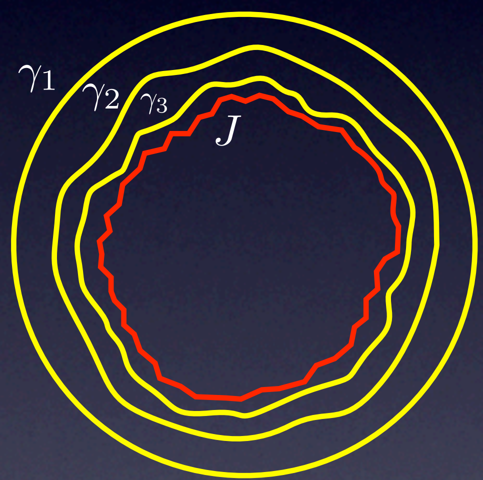

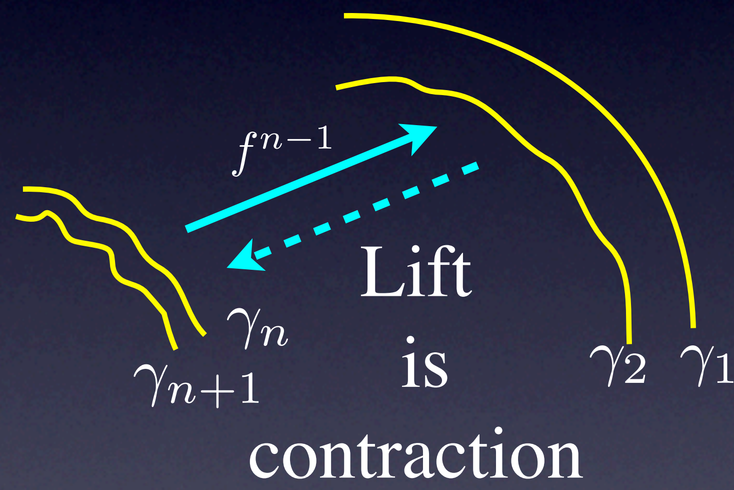

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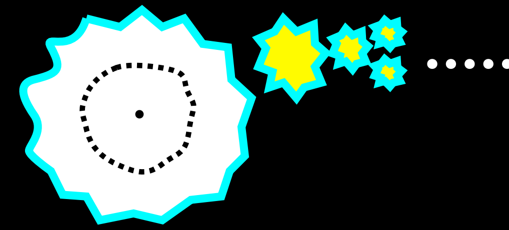

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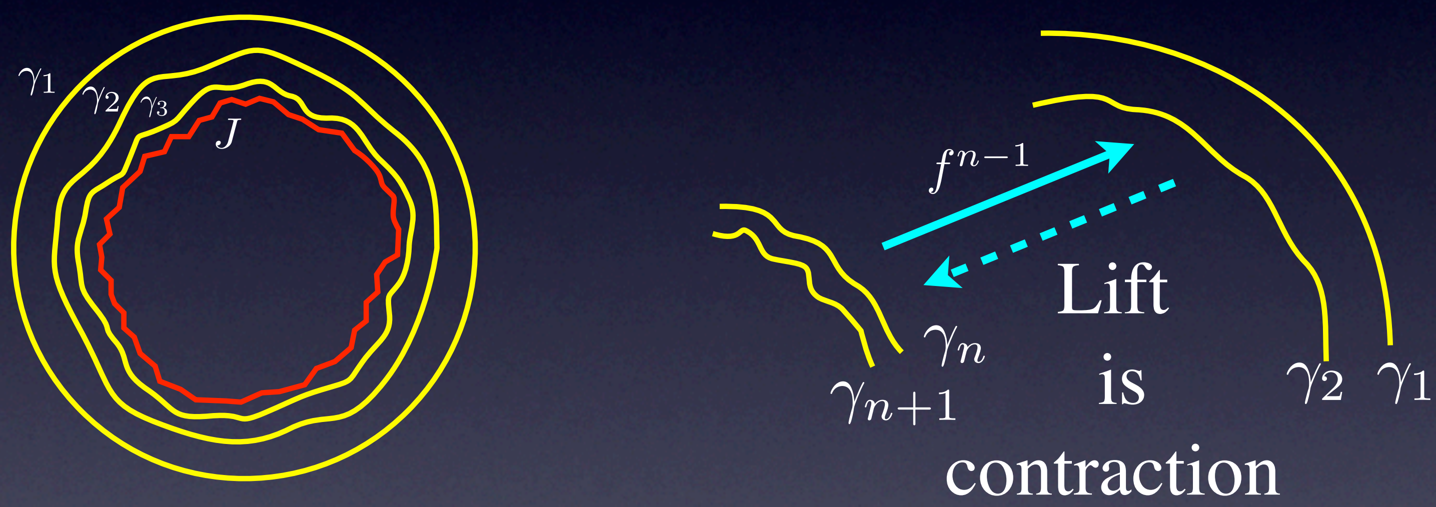

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Gondola, T-bar, . . . *use renormalization!*

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α irrational

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$(\Sigma_g \text{ surface } g \geq 2)$

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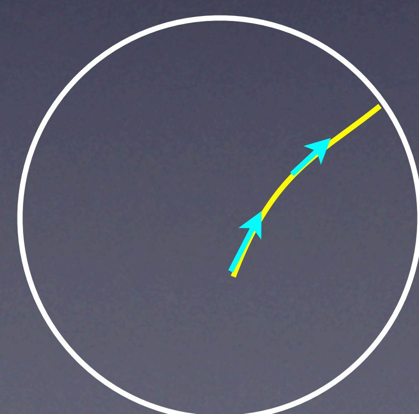

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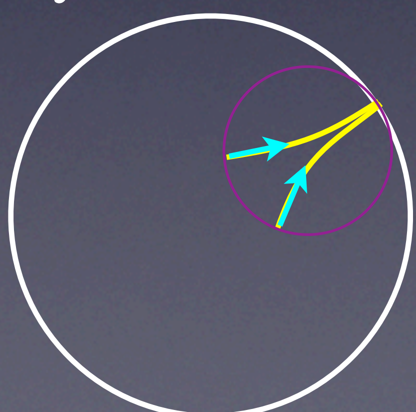

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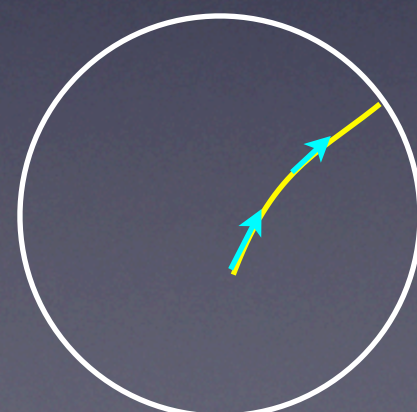

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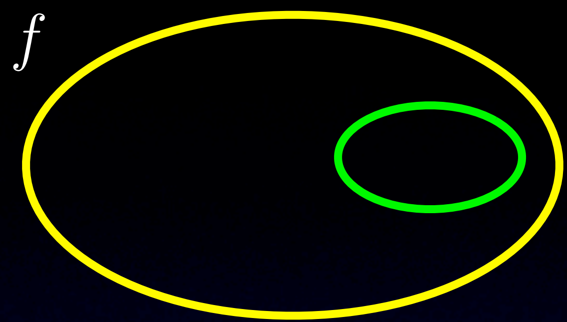

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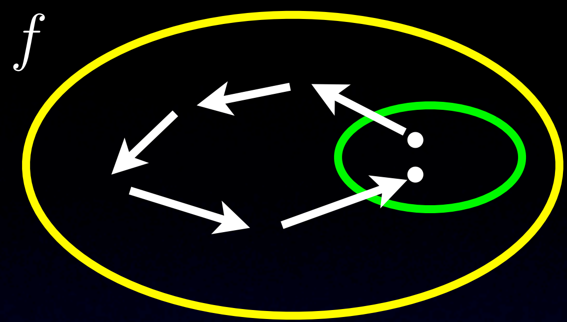

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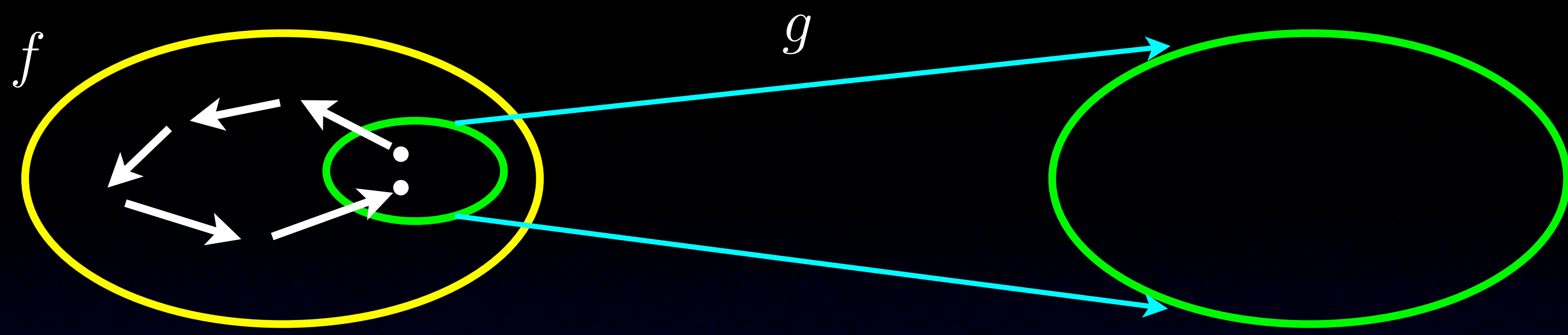

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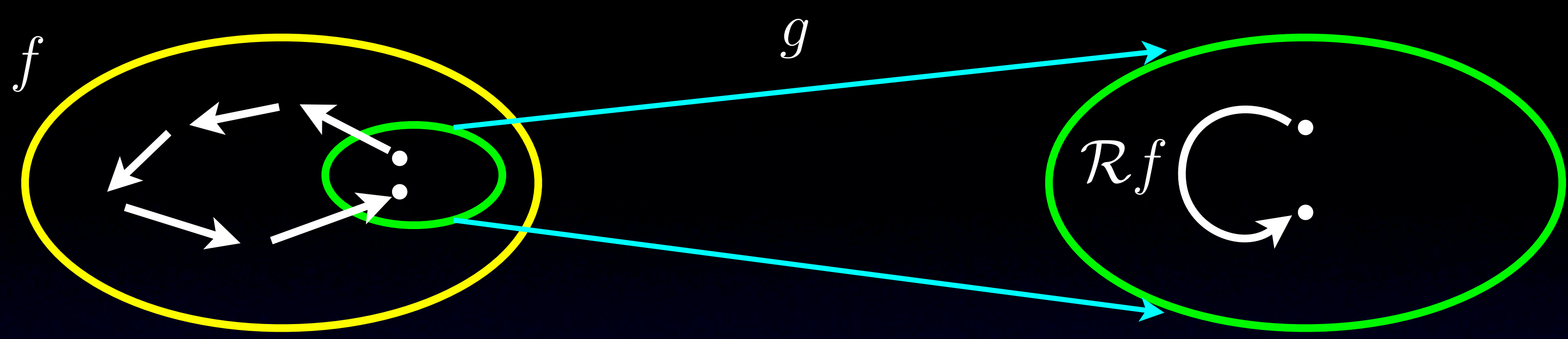

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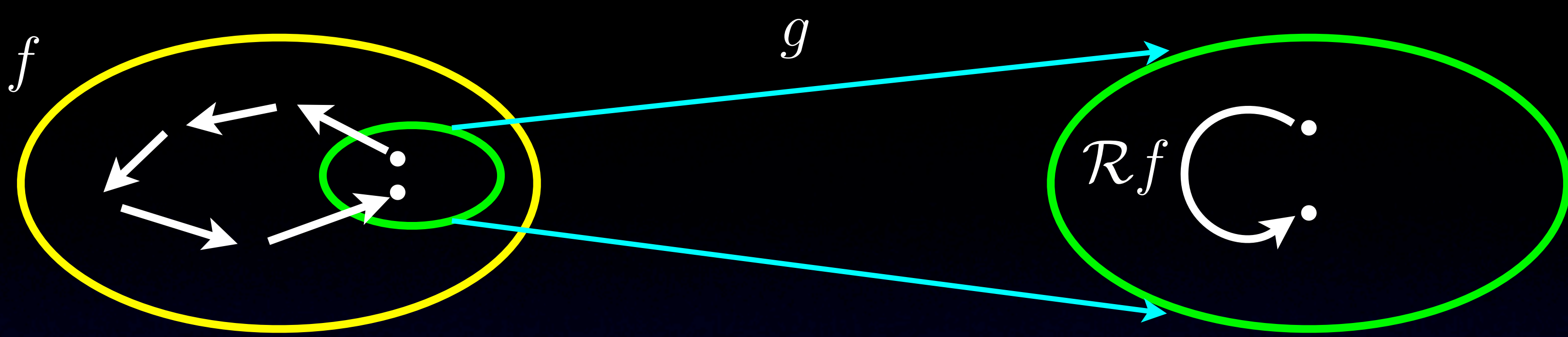

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$$= g \circ f^k \circ g^{-1} \quad (\text{if return time} \equiv k)$$

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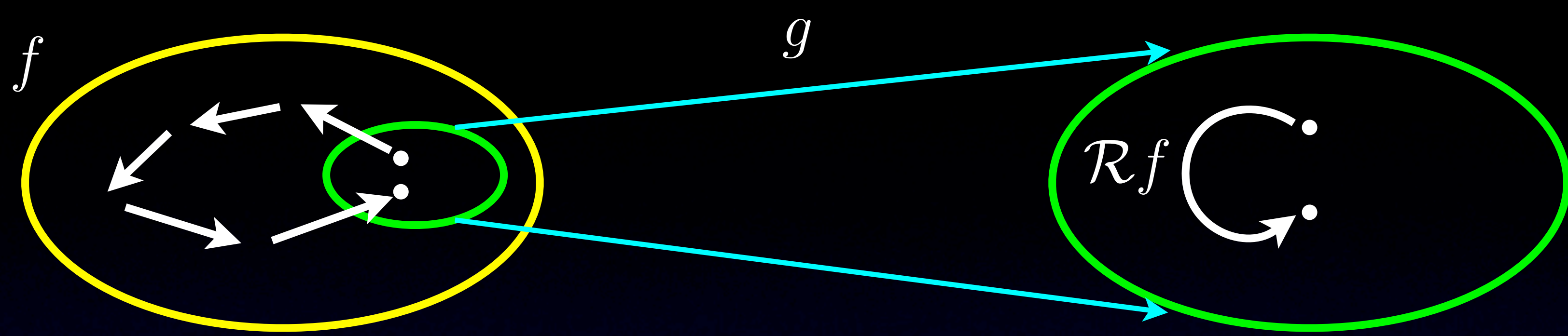

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Renormalization

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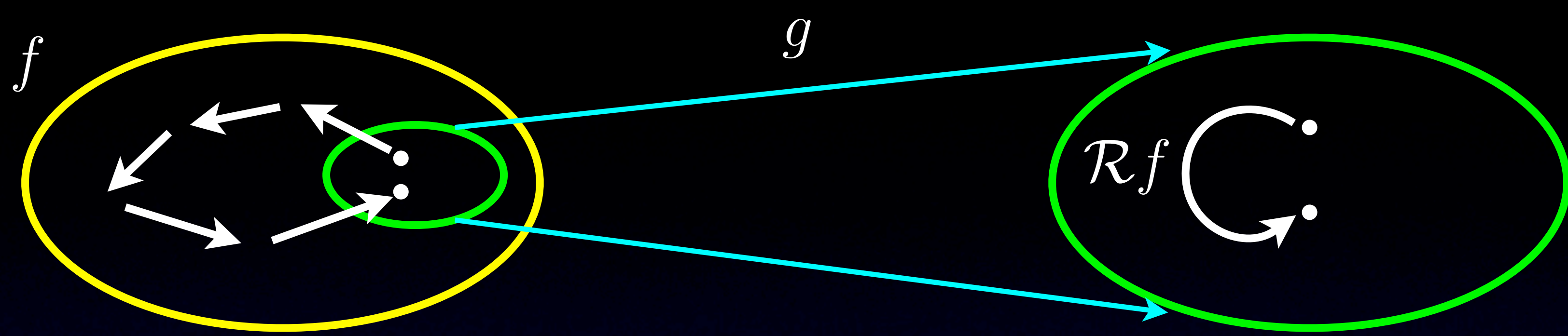

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$$= g \circ f^k \circ g^{-1} \quad (\text{if return time} \equiv k)$$

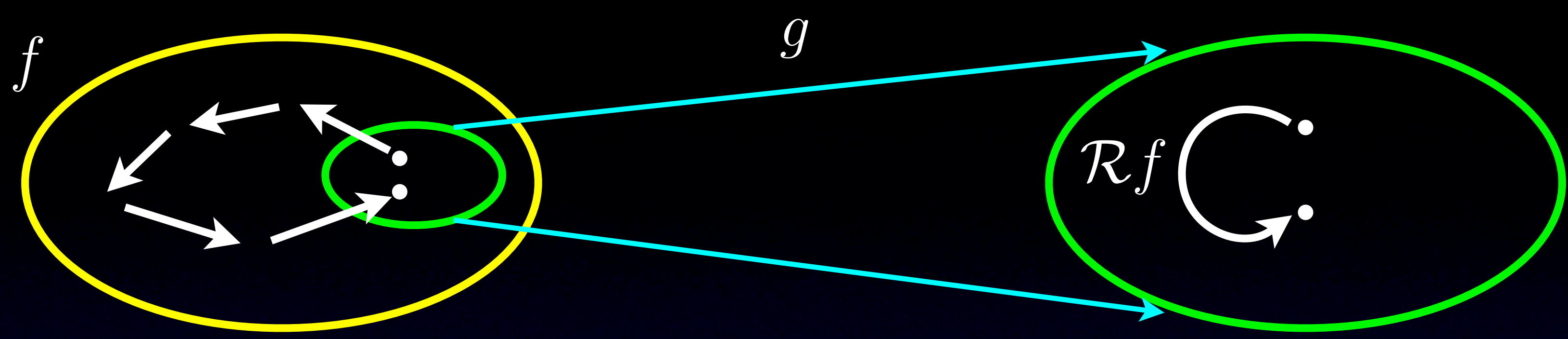
Renormalization

high iterates of f $\longleftrightarrow$ fewer iterates of $\mathcal{R}f$

fine orbit structure for f $\longleftrightarrow$ large scale orbit structure for $\mathcal{R}f$

Successive construction of $\mathcal{R}f$, $\mathcal{R}^2 f$, ..., helps to understand the dynamics of f (orbits, invariant sets, rigidity, bifurcation, ...)

Return map and renormalization



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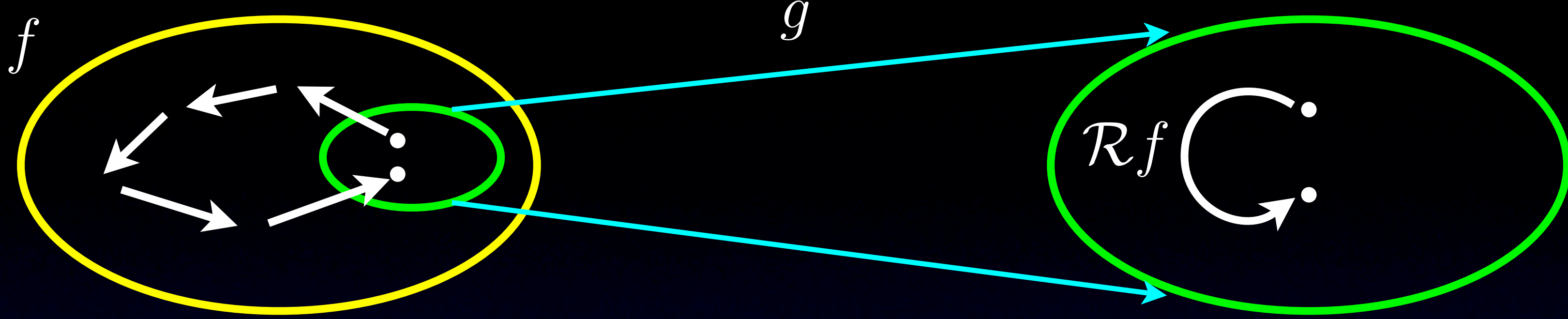
If f is a fixed point of renormalization $\mathcal{R}$ (with return time $\equiv k$), then $g \circ f^k \circ g^{-1} = f$, i.e. $g \circ f^k = f \circ g$ (intertwining relation).

Return map and renormalization



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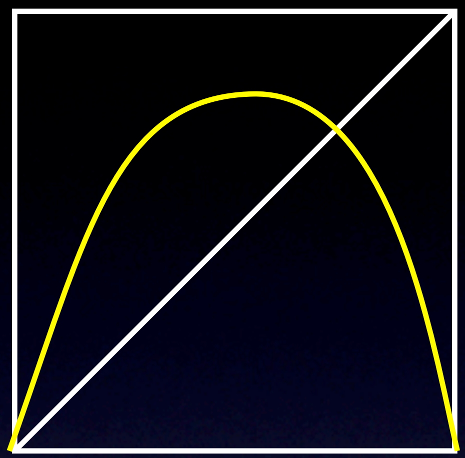
Usually f tame and g expanding (chaotic).

Feigenbaum-Coullet-Tresser for unimodal maps

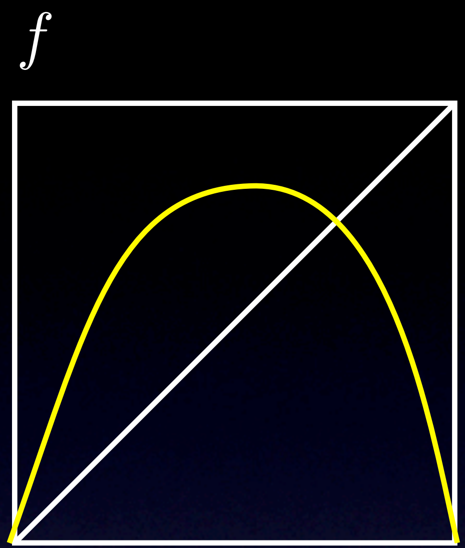
Feigenbaum-Coullet-Tresser for unimodal maps

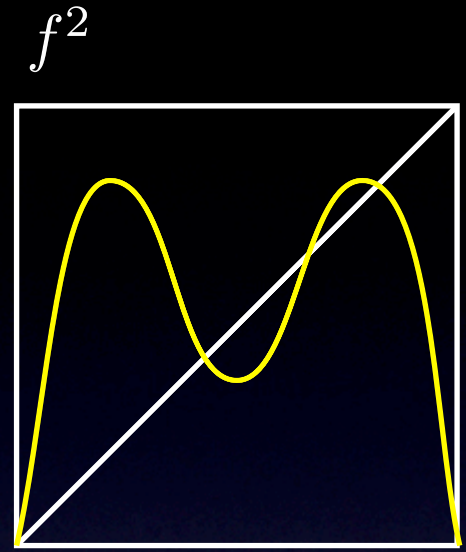
f



$I = [0, 1]$

Feigenbaum-Coullet-Tresser for unimodal maps





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Feigenbaum-Coullet-Tresser for unimodal maps

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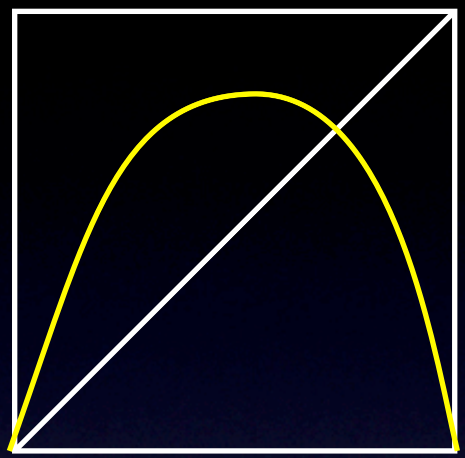
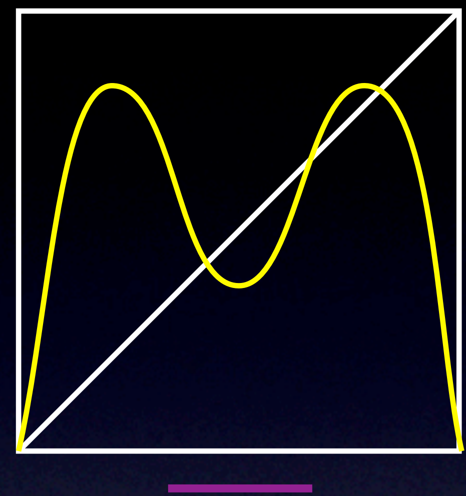


$I = [0, 1]$

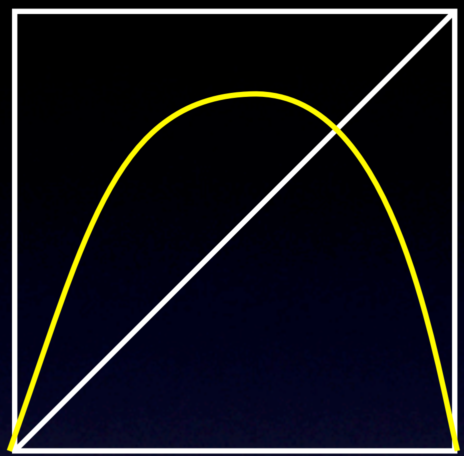
f^2



$J \subset I \text{ s.t. } f^2(J) \subset J$

Feigenbaum-Coullet-Tresser for unimodal maps

f



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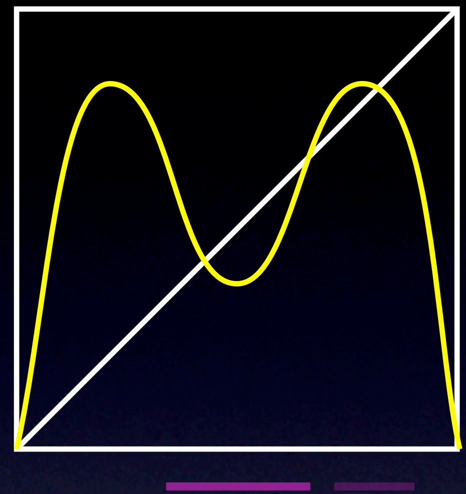

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Feigenbaum-Coullet-Tresser for unimodal maps

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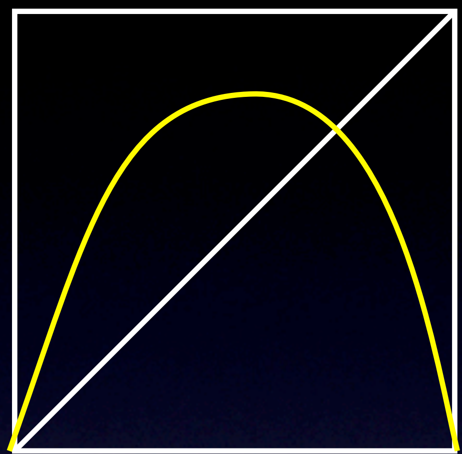

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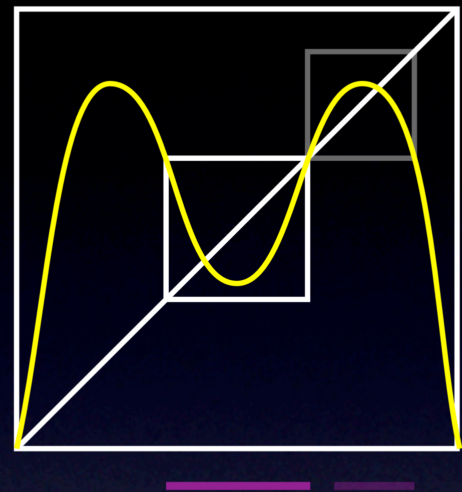

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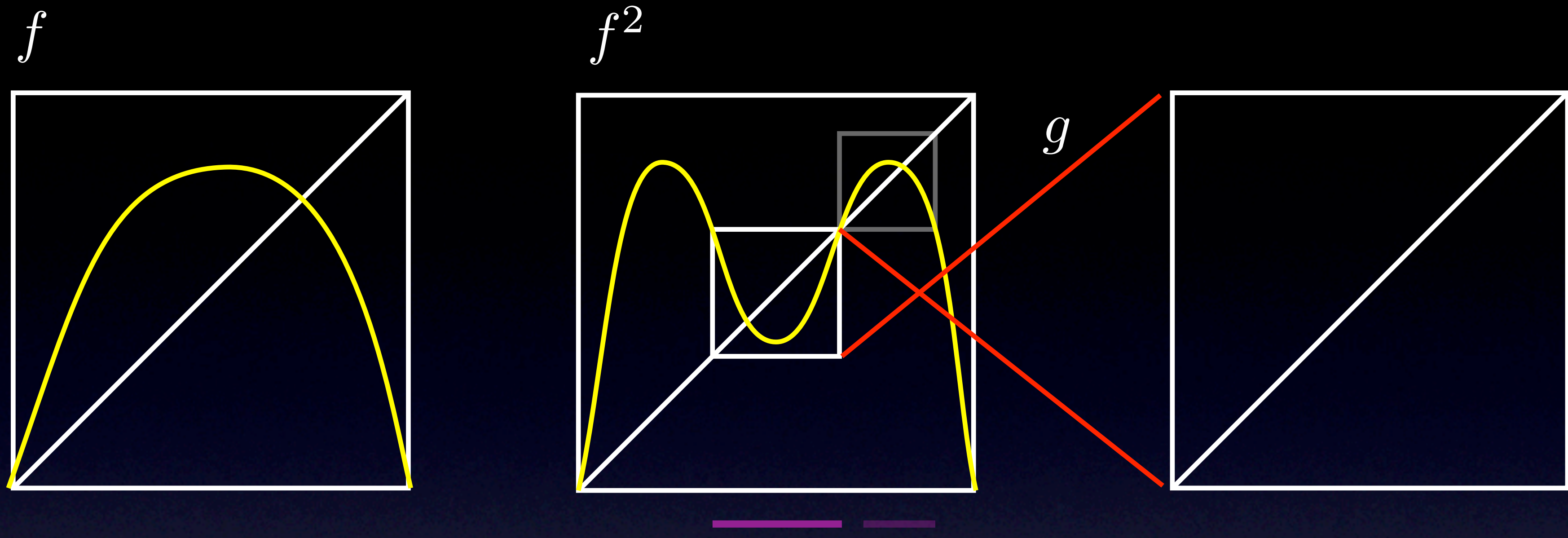
Feigenbaum-Coullet-Tresser for unimodal maps



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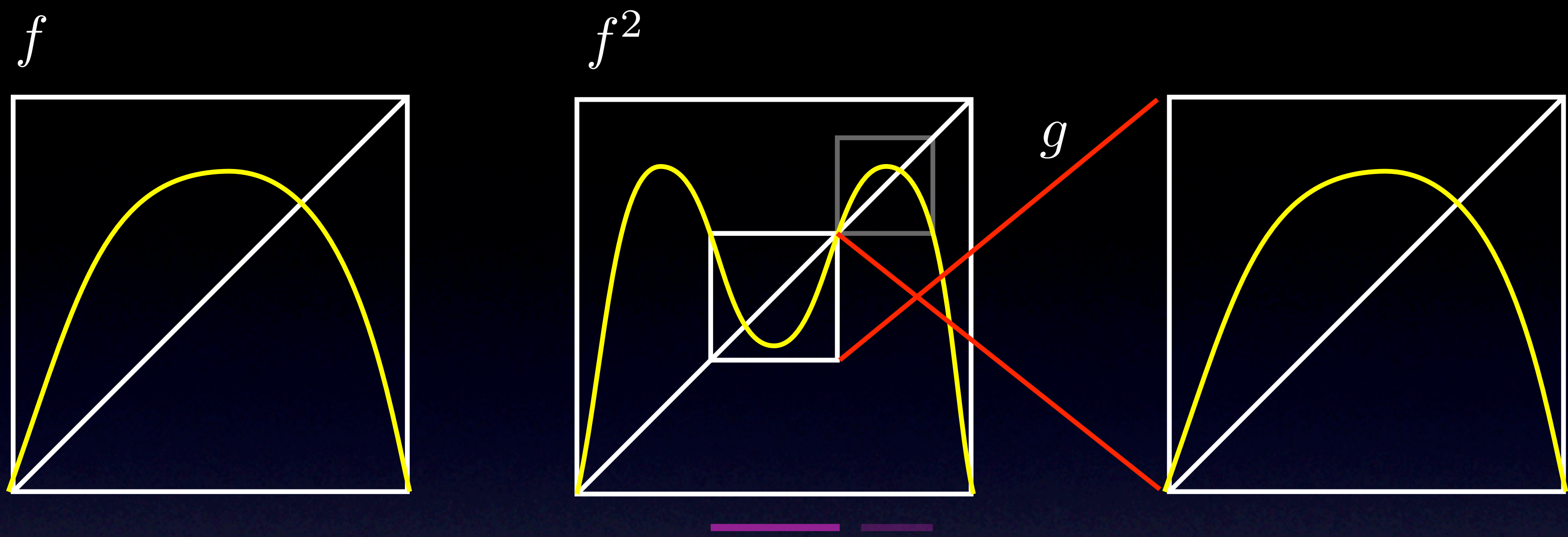

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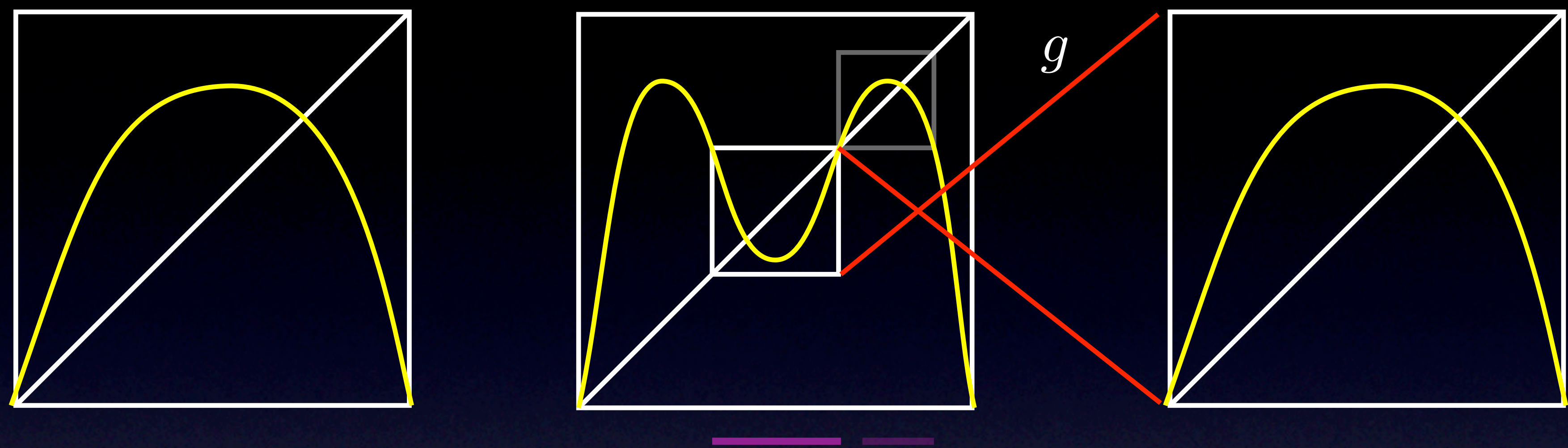
$J \subset I$ s.t. $f^2(J) \subset J$

Feigenbaum-Coullet-Tresser for unimodal maps

f $\qquad$ f^2 $\qquad$ $\mathcal{R}f = g \circ (f^2|_J) \circ g^{-1}$



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Renormalization: Meta-dynamics
Dynamics *on the space of certain dynamical systems*

Feigenbaum-Coullet-Tresser for unimodal maps

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Renormalization: Meta-dynamics
Dynamics *on the space of certain dynamical systems*

Hyperbolic fixed point or hyperbolic horseshoe of the meta-dynamics imply conclusion on rigidity and structure of parameter space

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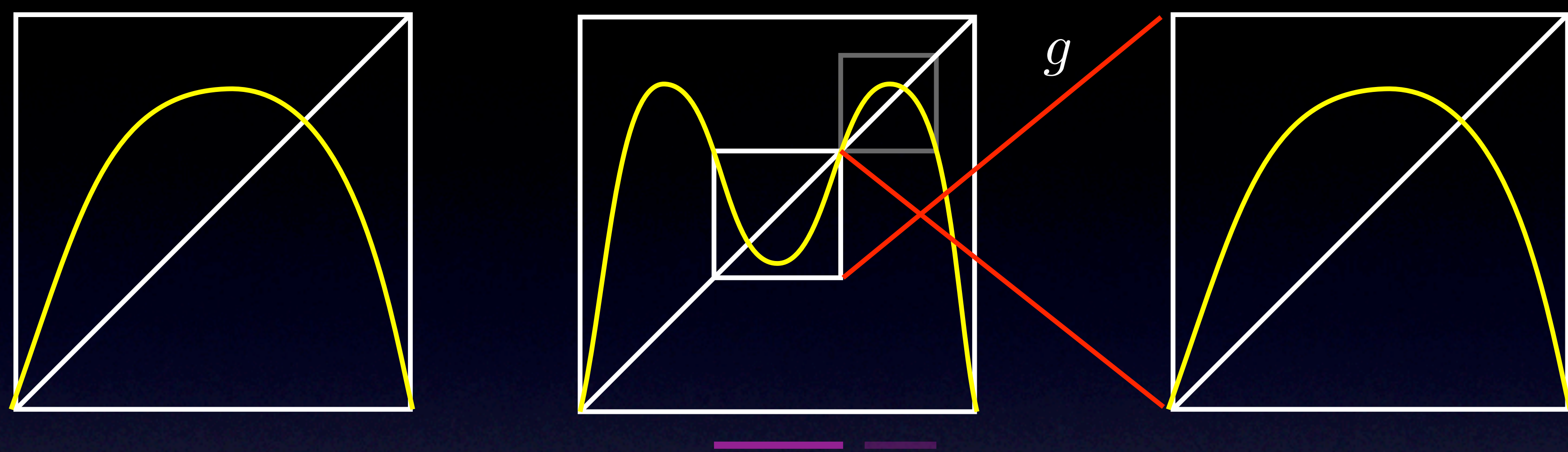

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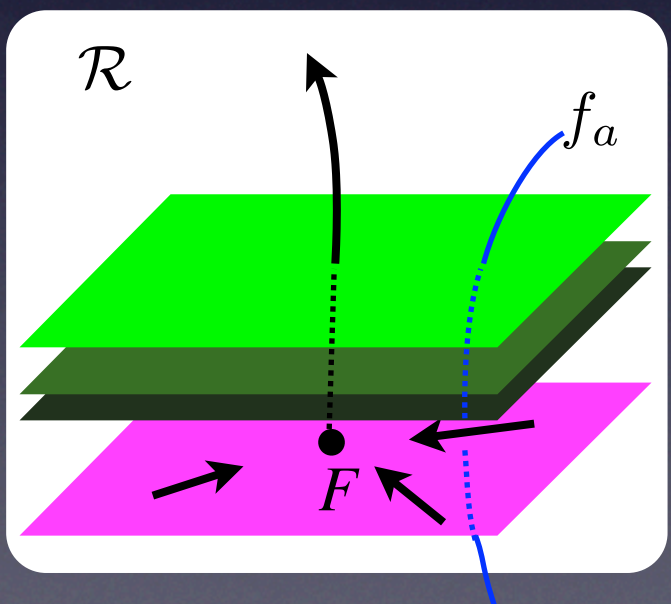

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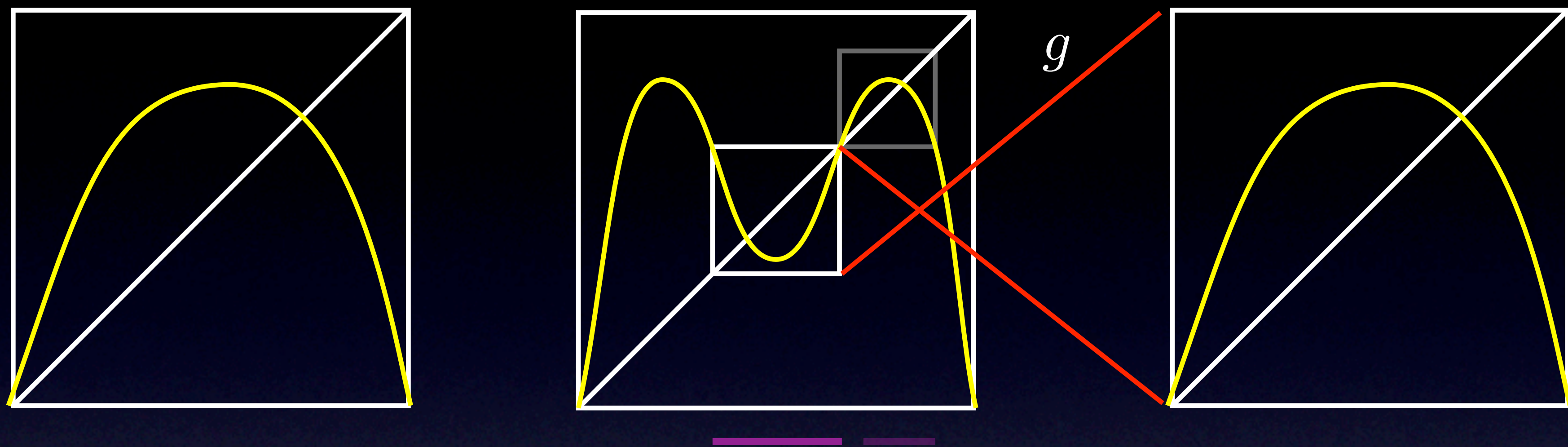

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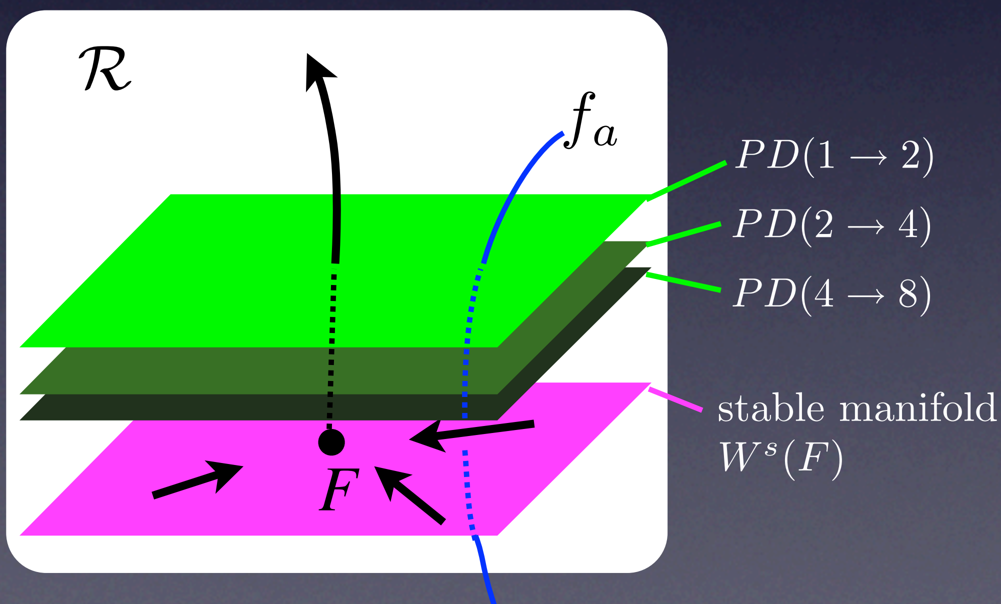


Renormalization: Meta-dynamics
Dynamics *on the space of certain dynamical systems*

Hyperbolic fixed point or hyperbolic horseshoe of the meta-dynamics imply conclusion on rigidity and structure of parameter space

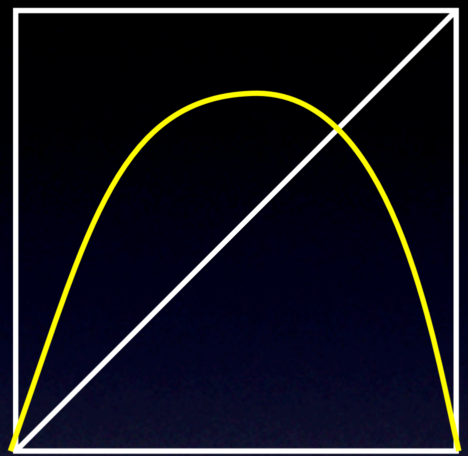
Various Renormalizations

Various Renormalizations

Feigenbaum

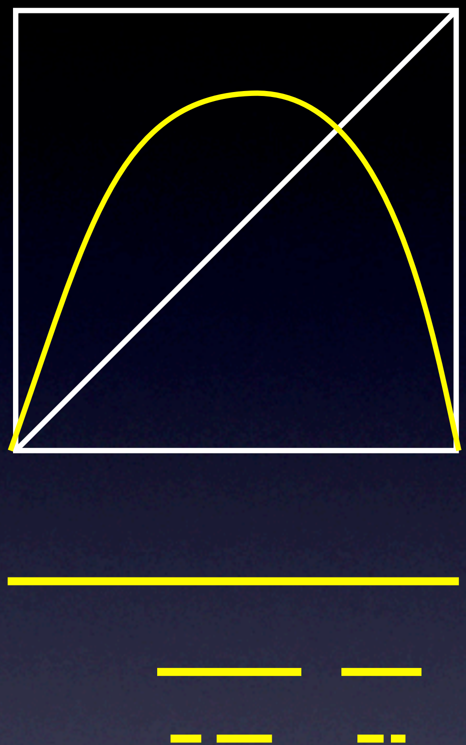
Various Renormalizations

Feigenbaum



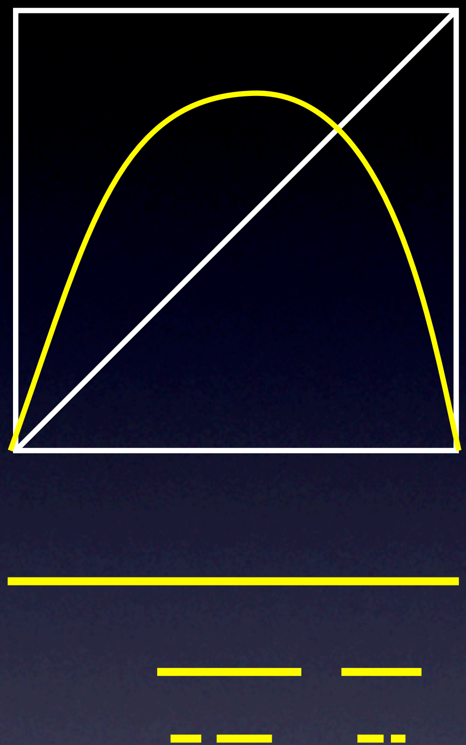
Various Renormalizations

Feigenbaum



Various Renormalizations

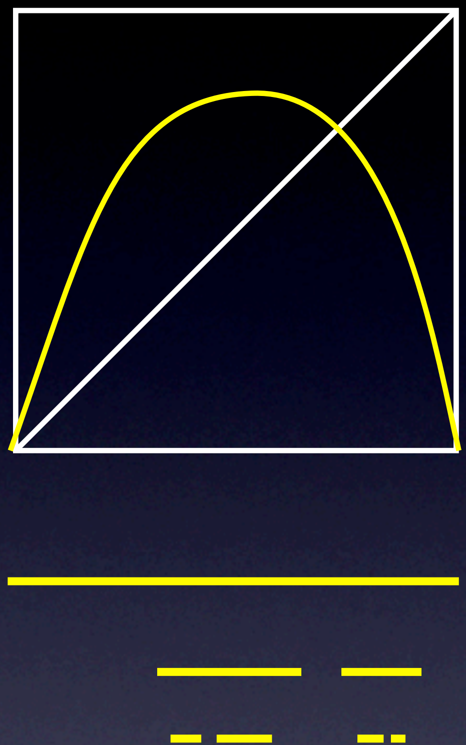
Feigenbaum



proper subintervals

Various Renormalizations

Feigenbaum

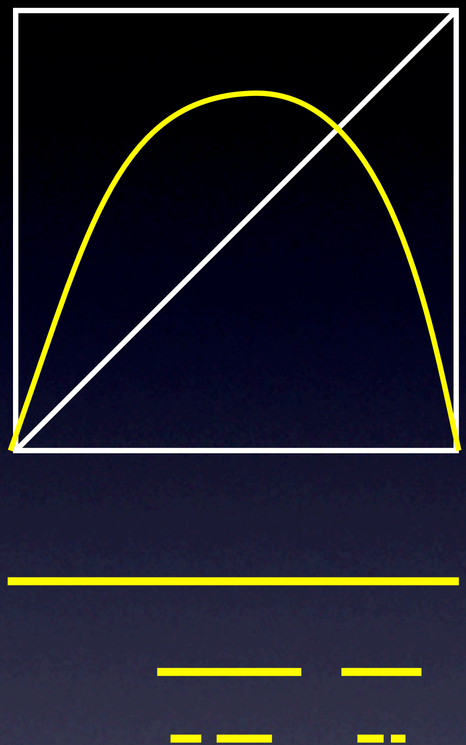


proper subintervals

-> Cantor set

Various Renormalizations

Feigenbaum



proper subintervals

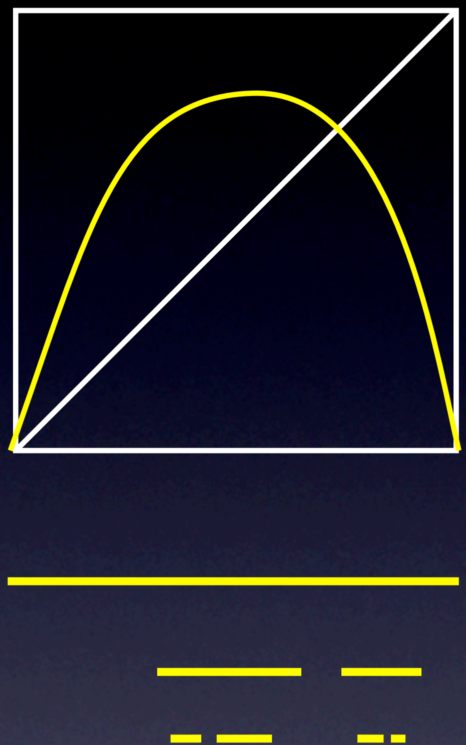
-> Cantor set

Feigenbaum, Coullet-Tresser,
Lanford, H. Epstein,
Polynomial-like maps:
Douady-Hubbard, Sullivan,
McMullen, Lyubich

Various Renormalizations

Feigenbaum

Circle map



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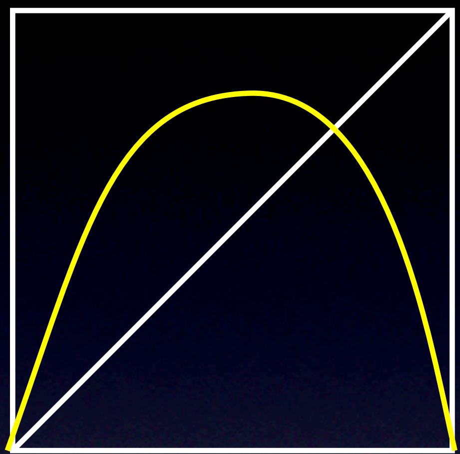
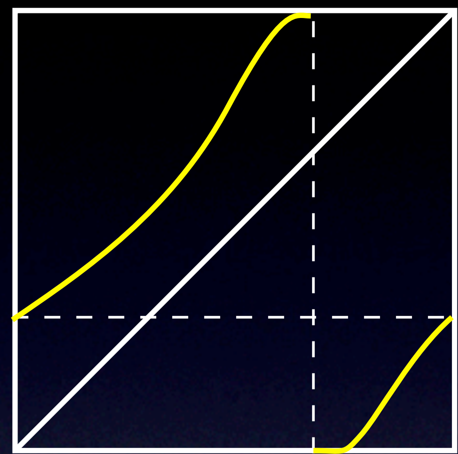
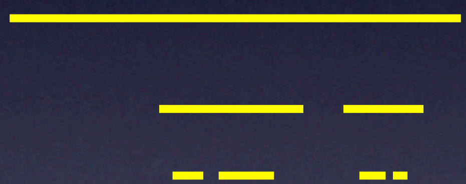
Various Renormalizations

Feigenbaum



Circle map





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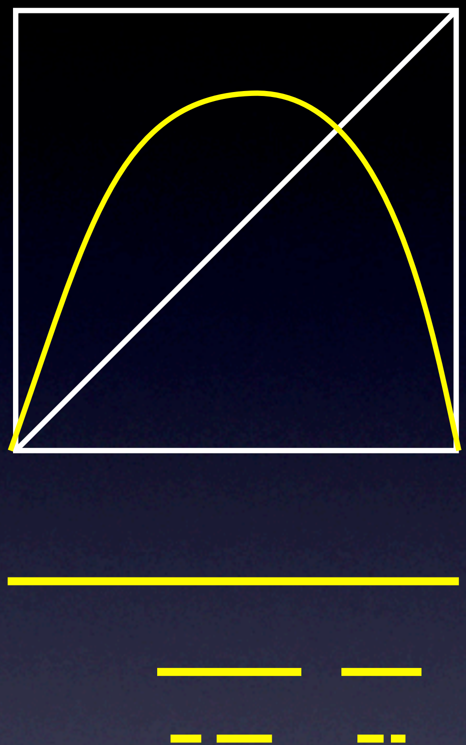
Various Renormalizations

Feigenbaum



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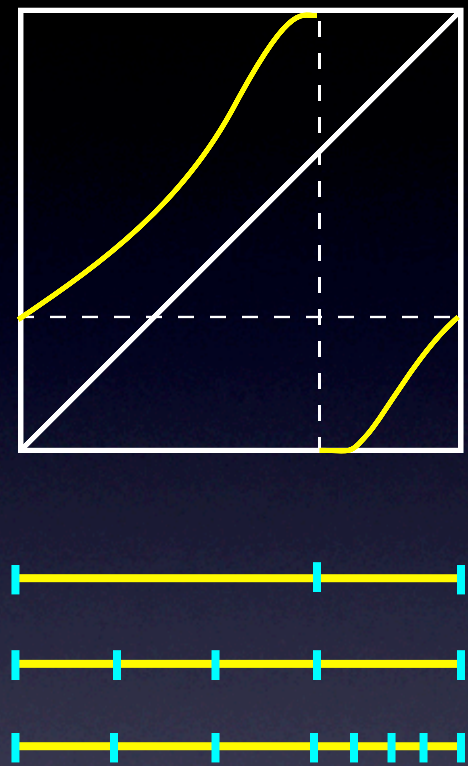

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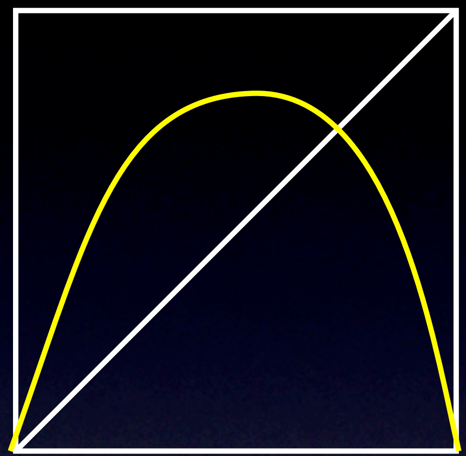
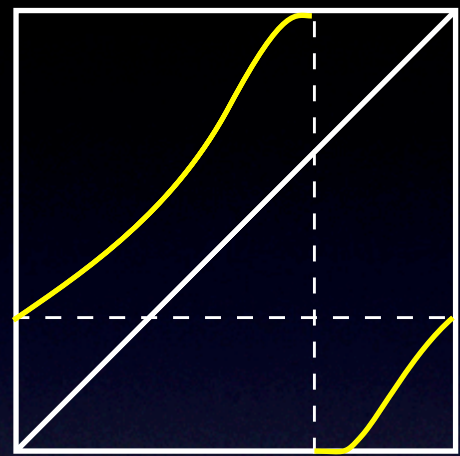
Various Renormalizations

Feigenbaum



Circle map



proper subintervals

-> Cantor set

partition of interval

Feigenbaum, Coullet-Tresser,
Lanford, H. Epstein,
Polynomial-like maps:
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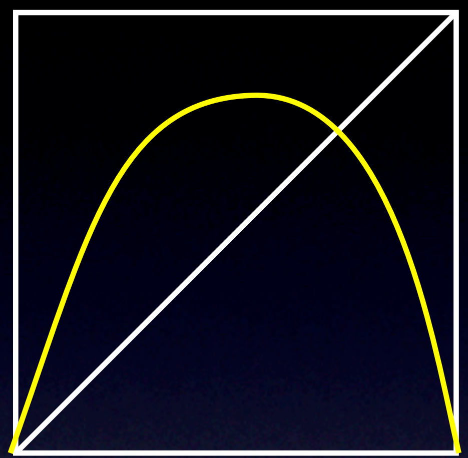
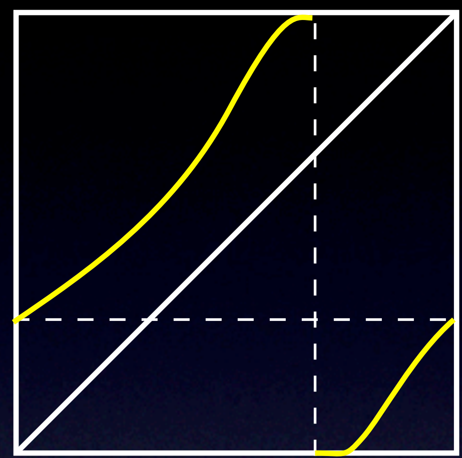
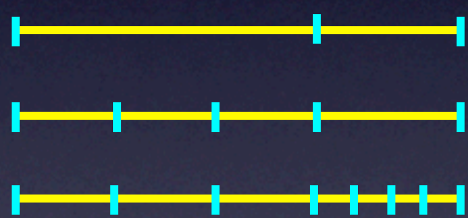
Various Renormalizations

Feigenbaum	Circle map







proper subintervals

-> Cantor set

partition of interval

Feigenbaum, Coullet-Tresser,
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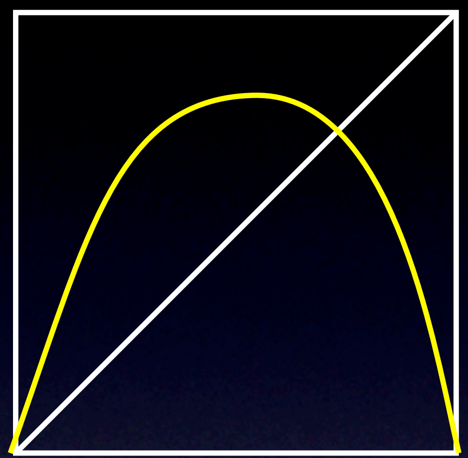
Rand,
Khanin-Sinai,
de Faria, Yampolsky,
A. Epstein-Yamplosky

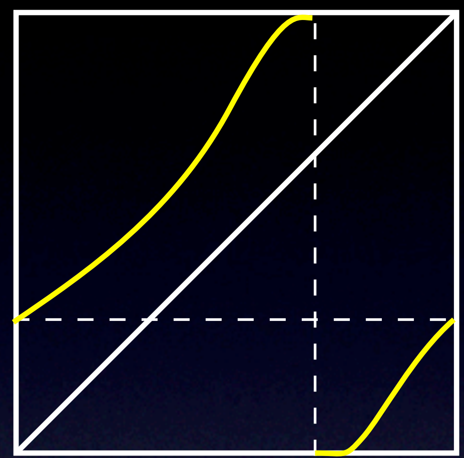
Various Renormalizations

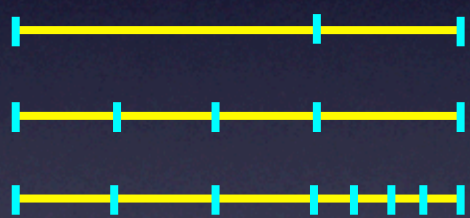
Feigenbaum

Circle map

Sector/Near-parabolic







proper subintervals

-> Cantor set

partition of interval

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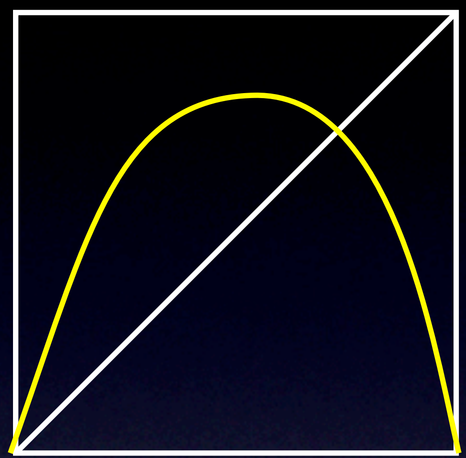
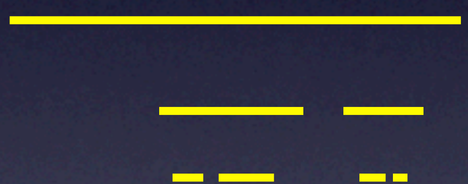
Various Renormalizations

Feigenbaum





proper subintervals

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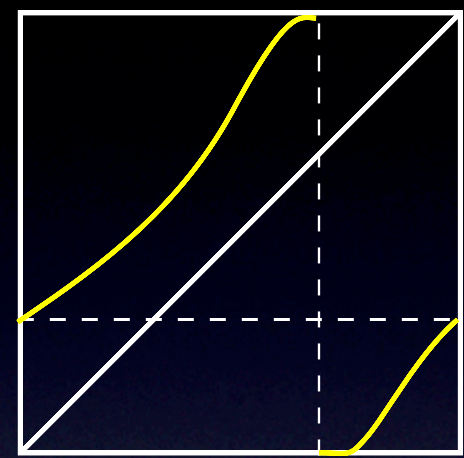
Circle map



partition of interval

Rand,
Khanin-Sinai,
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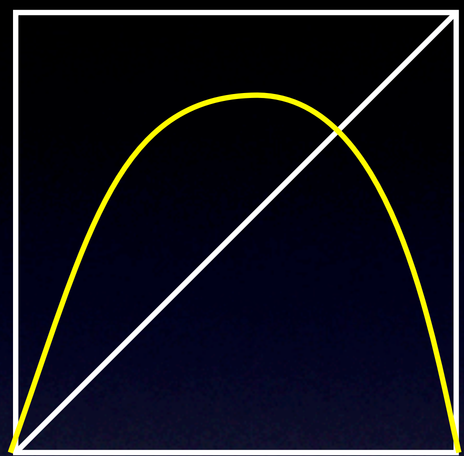
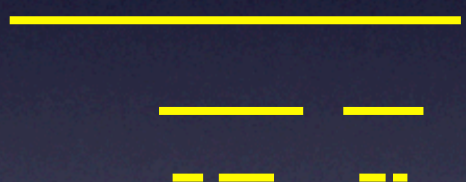
A. Epstein-Yamplosky

Sector/Near-parabolic

Various Renormalizations

Feigenbaum





proper subintervals

-> Cantor set

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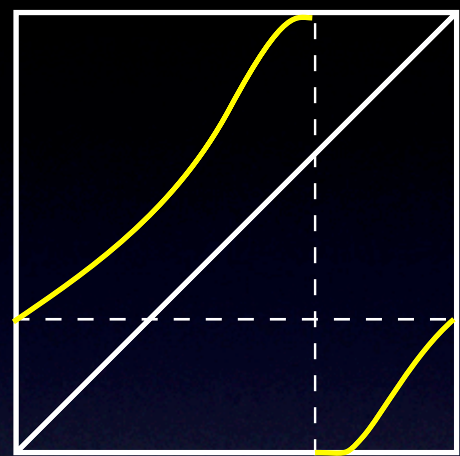
Circle map



partition of interval

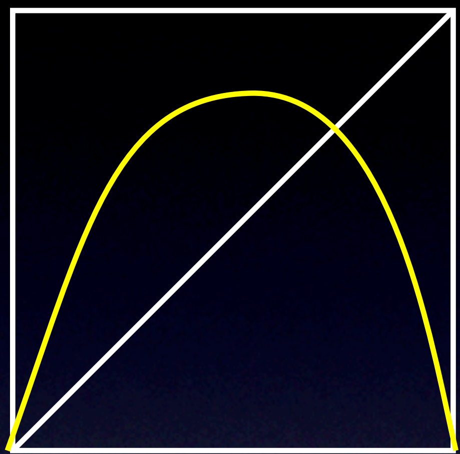
Rand,
Khanin-Sinai,
de Faria, Yampolsky,
A. Epstein-Yamplosky

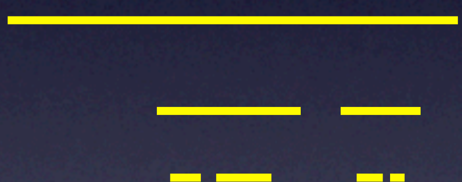
Sector/Near-parabolic

covering by sector or
croissant-like domains

Various Renormalizations

Feigenbaum





proper subintervals

-> Cantor set

Feigenbaum, Coullet-Tresser,
Lanford, H. Epstein,
Polynomial-like maps:
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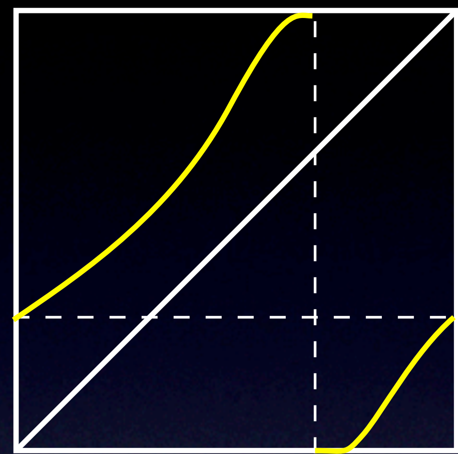
Circle map



partition of interval

Rand,
Khanin-Sinai,
de Faria, Yampolsky,
A. Epstein-Yamplosky

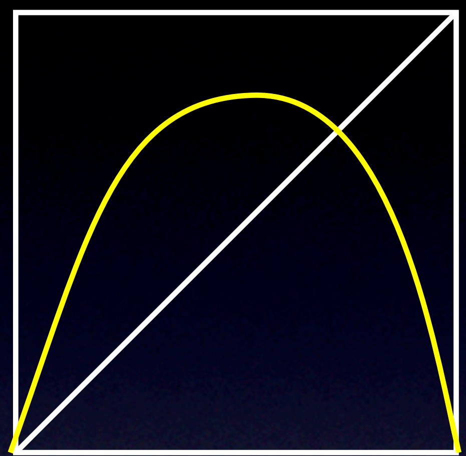
Sector/Near-parabolic

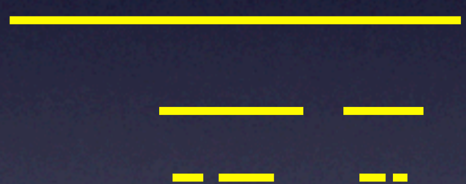
covering by sector or croissant-like domains

gluing/identification needed to define the renormalization

Various Renormalizations

Feigenbaum





proper subintervals

-> Cantor set

Feigenbaum, Coullet-Tresser,
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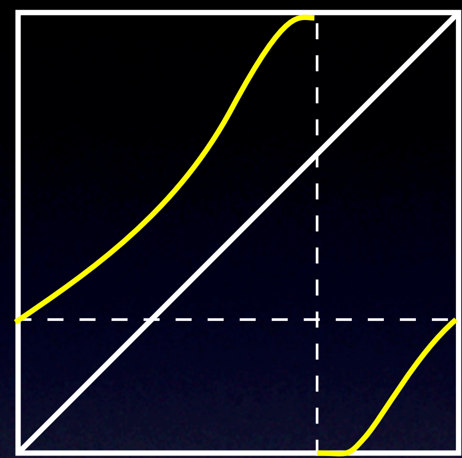
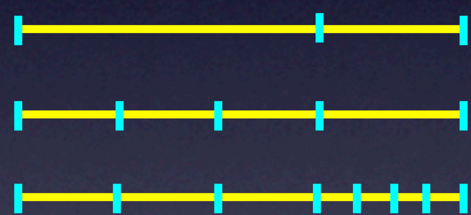
Circle map





partition of interval

Rand,
Khanin-Sinai,
de Faria, Yampolsky,
A. Epstein-Yamplosky

Sector/Near-parabolic

covering by sector or croissant-like domains

gluing/identification needed to define the renormalization

Yoccoz, Perez-Marco,
Inou-S.

Irrationally indifferent fixed points

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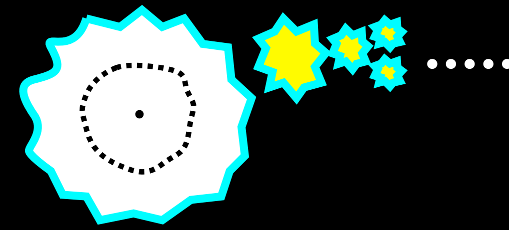

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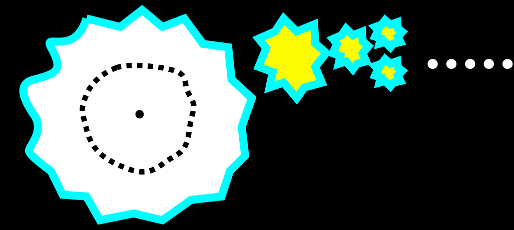

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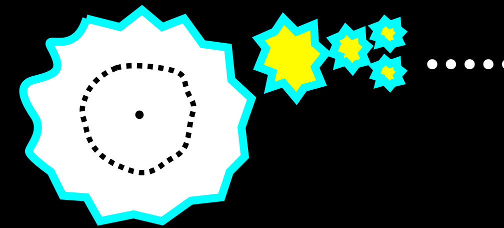

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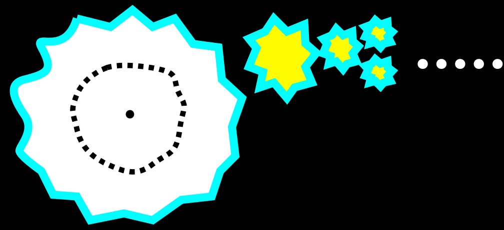

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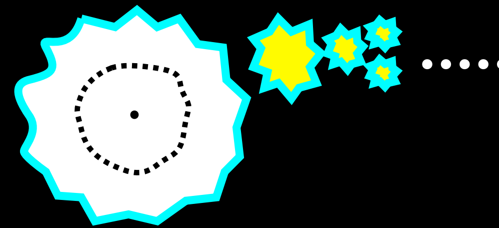

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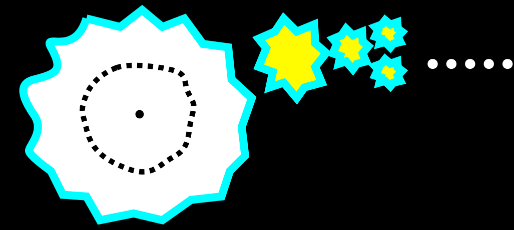

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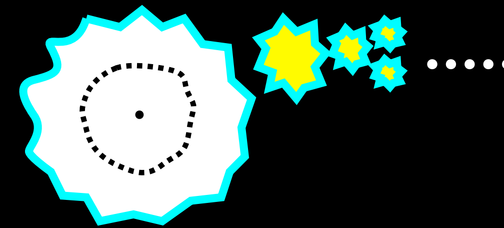

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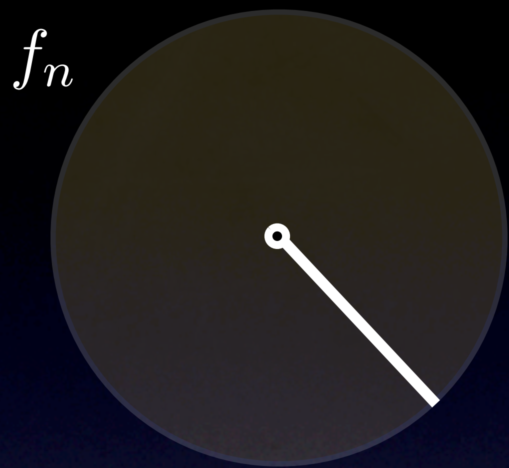

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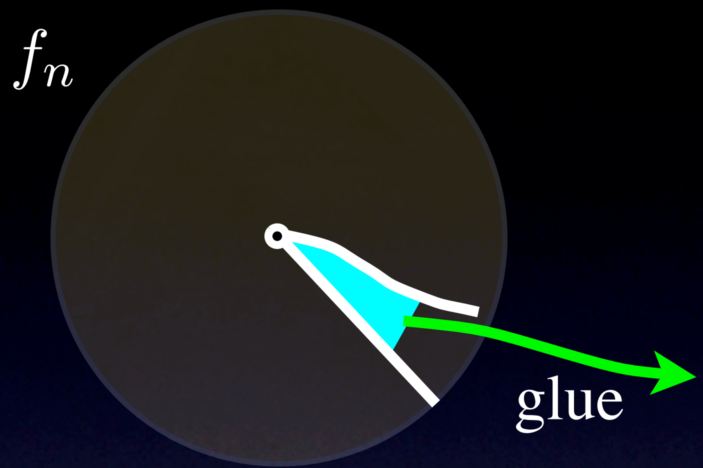

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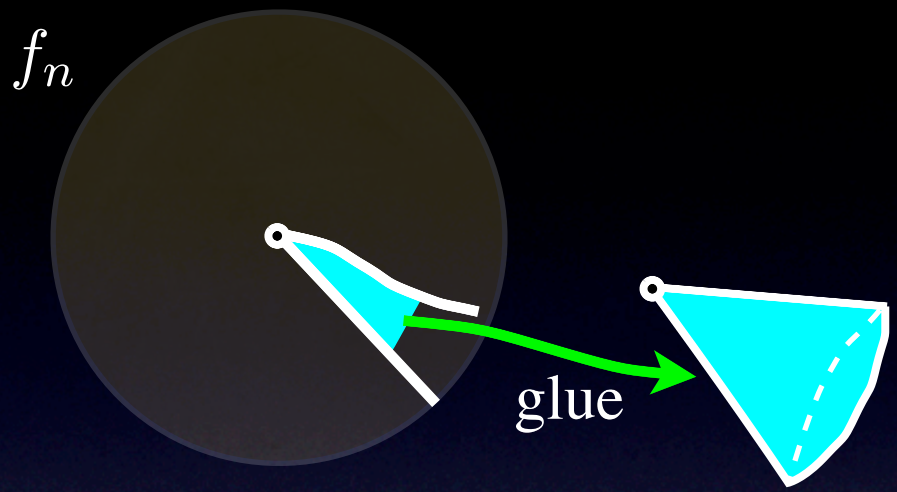

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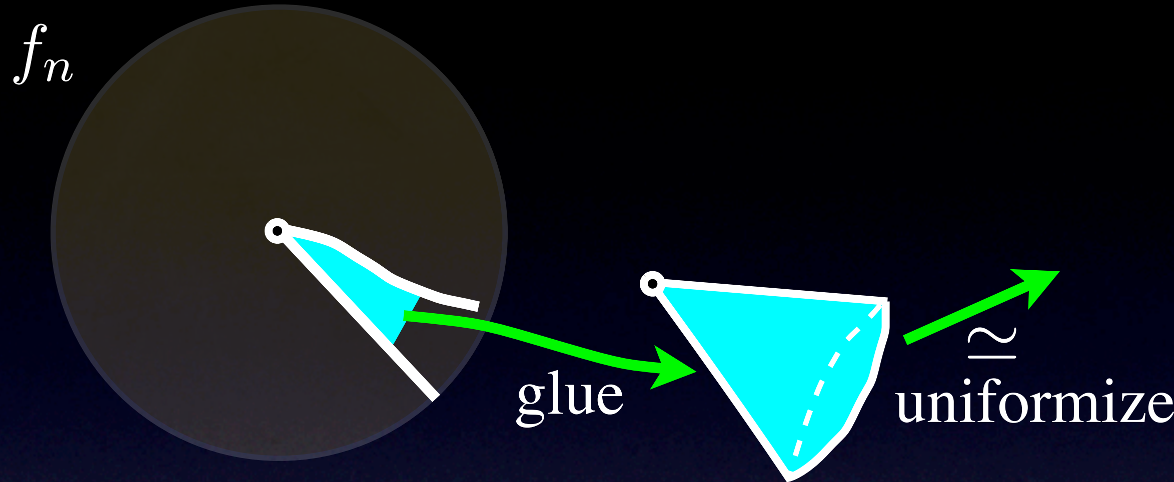

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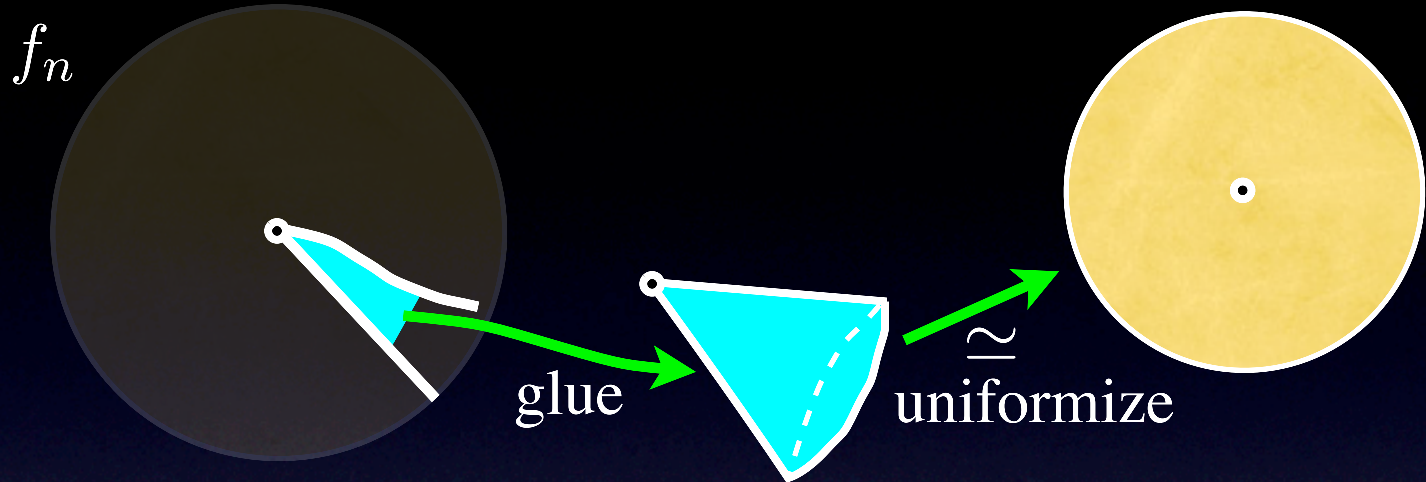

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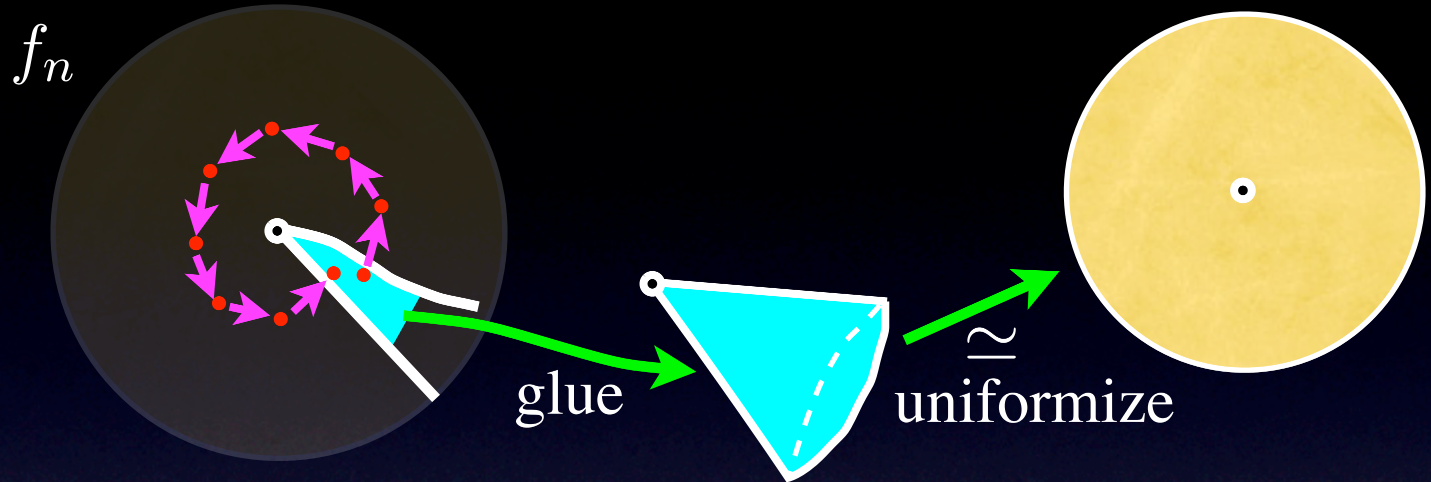

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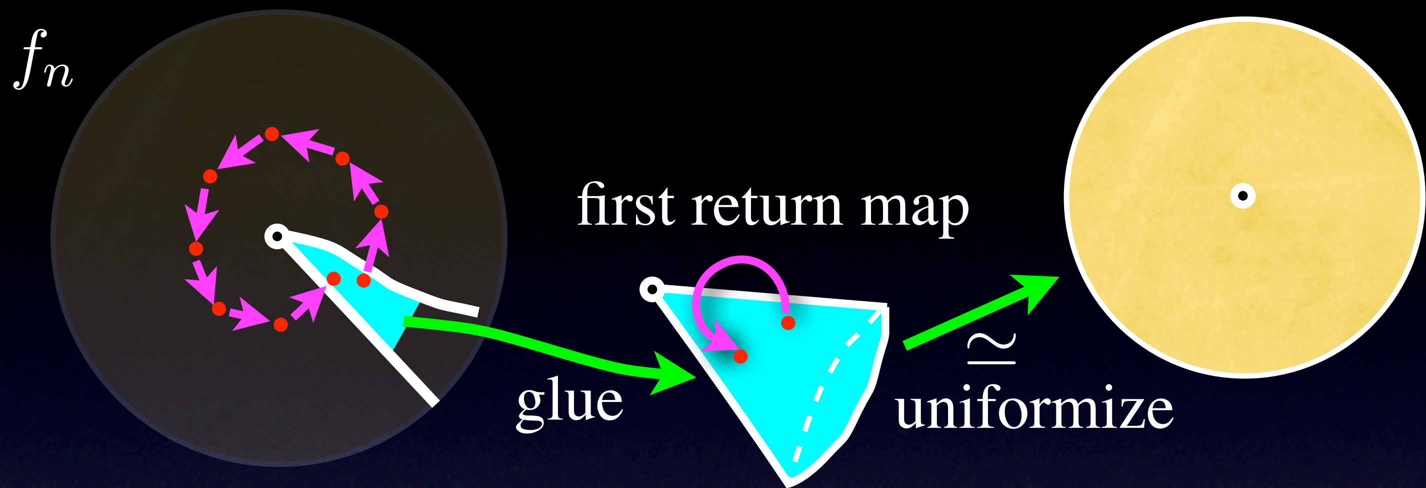

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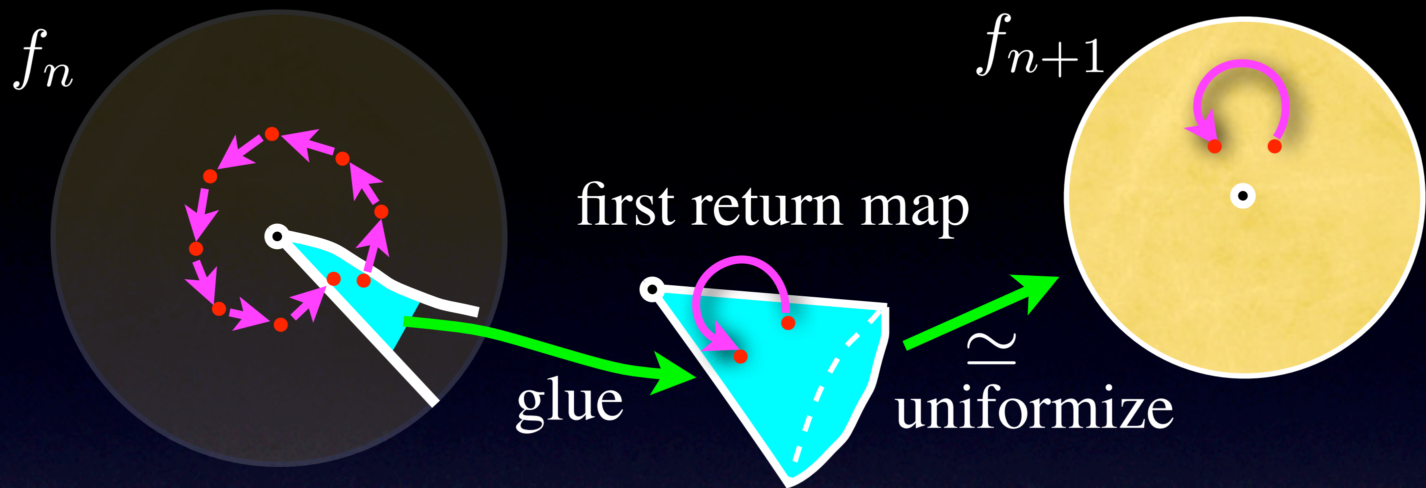

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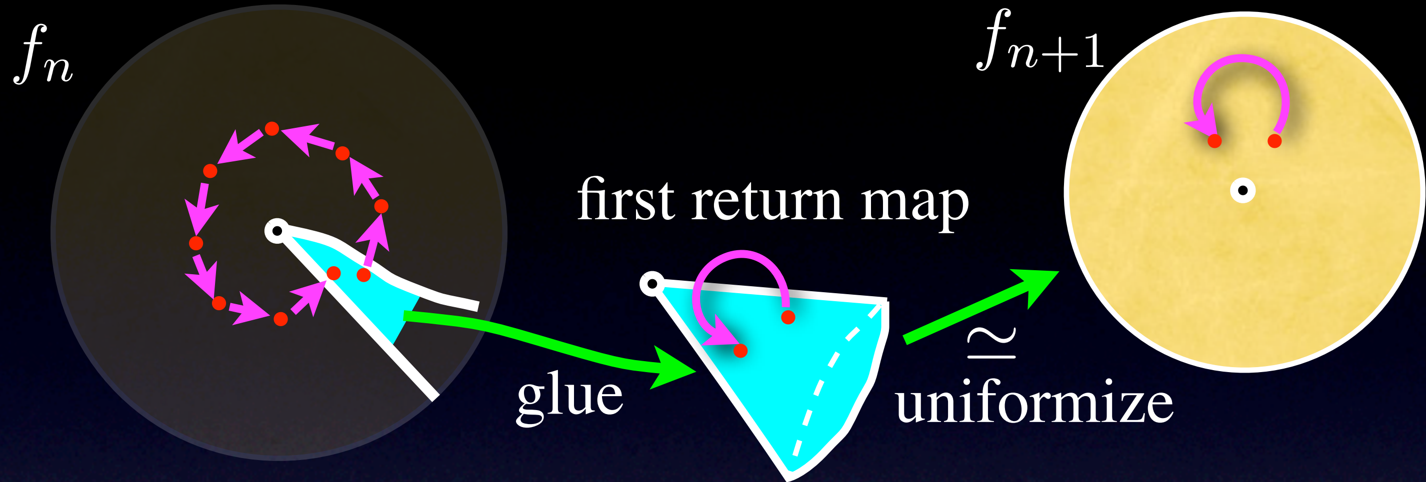

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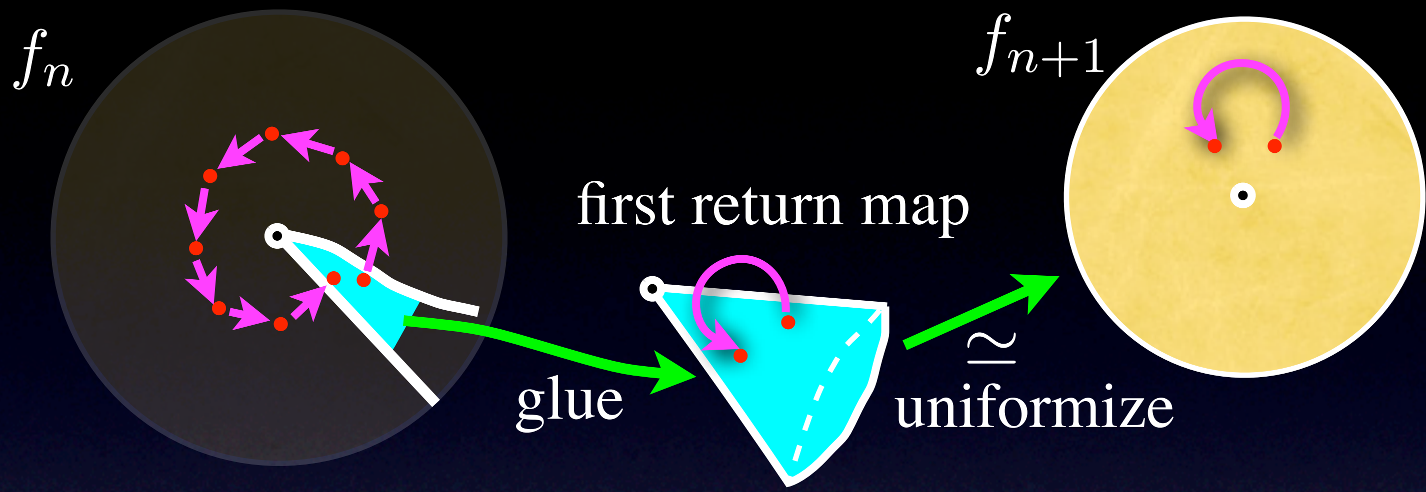

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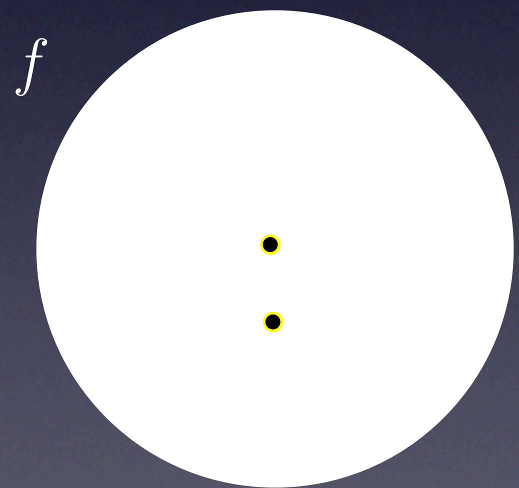

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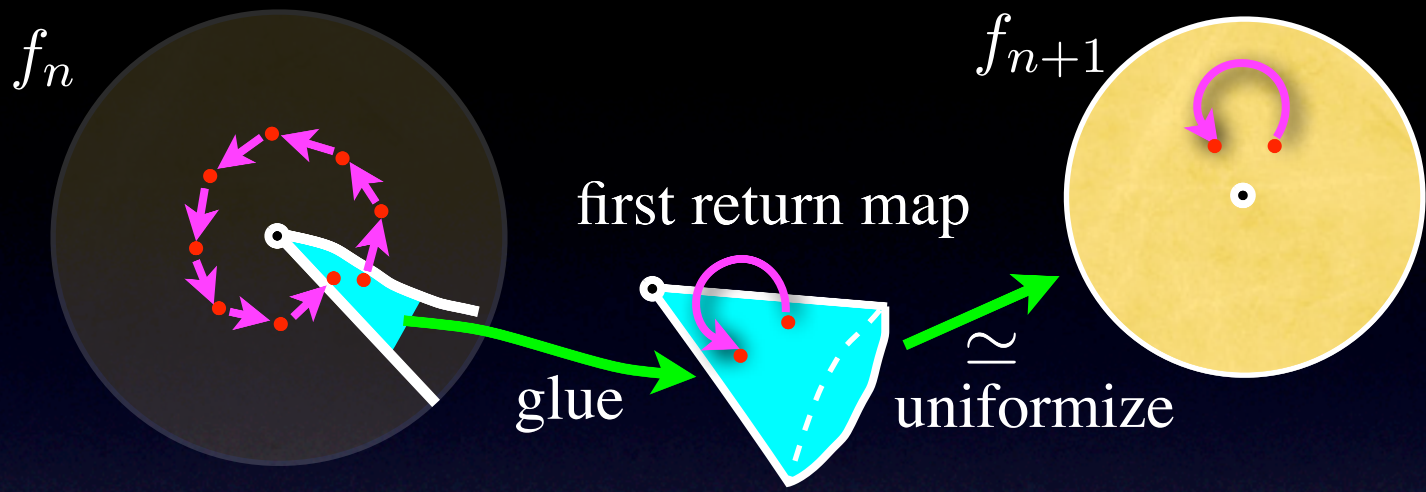

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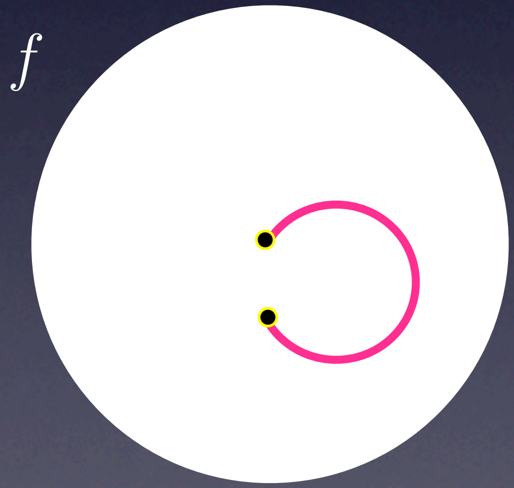

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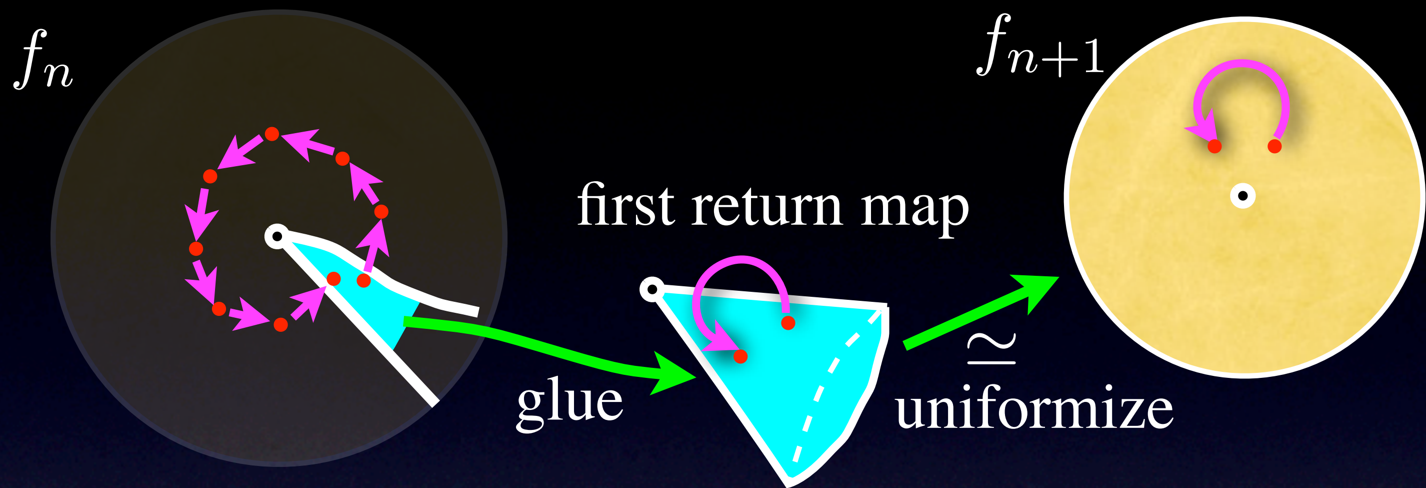

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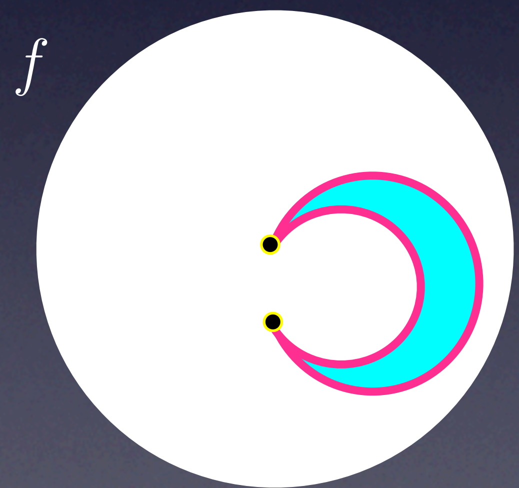

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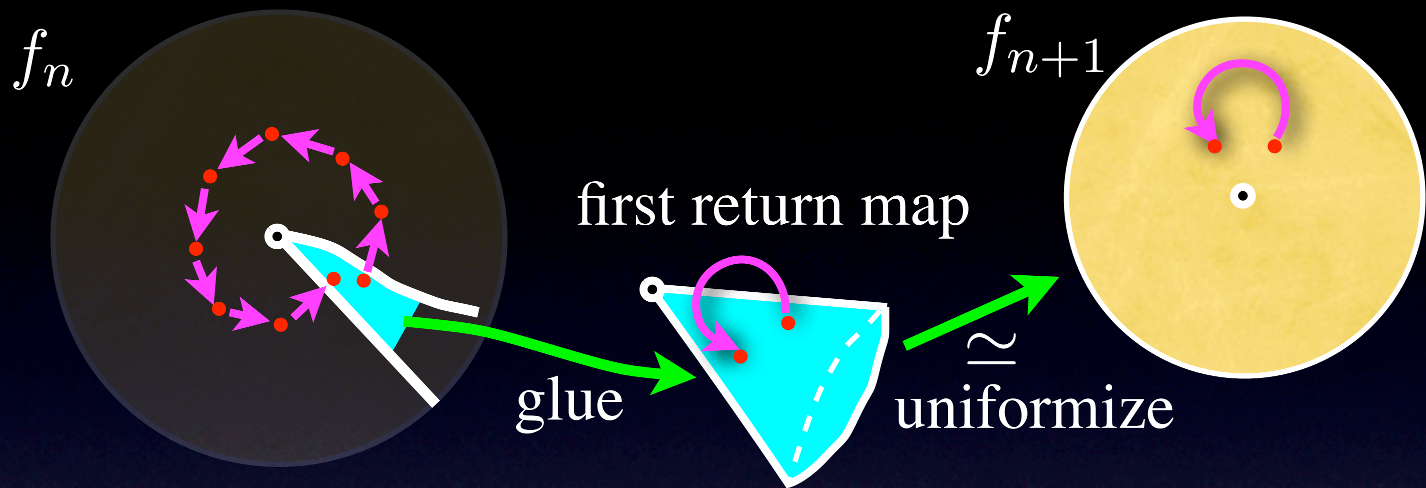

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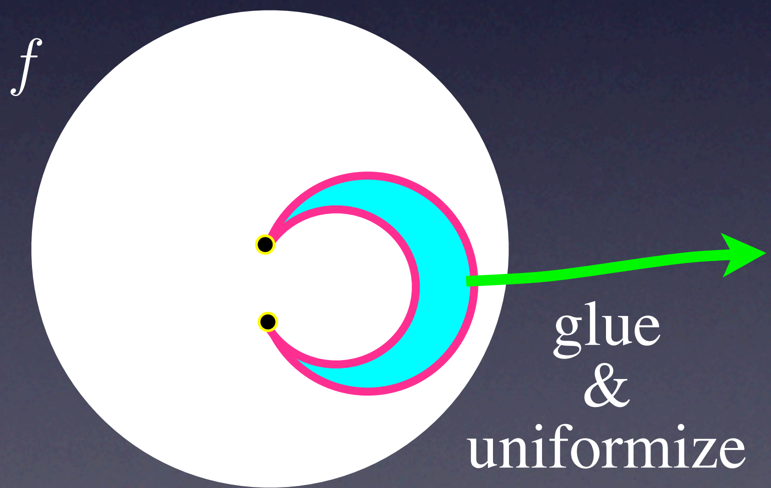

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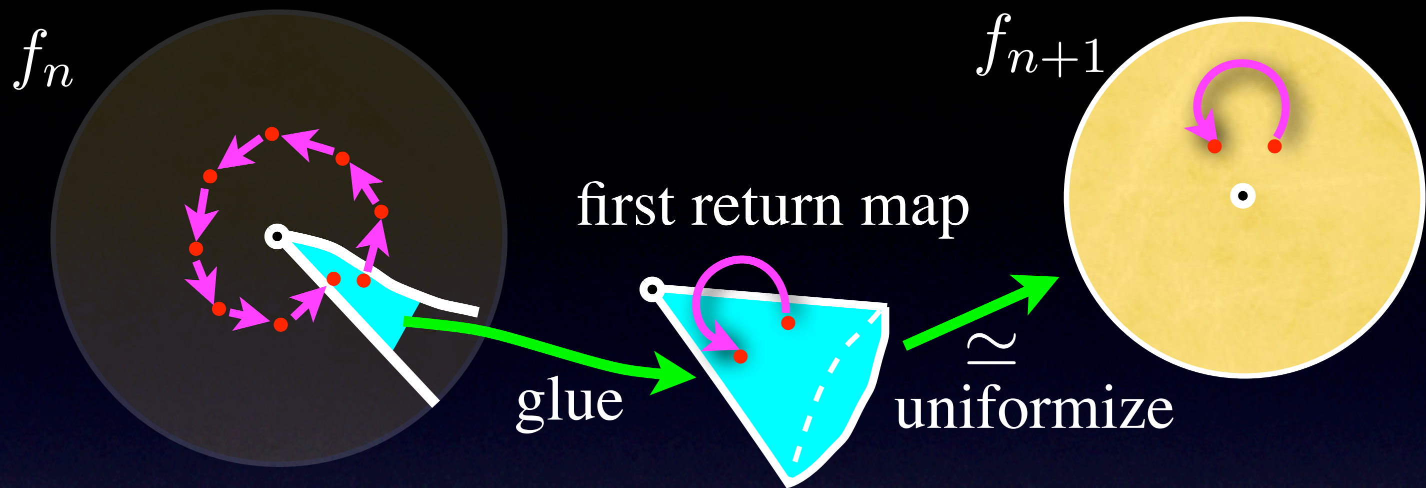

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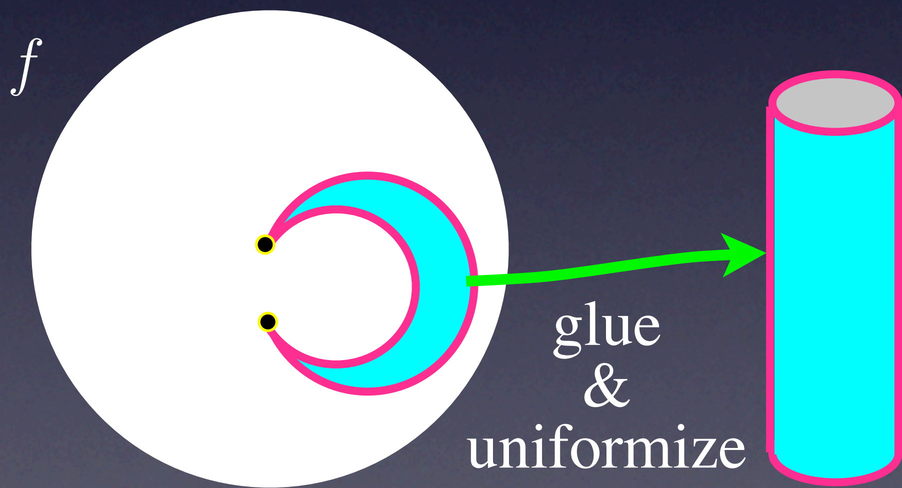

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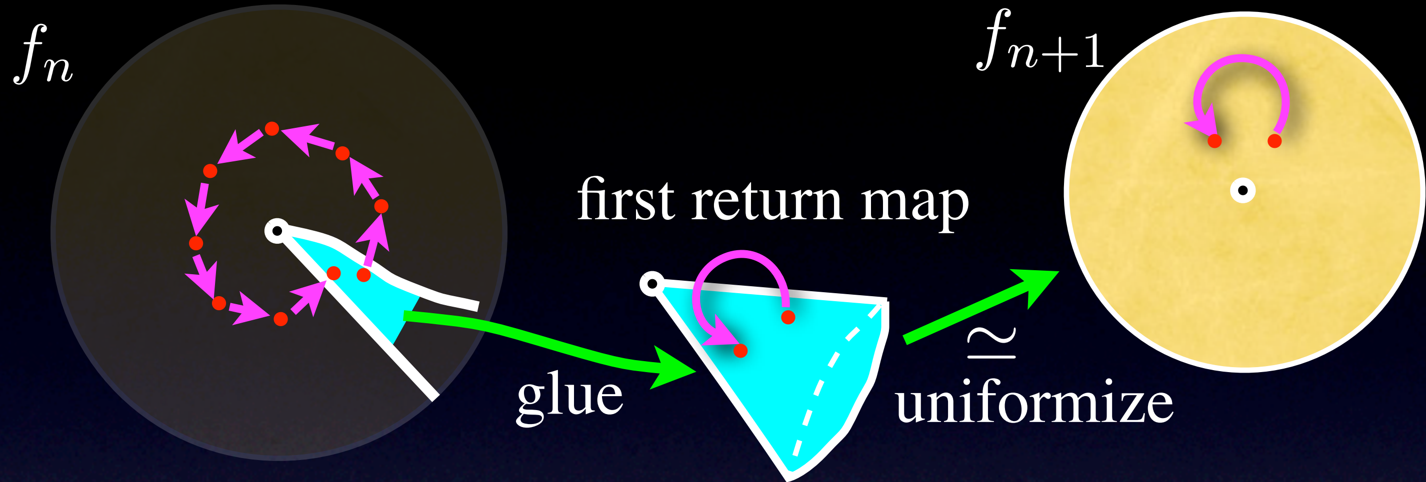

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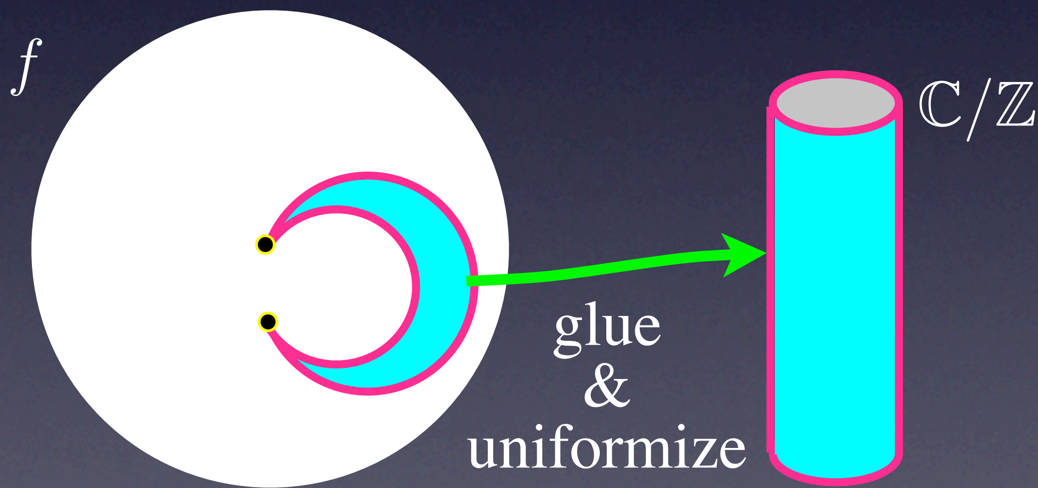

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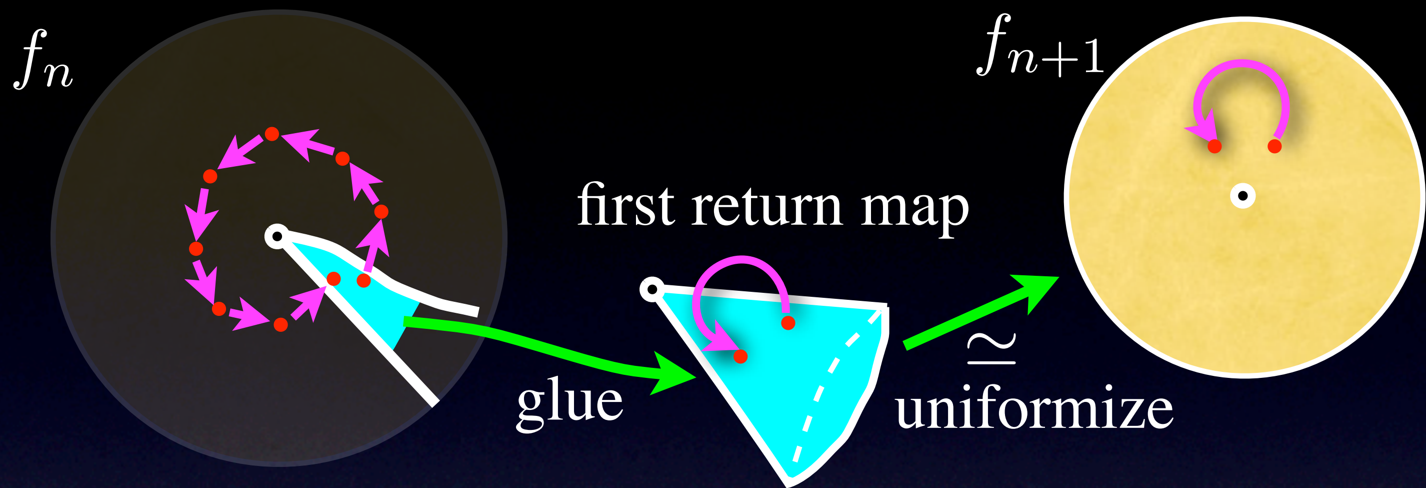

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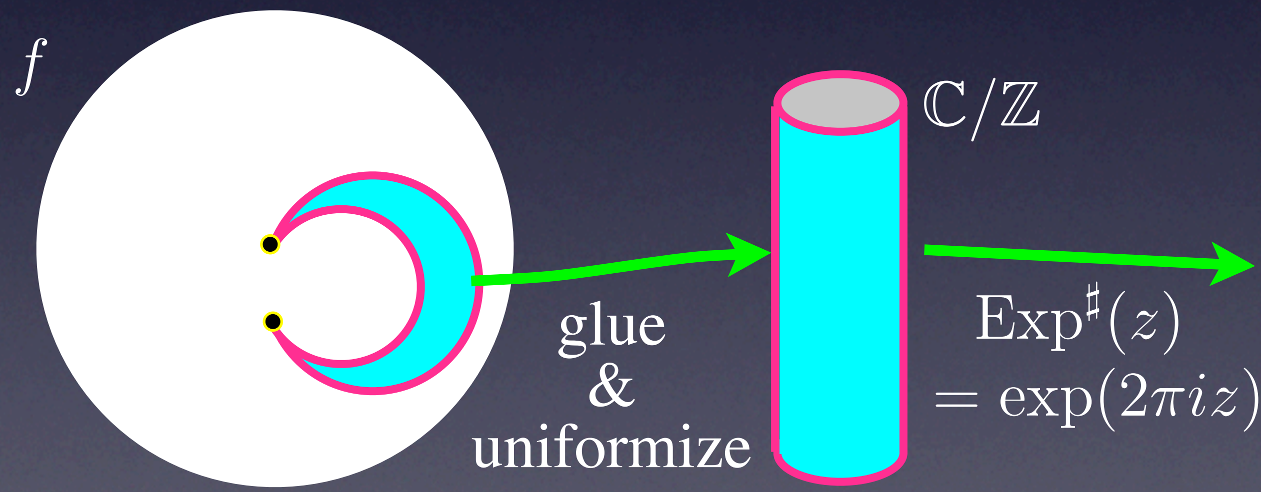

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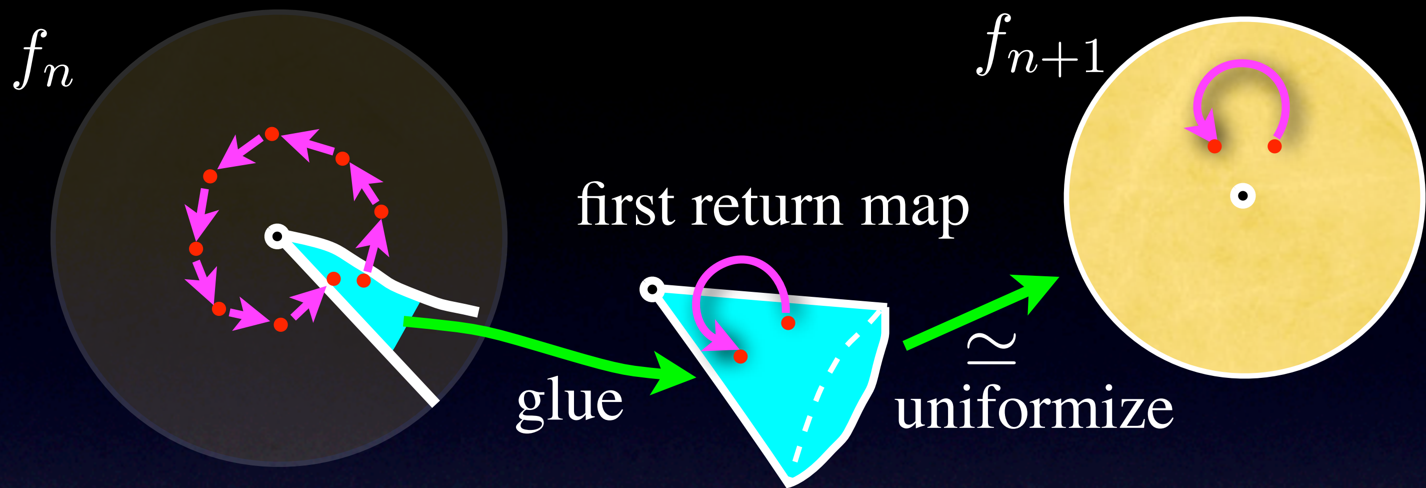

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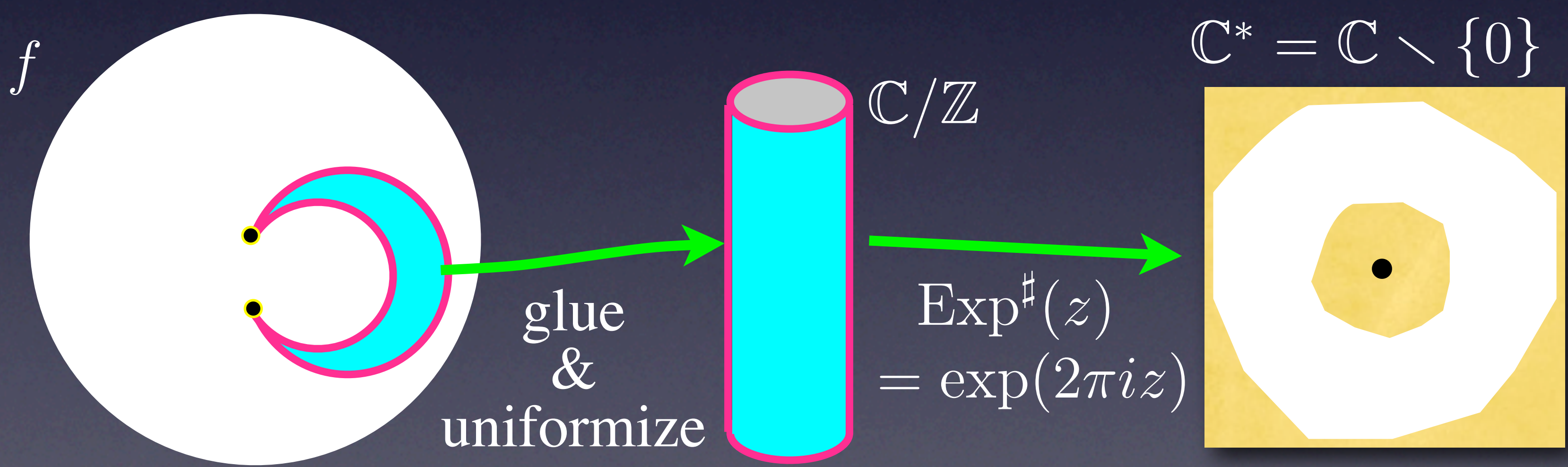

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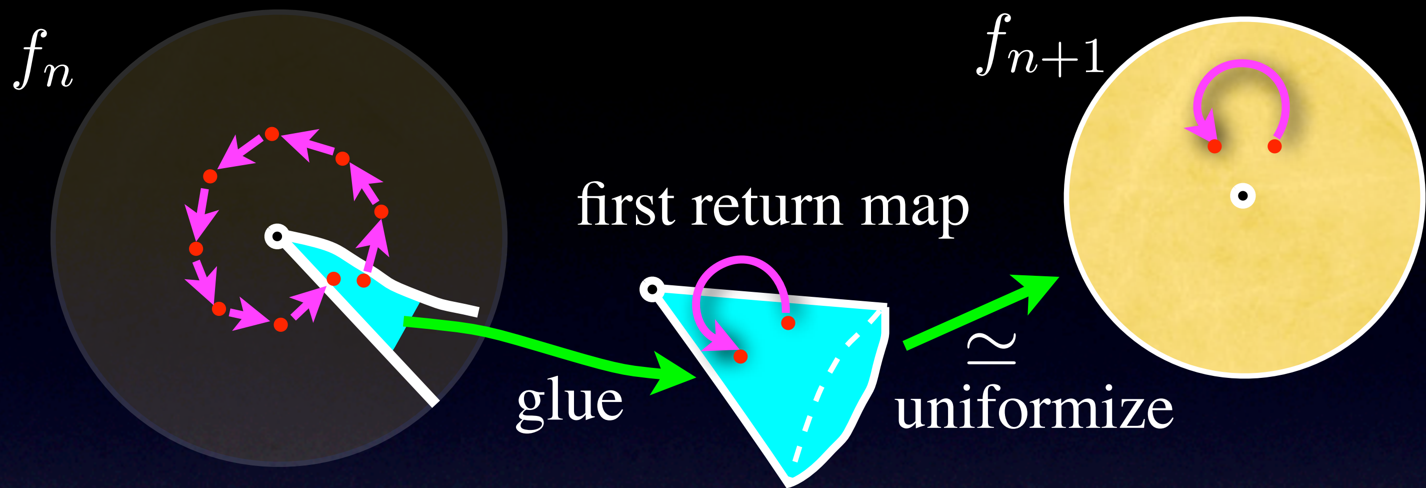

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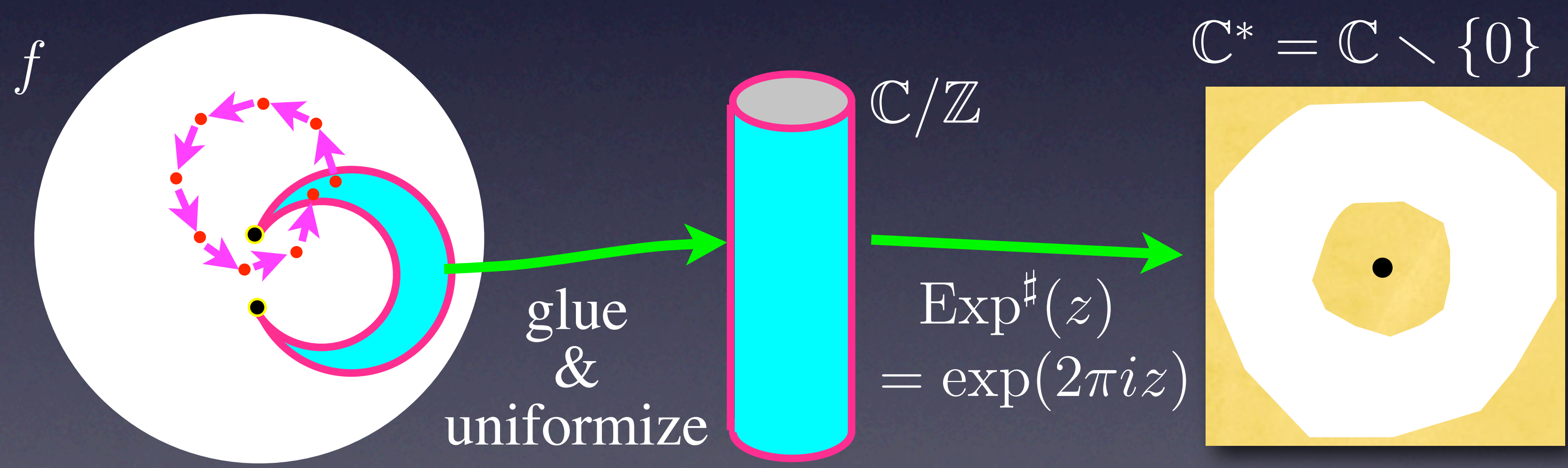

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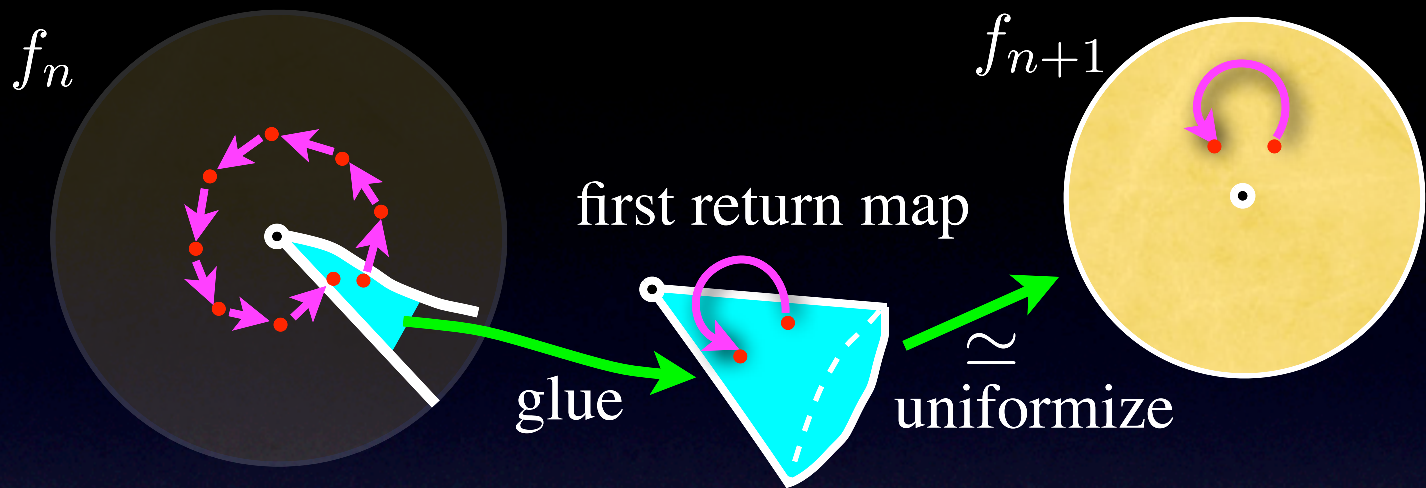

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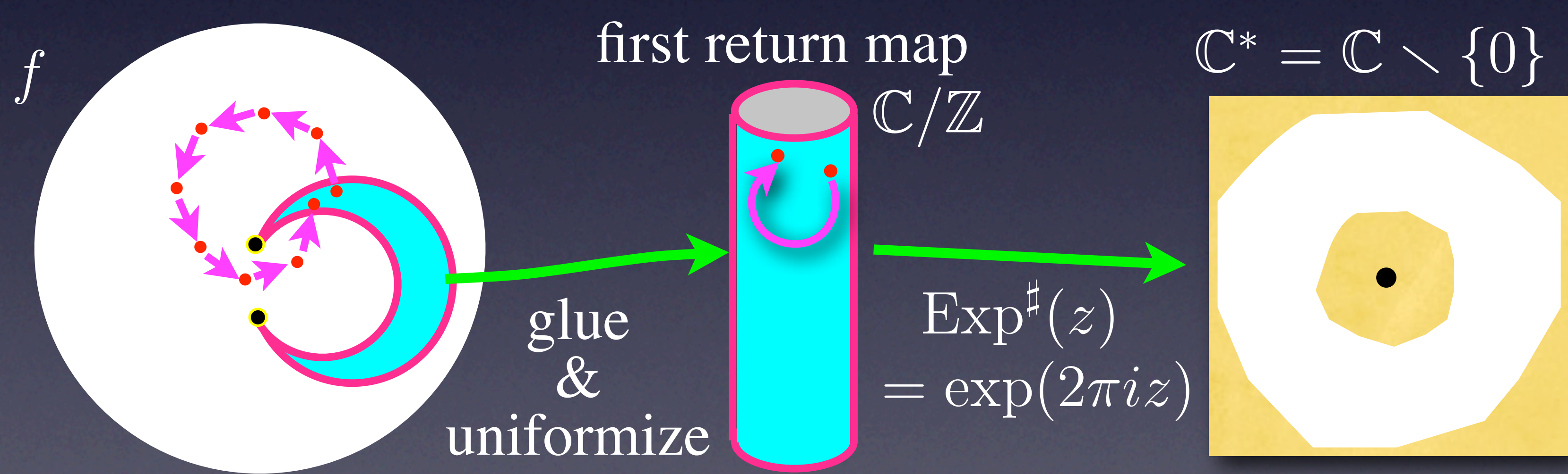

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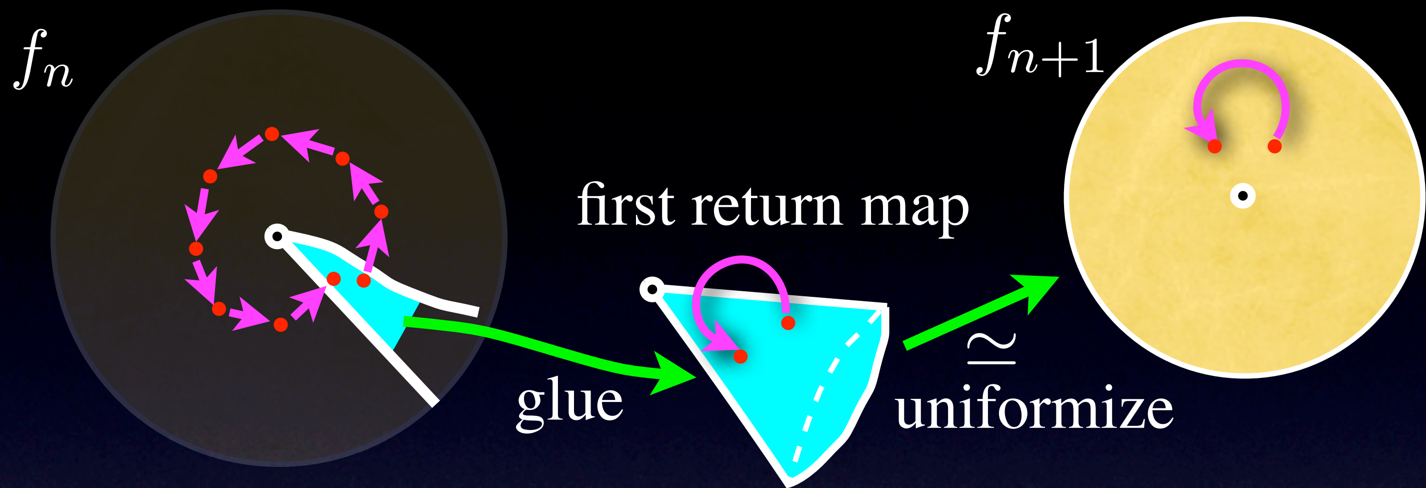

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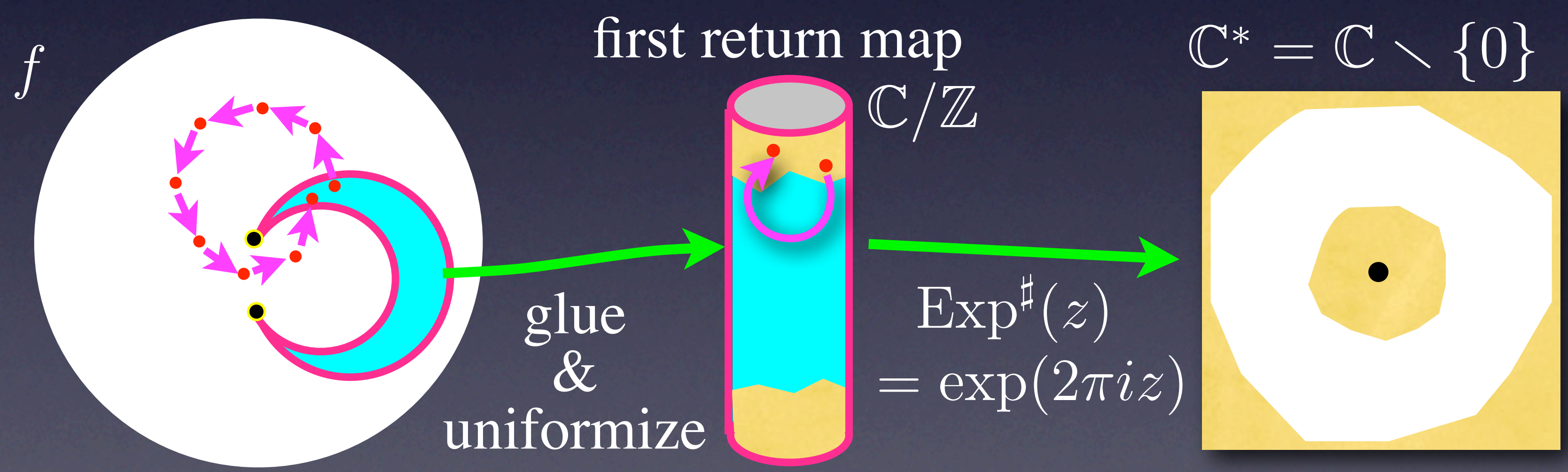

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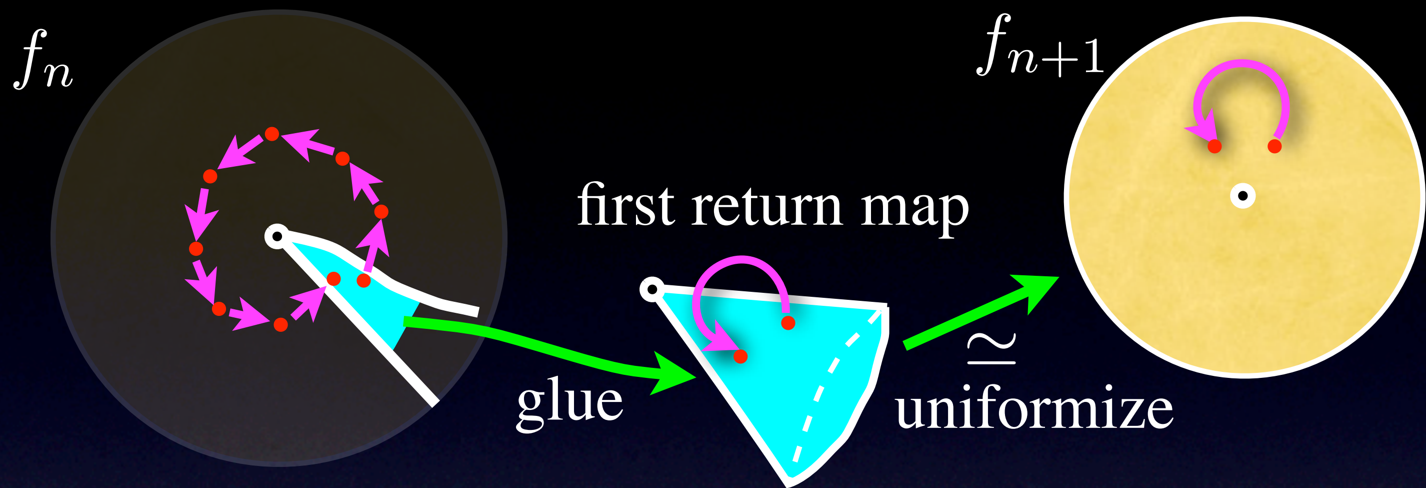
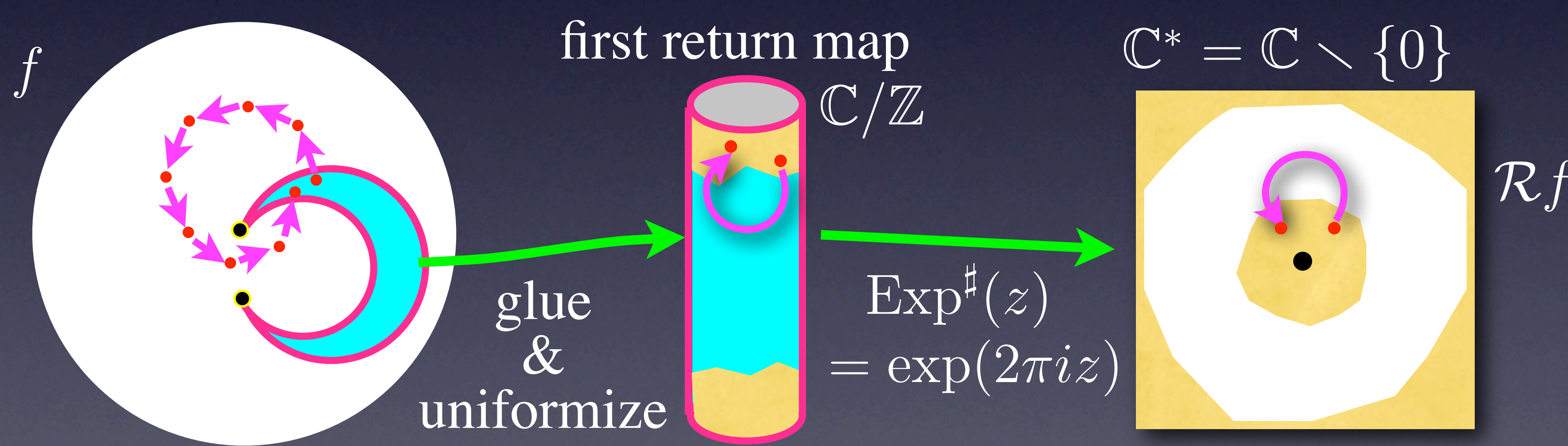
$$f_n(z) = e^{2\pi i \alpha_n} z + \dots \rightsquigarrow f_{n+1}(z) = e^{2\pi i \alpha_{n+1}} z + \dots$$



Cylinder/Near-parabolic renormalization



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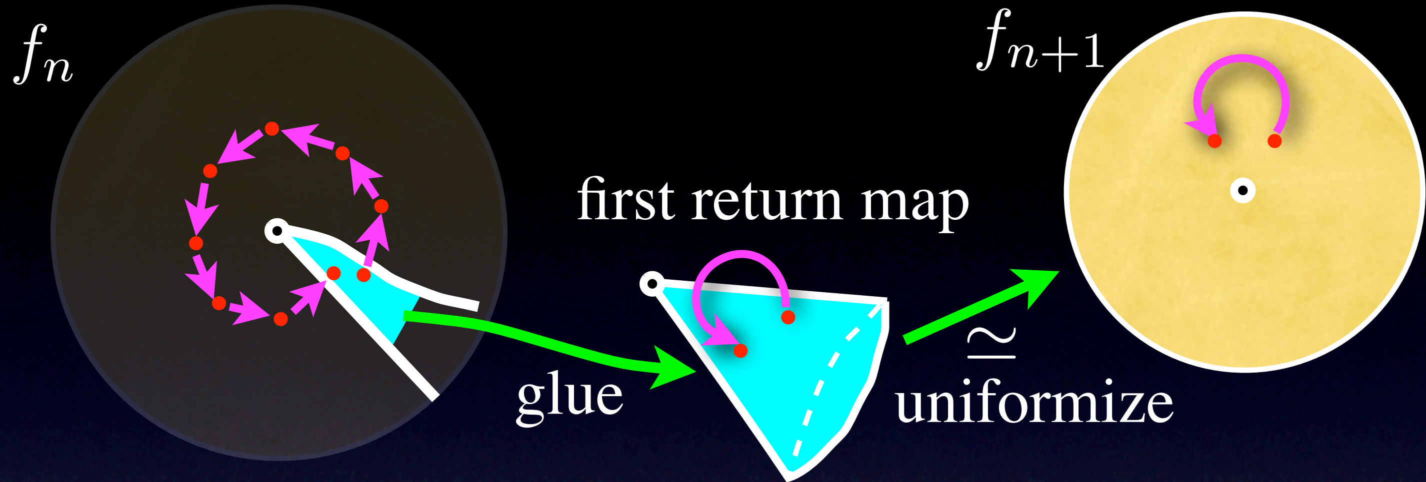

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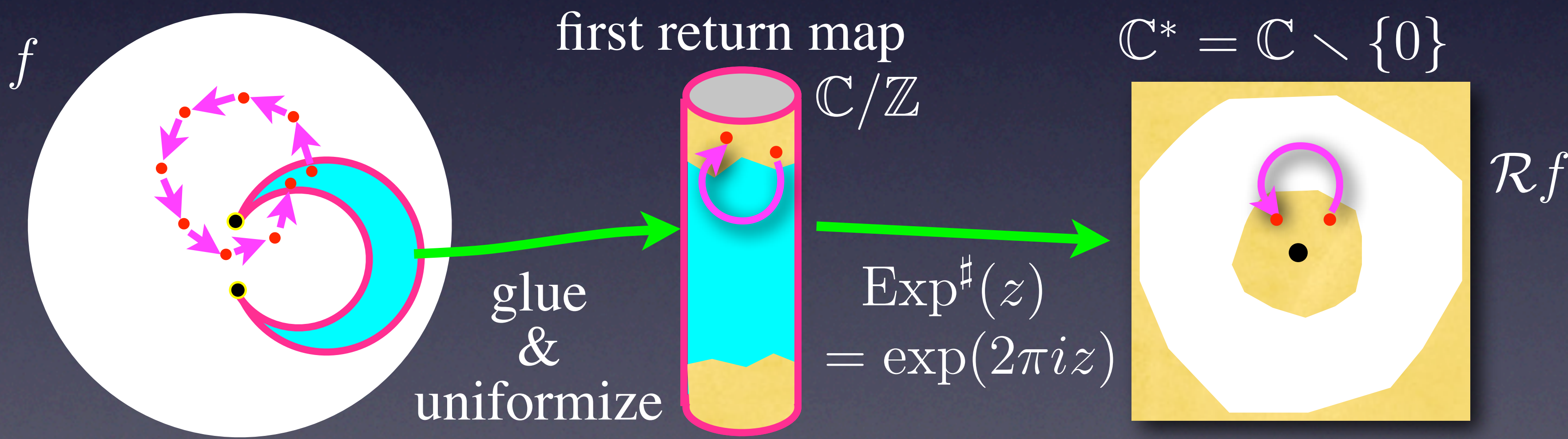

$\mathcal{R}f$ can be defined when $f(z) = e^{2\pi i \alpha} z + \dots$ is a small perturbation of $z + a_2 z^2 + \dots$ $(a_2 \neq 0)$ and $|\arg \alpha| < \pi/4$.

For $N \in \mathbb{N}$, let $Irrat_N$ be the set of irrational number of high type:

$$Irrat_N \ni \alpha = \pm\cfrac{1}{a_1 \pm \cfrac{1}{a_2 \pm \cfrac{1}{\ddots}}} \quad \text{where } a_i \in \mathbb{N} \text{ and } a_i \geq N,$$

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For a neighborhood V of 0, define $P(z) = z(1+z)^2$ and

$$\mathcal{F}_1 = \left\{ f = P \circ \varphi^{-1} \,\middle|\, \begin{array}{c} \varphi : V \to \mathbb{C} \text{ is univalent (with qc extension)} \\ \varphi(0) = 0,\ \varphi'(0) = 1 \end{array} \right\}$$

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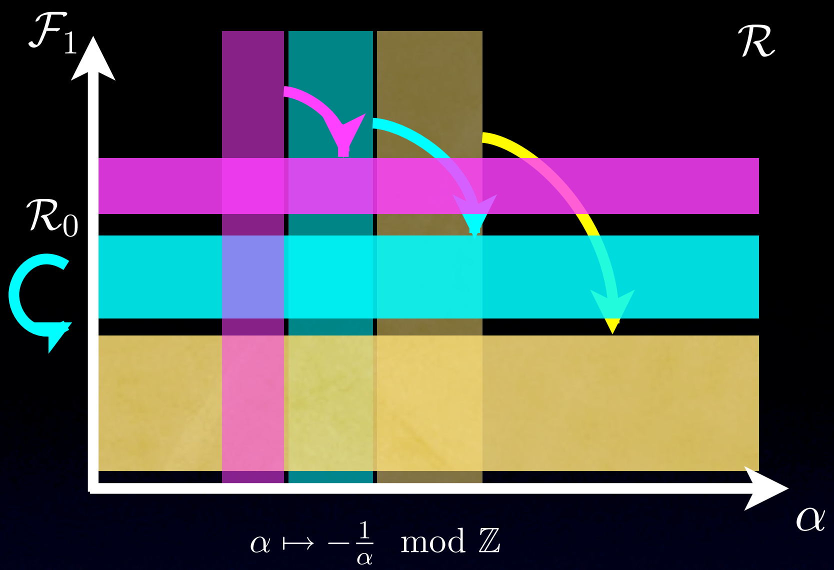
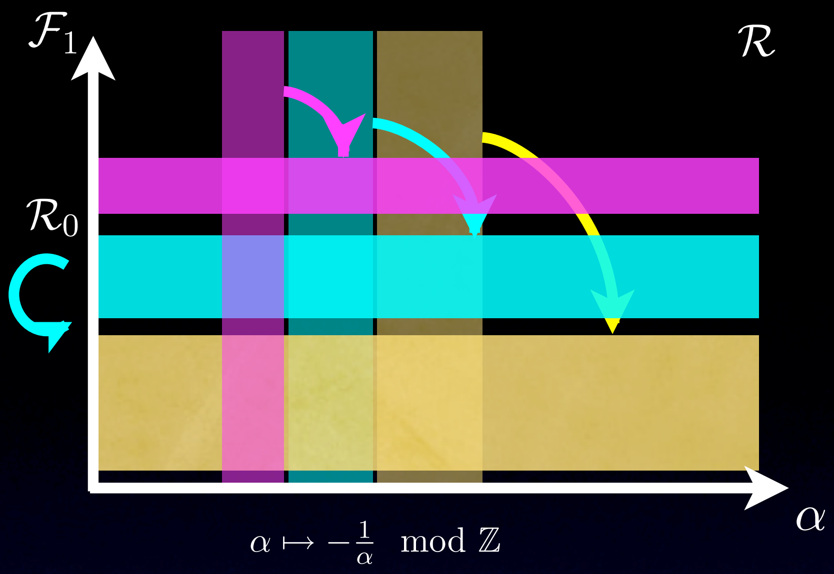
Theorem (Inou & S.): For some V and N, the near-parabolic renormalization $\mathcal{R}$ from

$$\{e^{2\pi i \alpha} f : \alpha \in Irrat_N, f \in \mathcal{F}_1\} = Irrat_N \times \mathcal{F}_1$$

is well defined and expanding along α direction and uniformly contracting along $\mathcal{F}_1$ direction. Moreover $\mathcal{R}(e^{2\pi i\alpha} z + z^2)$ belong to the above set for $\alpha \in Irrat_N$.



$\mathcal{F}_1$
$\mathcal{R}$
$\mathcal{R}_0$
α
$\alpha \mapsto -\frac{1}{\alpha} \mod \mathbb{Z}$



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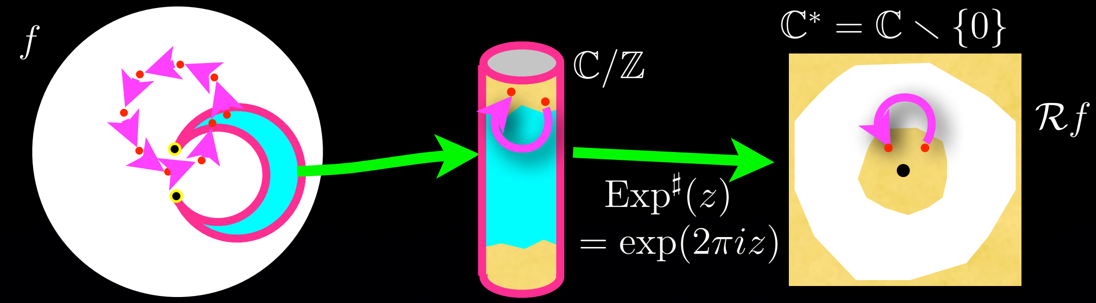
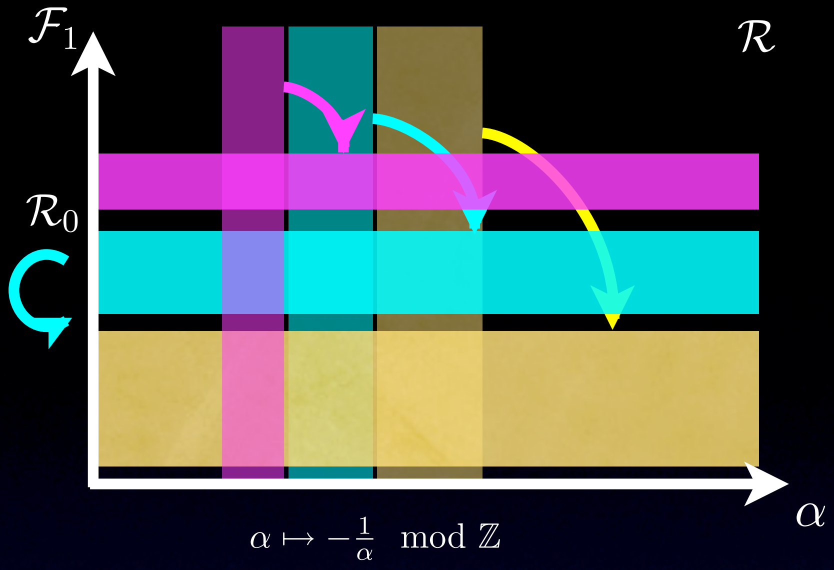
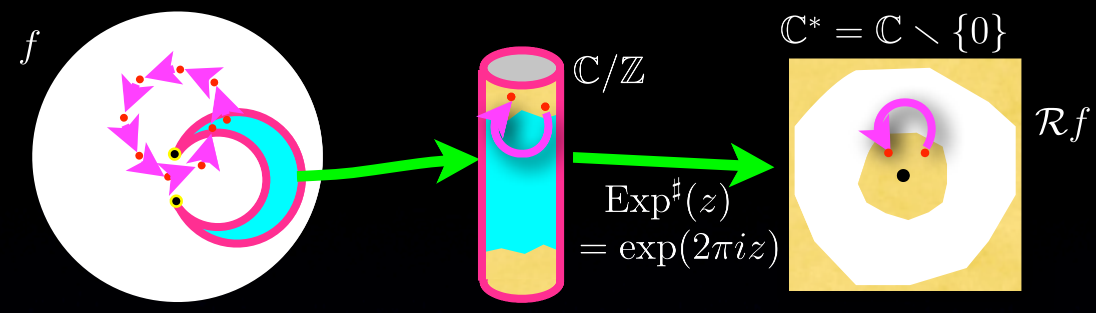


f
$\mathbb{C}/\mathbb{Z}$
$\mathbb{C}^* = \mathbb{C} \setminus \{0\}$
$\mathcal{R}f$
$\mathrm{Exp}^\sharp(z)$
$= \exp(2\pi i z)$





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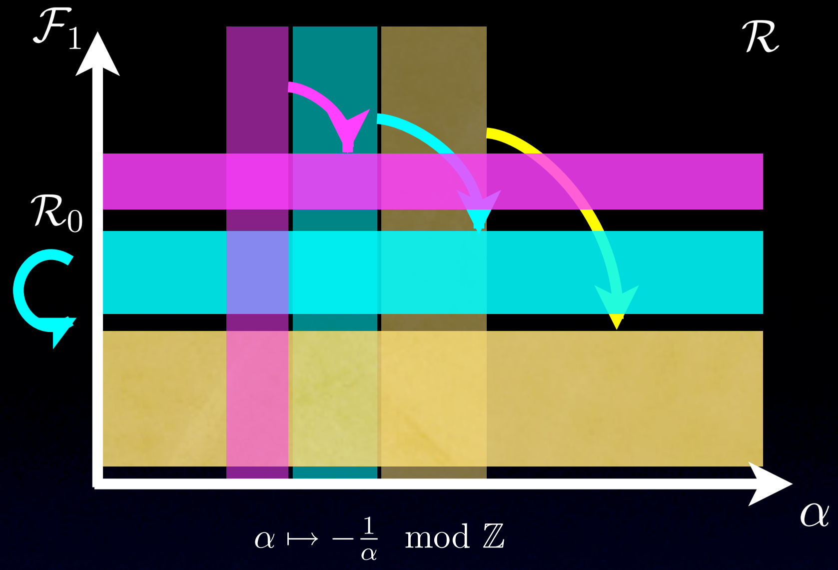

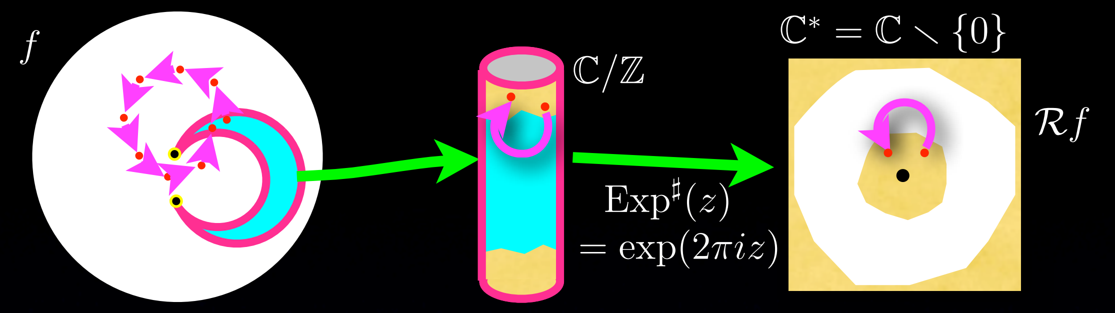

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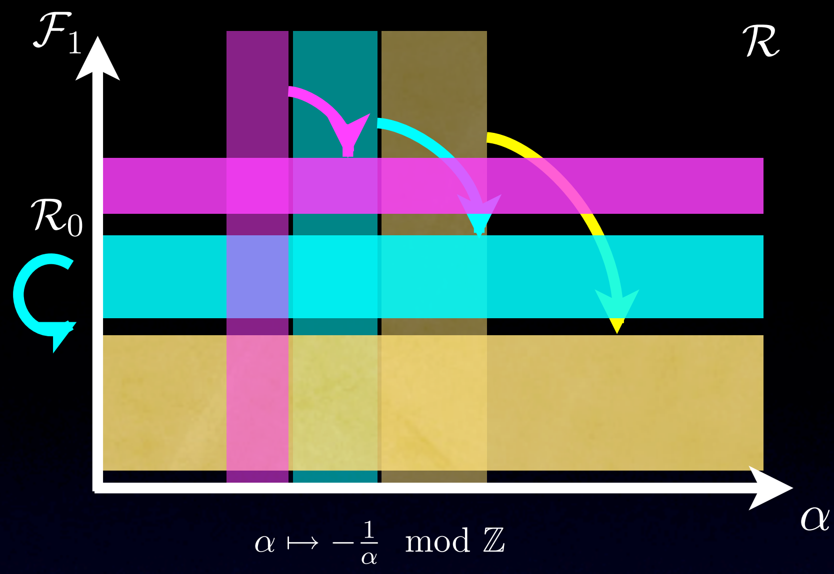

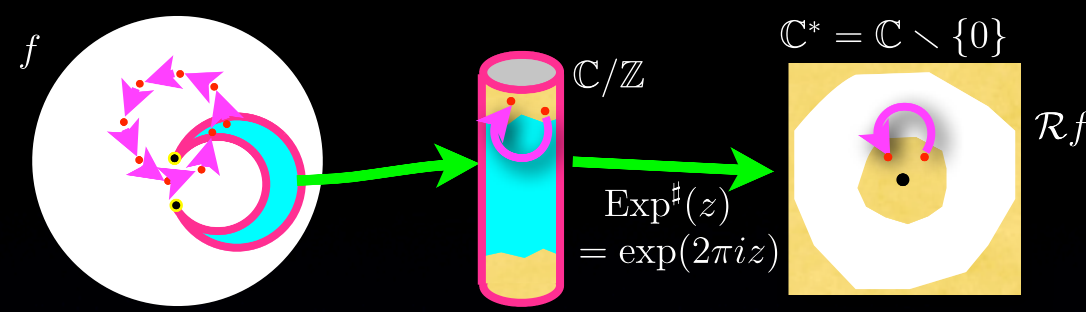

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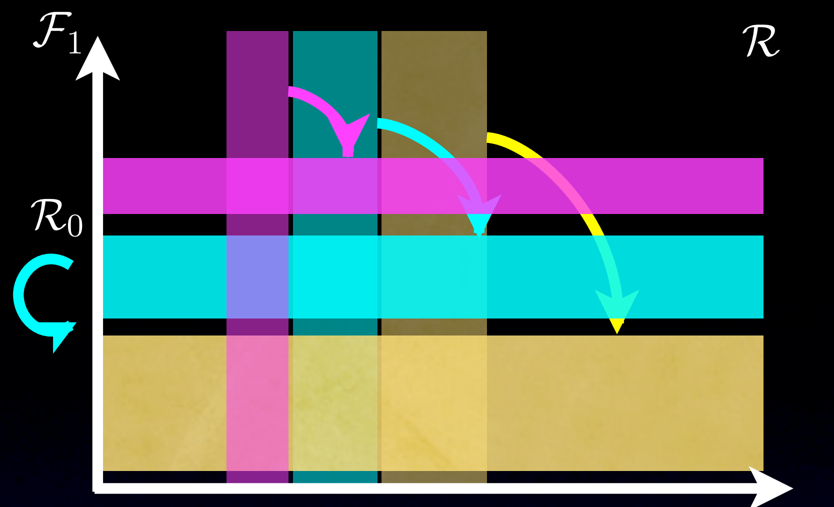
Sunday, February 27, 2011



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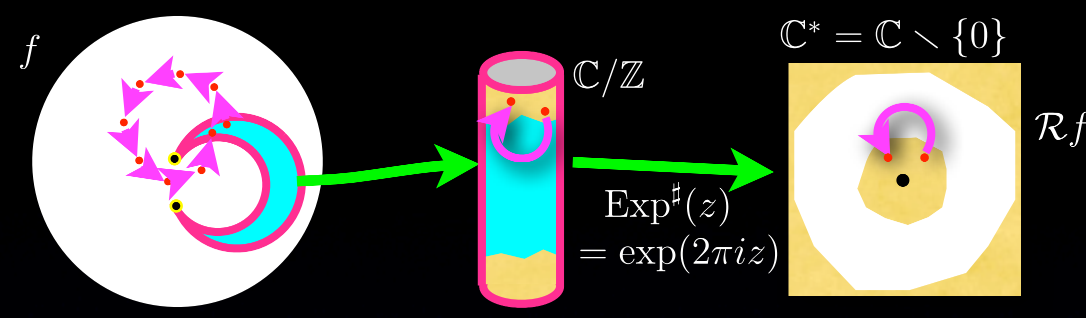

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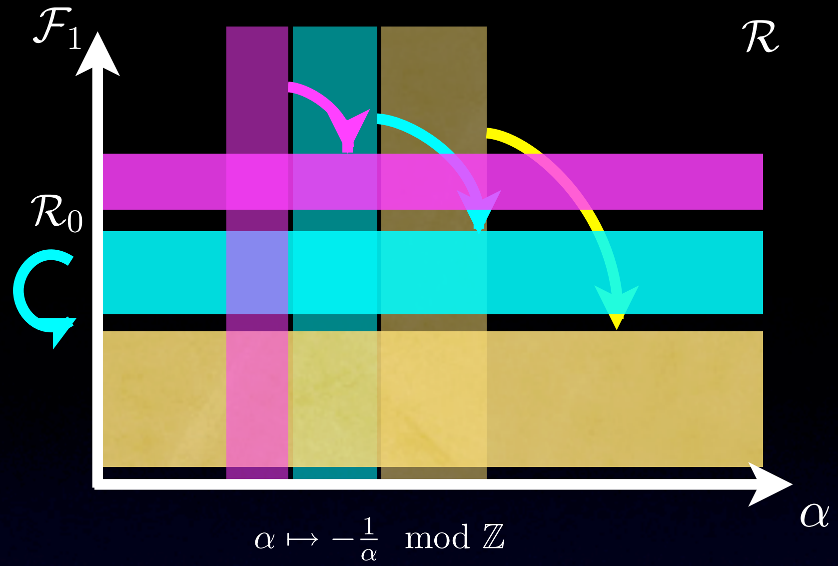

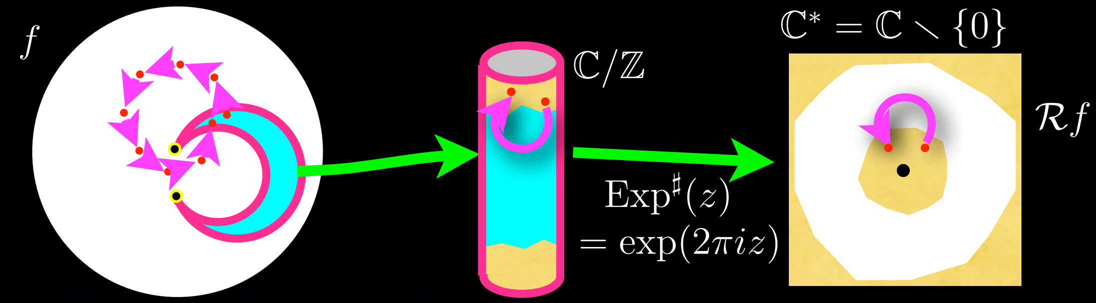

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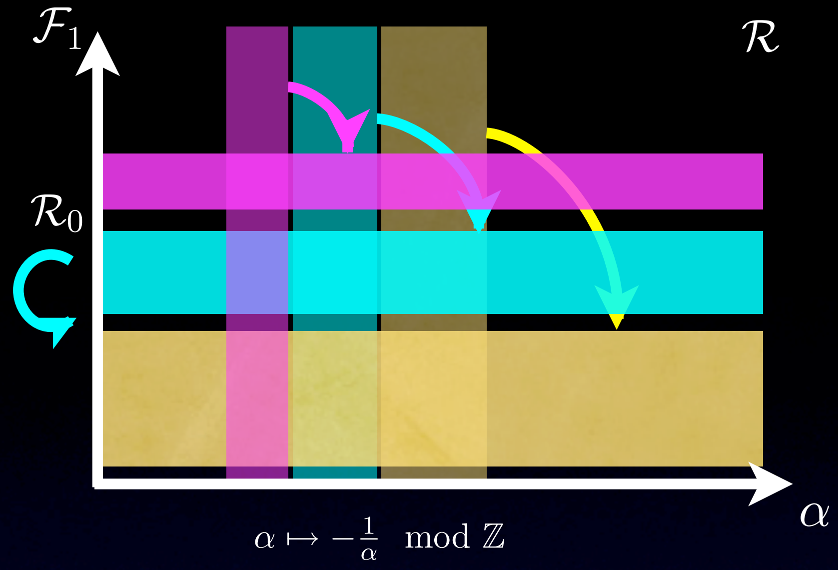

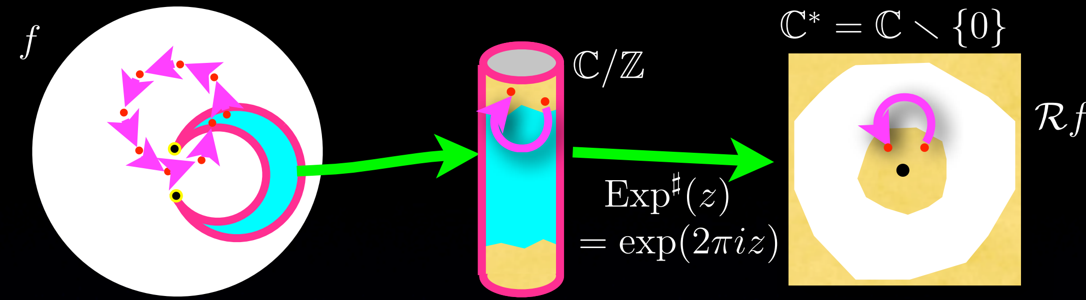

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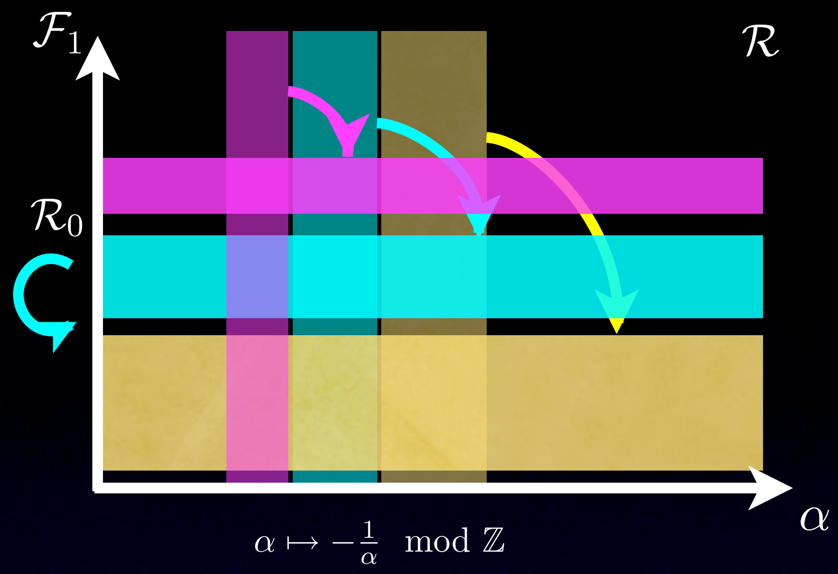

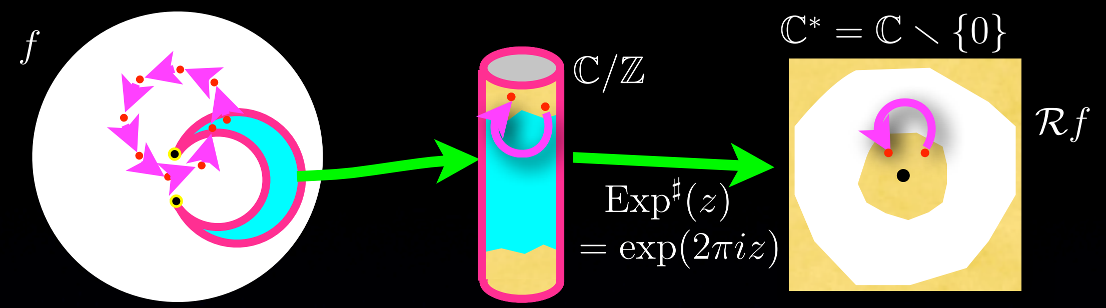

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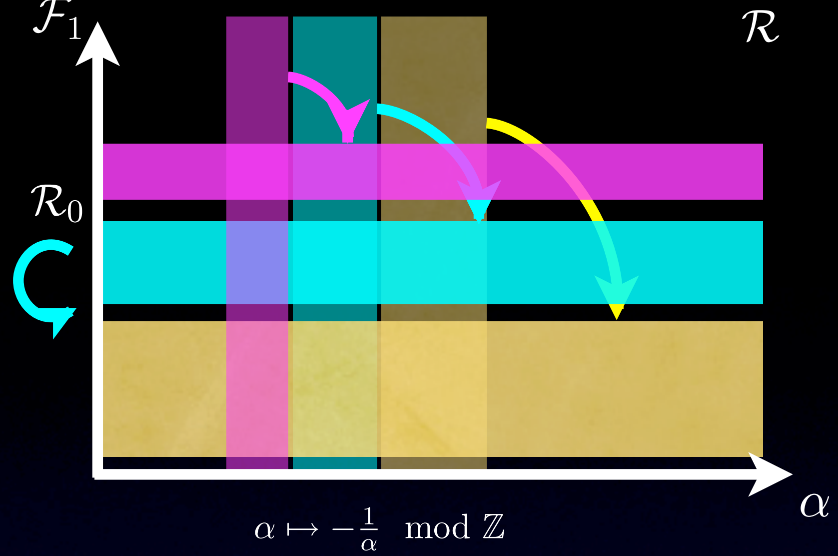
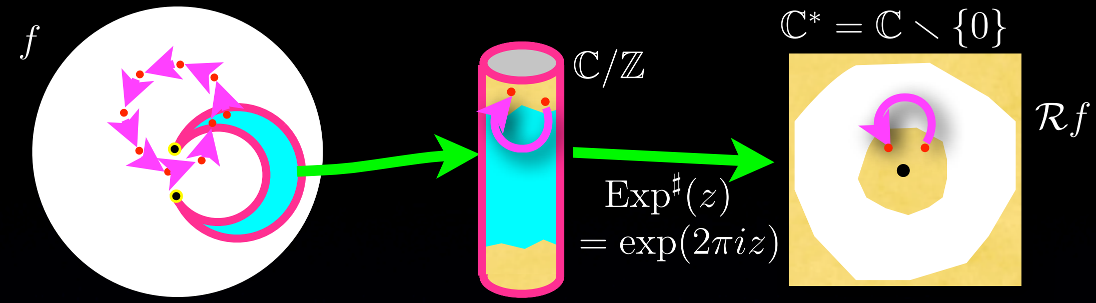
Royden-Gardiner: Holomorphic map between Teichmüller spaces are weakly contracting.





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$\tilde{\psi}$ conformal near $\partial V \implies$ strict contraction.

More on a priori bound

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$\alpha = 0$

parabolic fixed point

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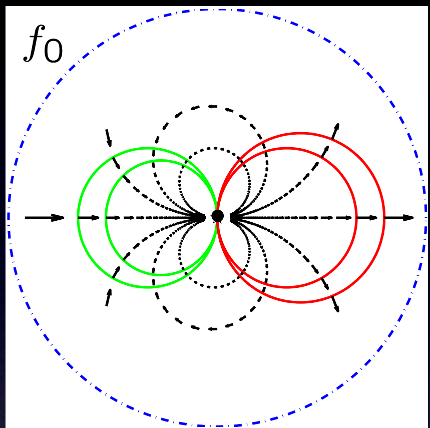
parabolic fixed point



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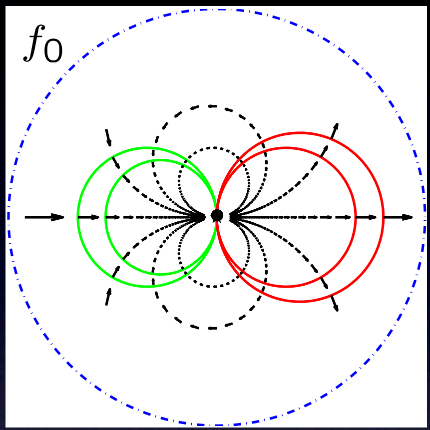

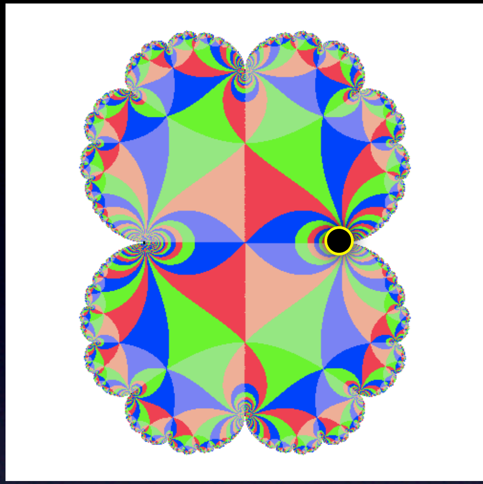

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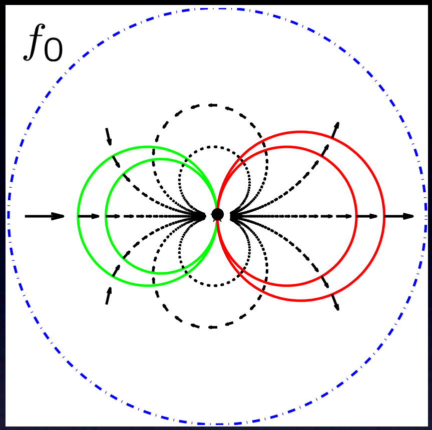
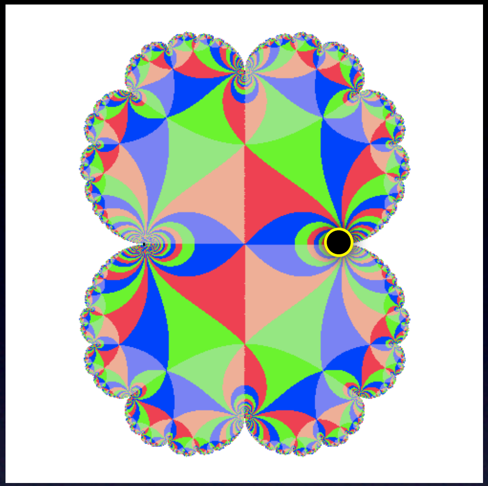
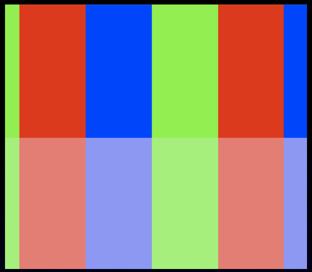




Φ_{attr}



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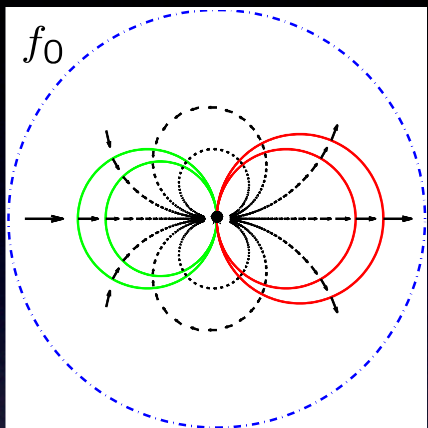

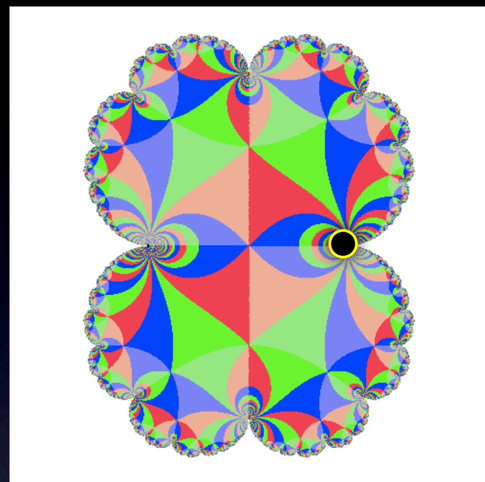

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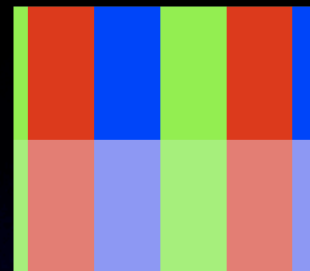

F_{can} $(\infty = \text{fixed pt})$

More on a priori bound

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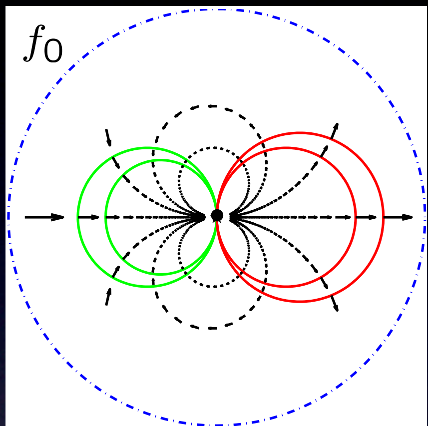

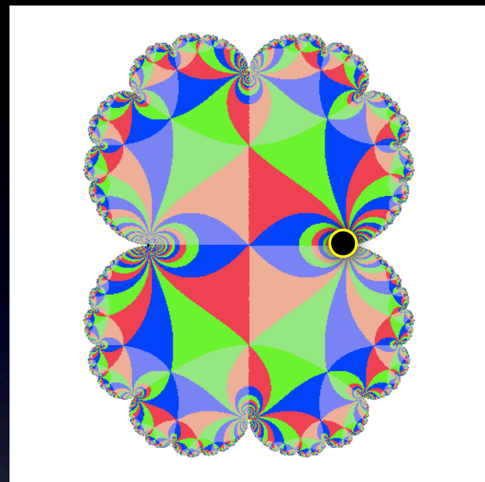

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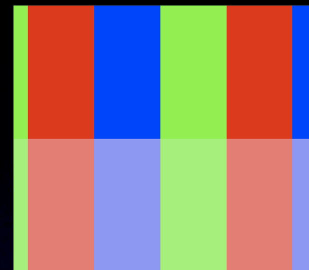

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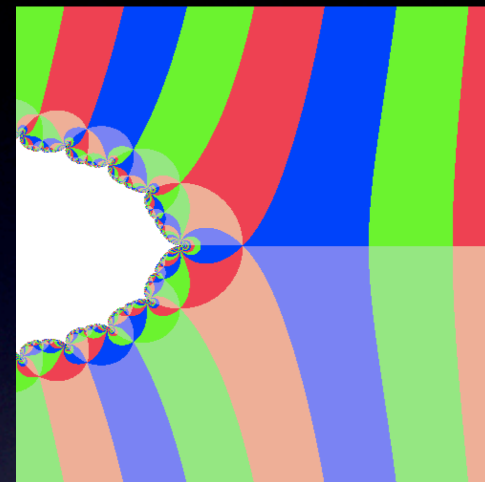

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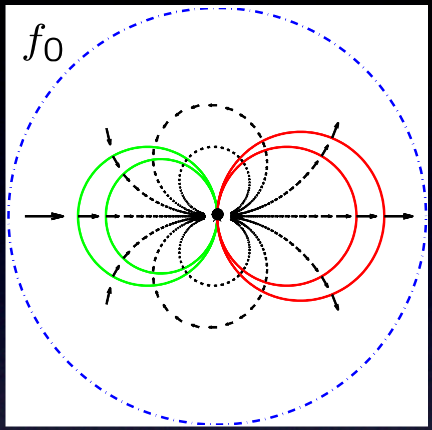

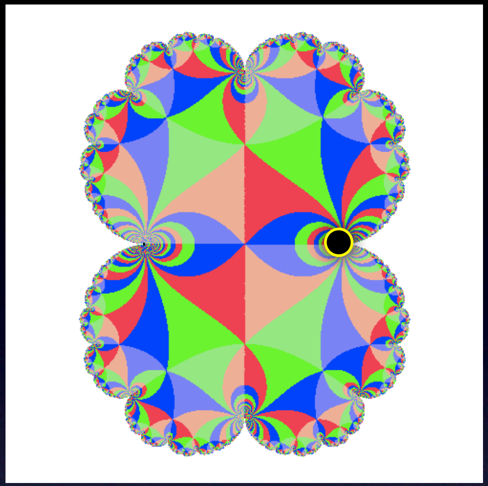

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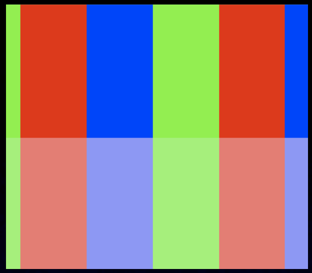

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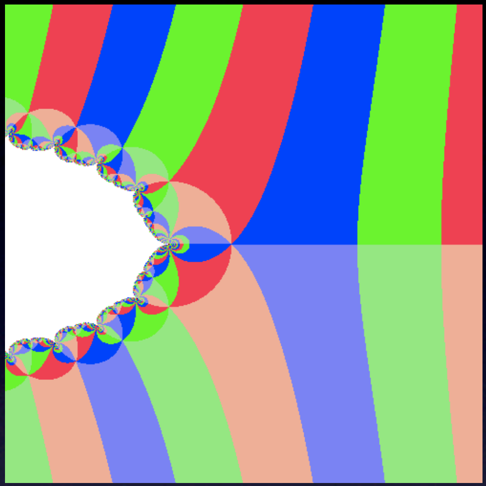

irrat. indiff. (near-parabolic)

More on a priori bound

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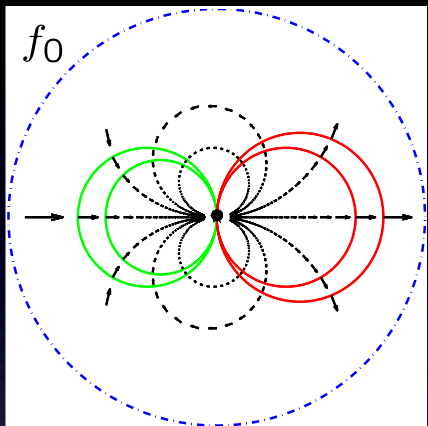
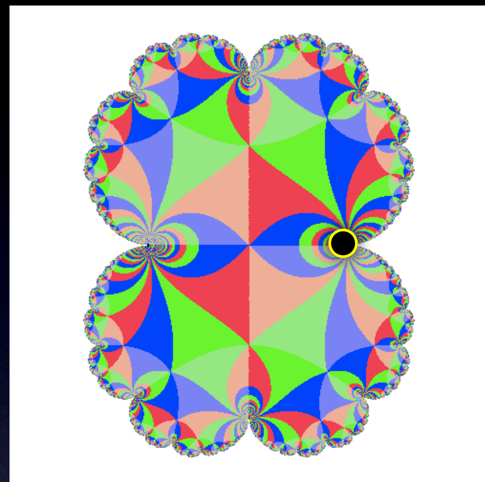
parabolic fixed point





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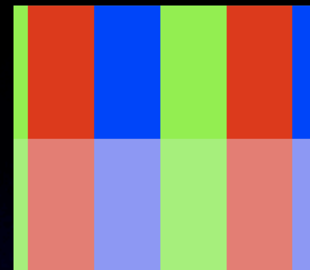

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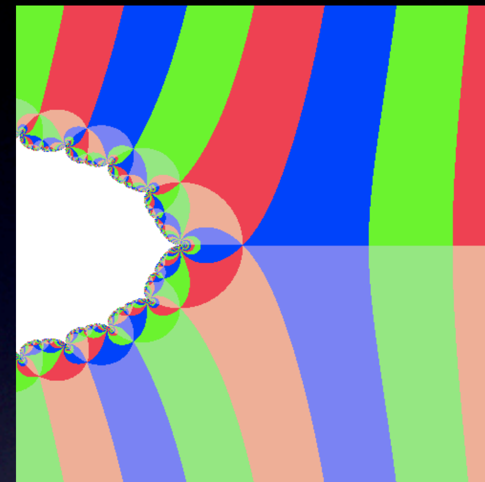

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use Douady-Hubbard-Lavaurs theory of parabolic implosion

More on a priori bound

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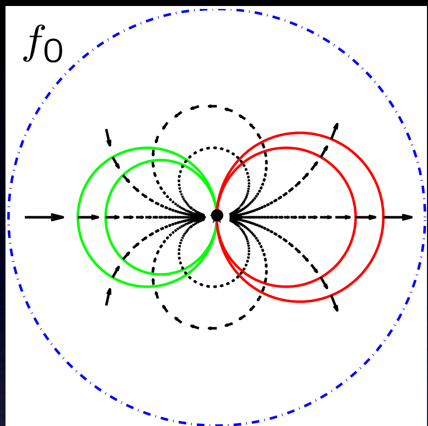
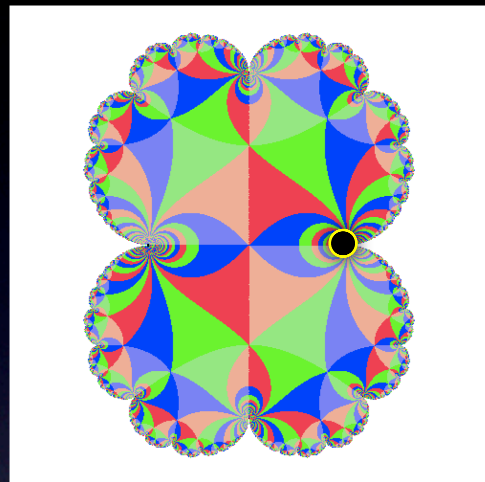
parabolic fixed point





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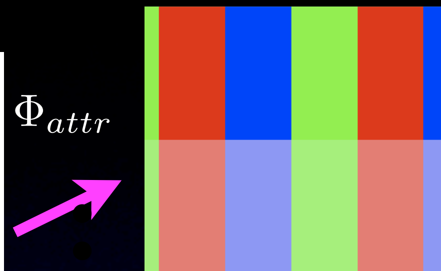

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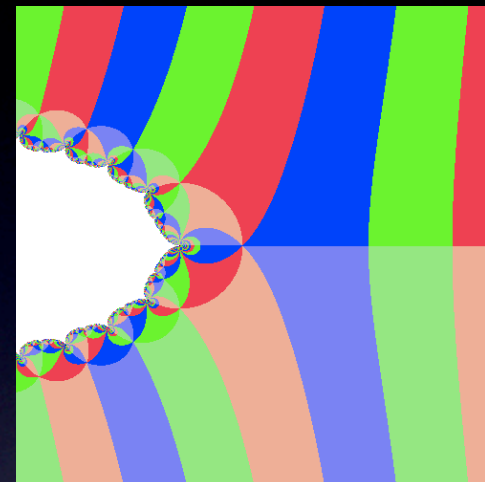

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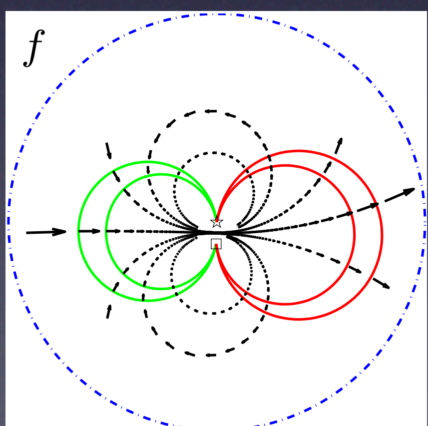

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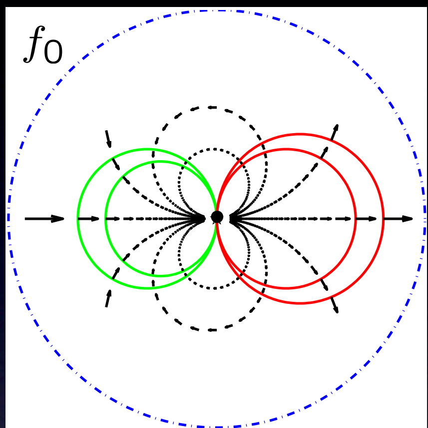
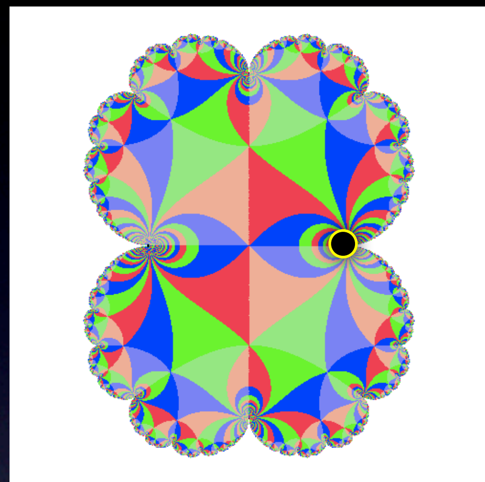
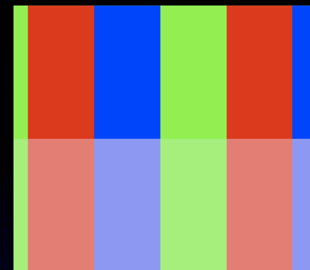
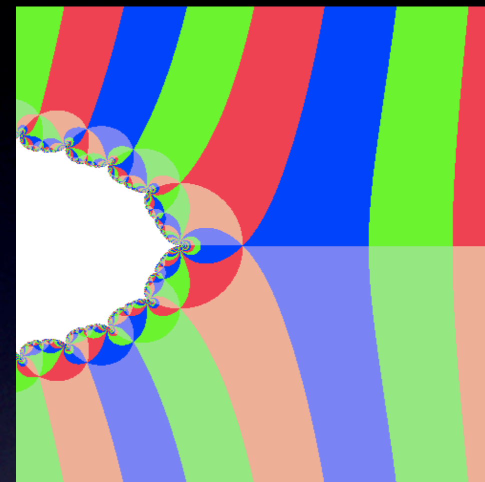




Φ_{attr}



F_{can} $(\infty = \text{fixed pt})$



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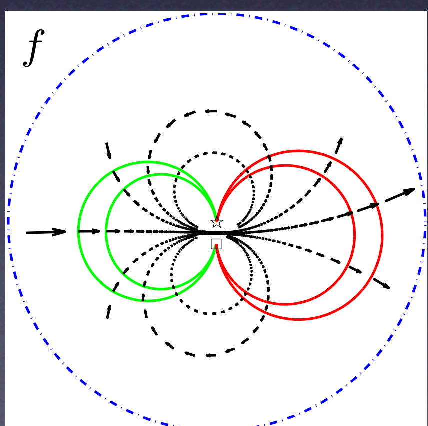
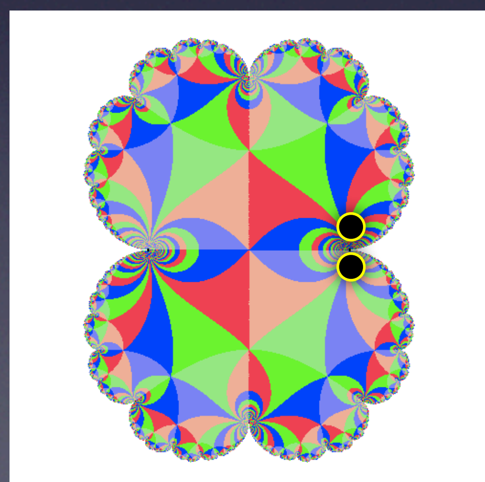
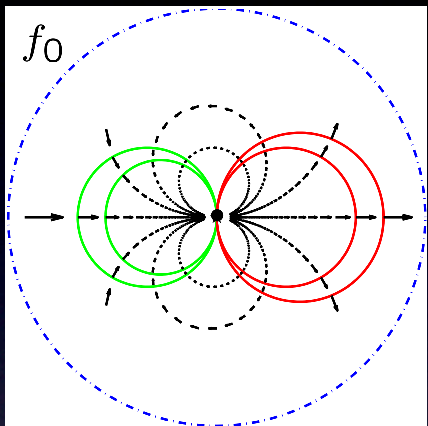


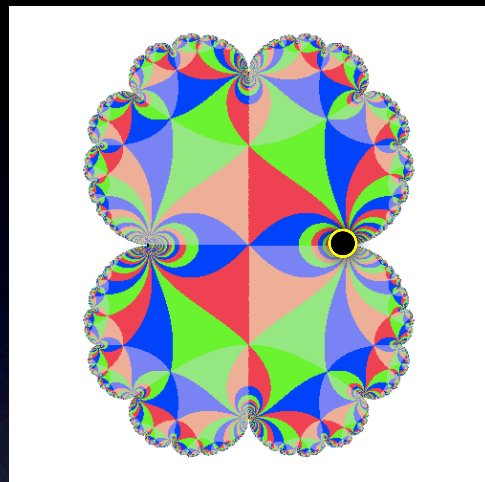


use Douady-Hubbard-Lavaurs theory of parabolic implosion

More on a priori bound

$\alpha = 0$
parabolic fixed point





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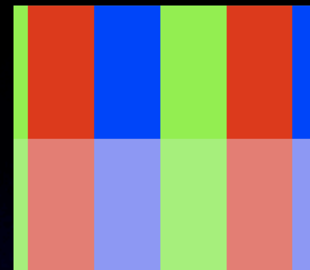

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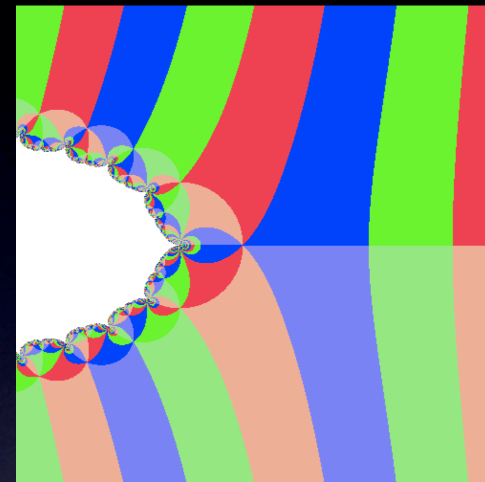

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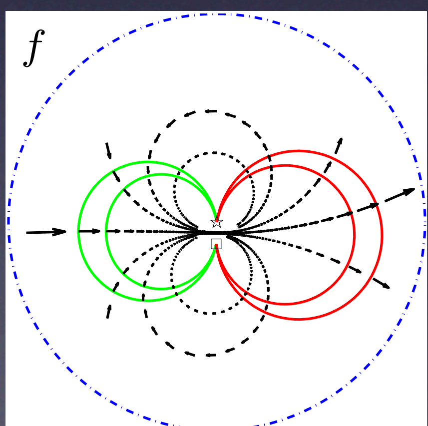

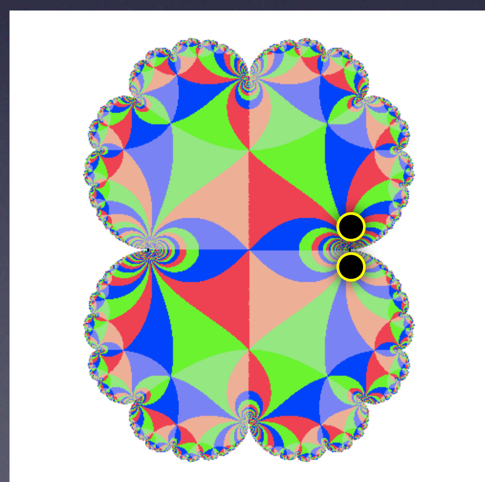

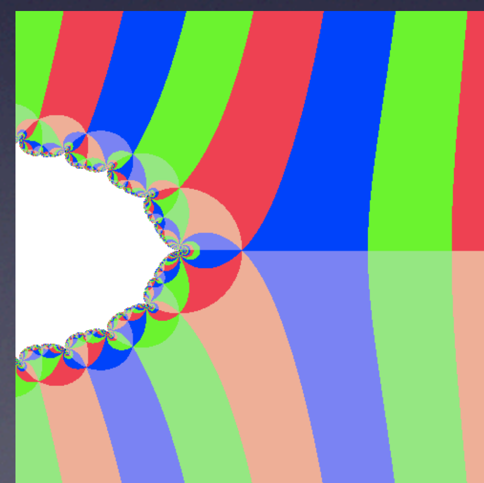

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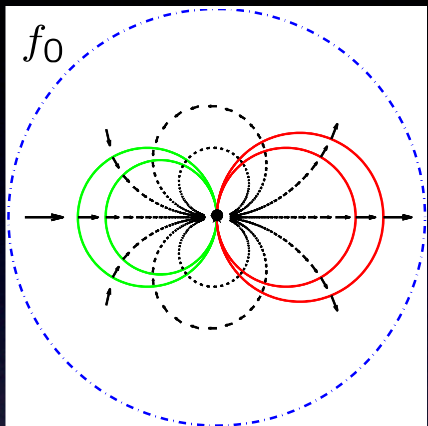

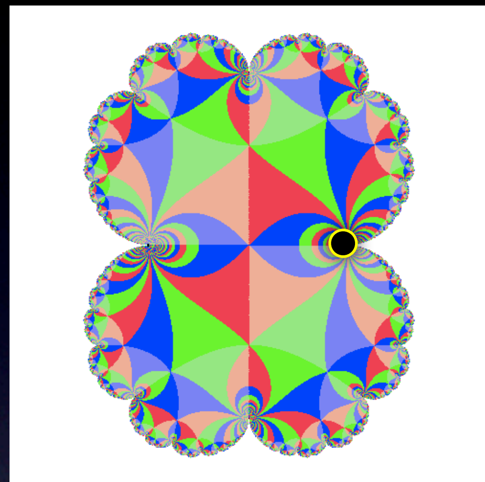

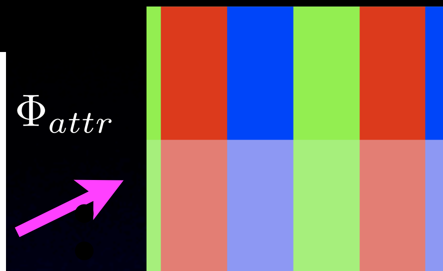

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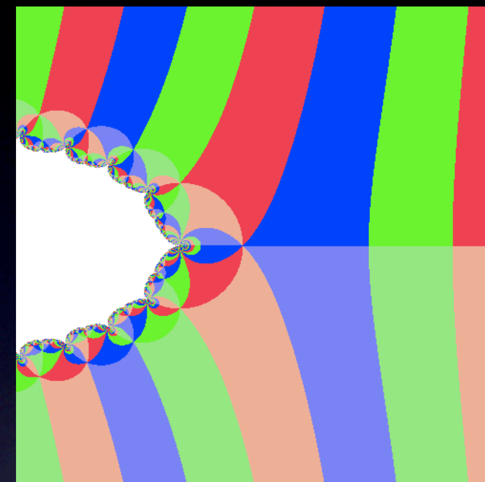

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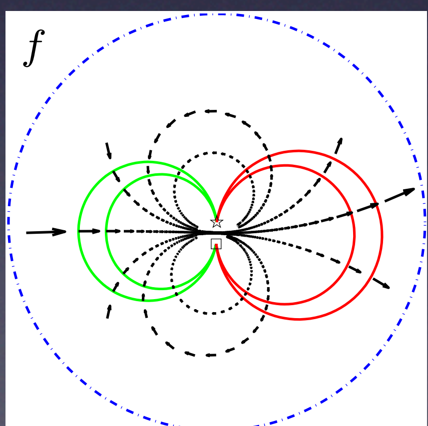

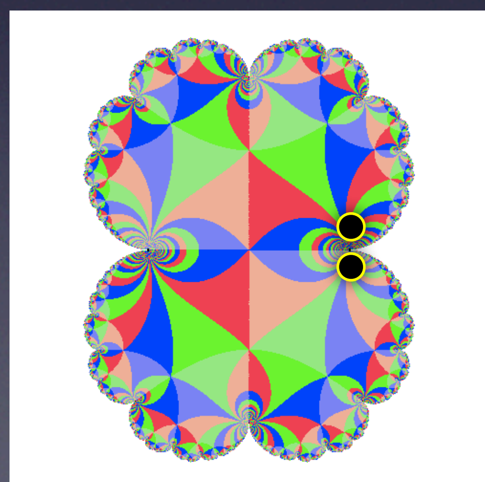

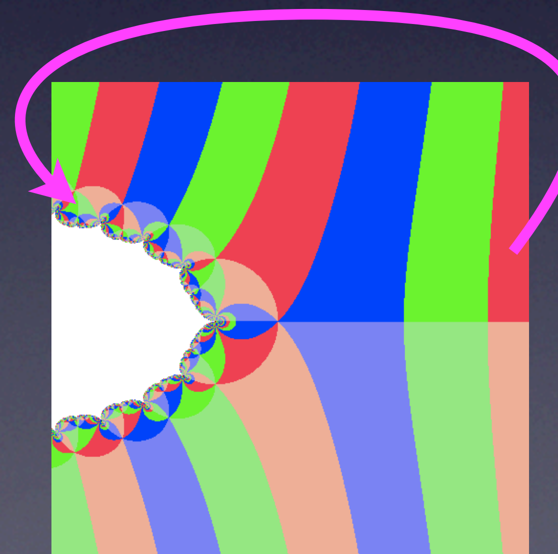

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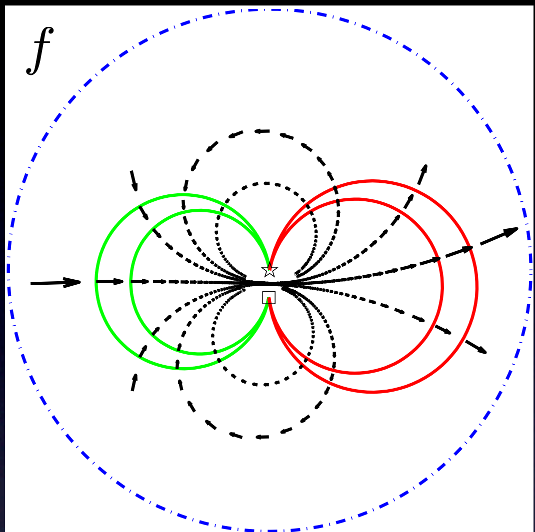

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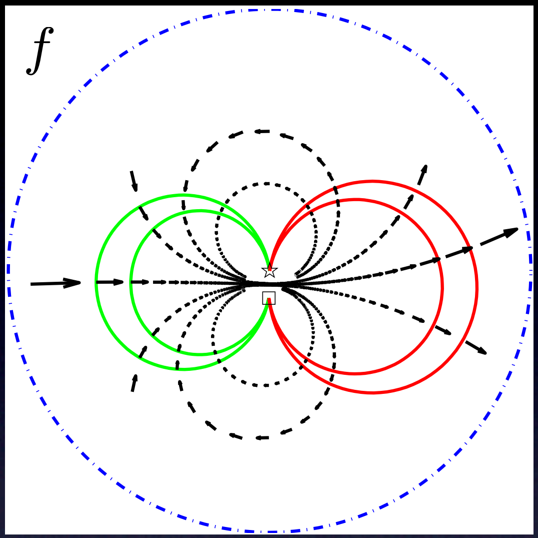

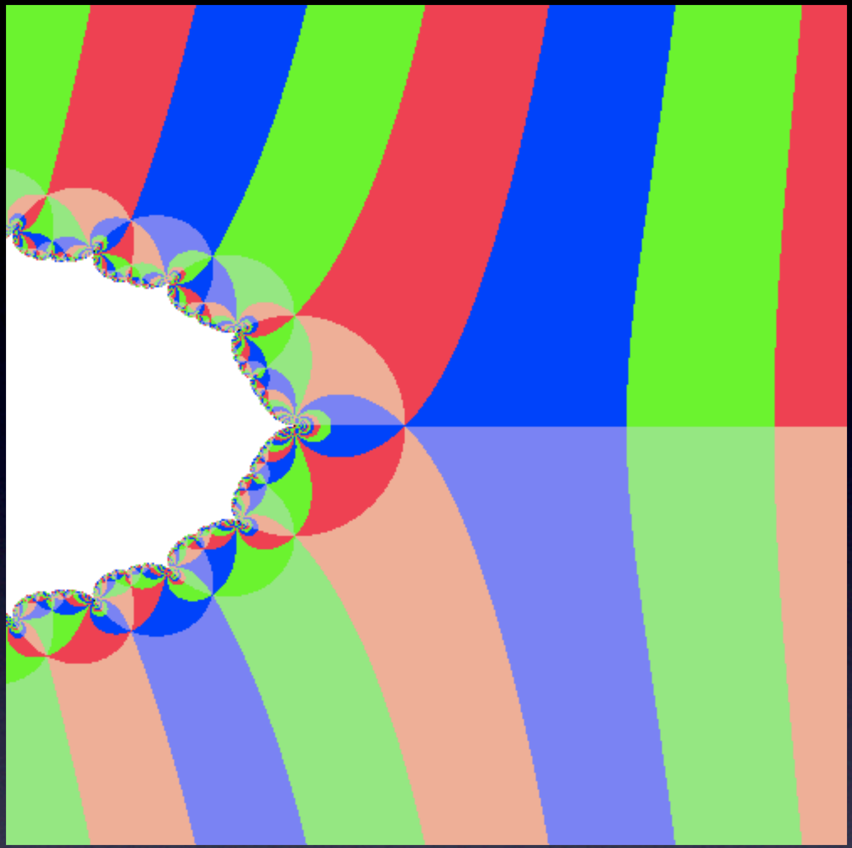

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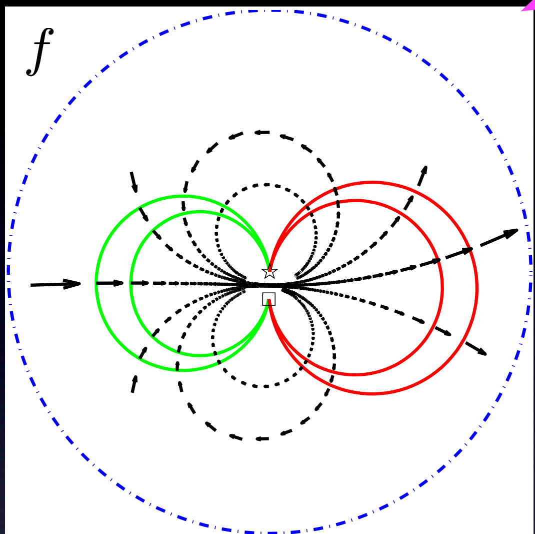

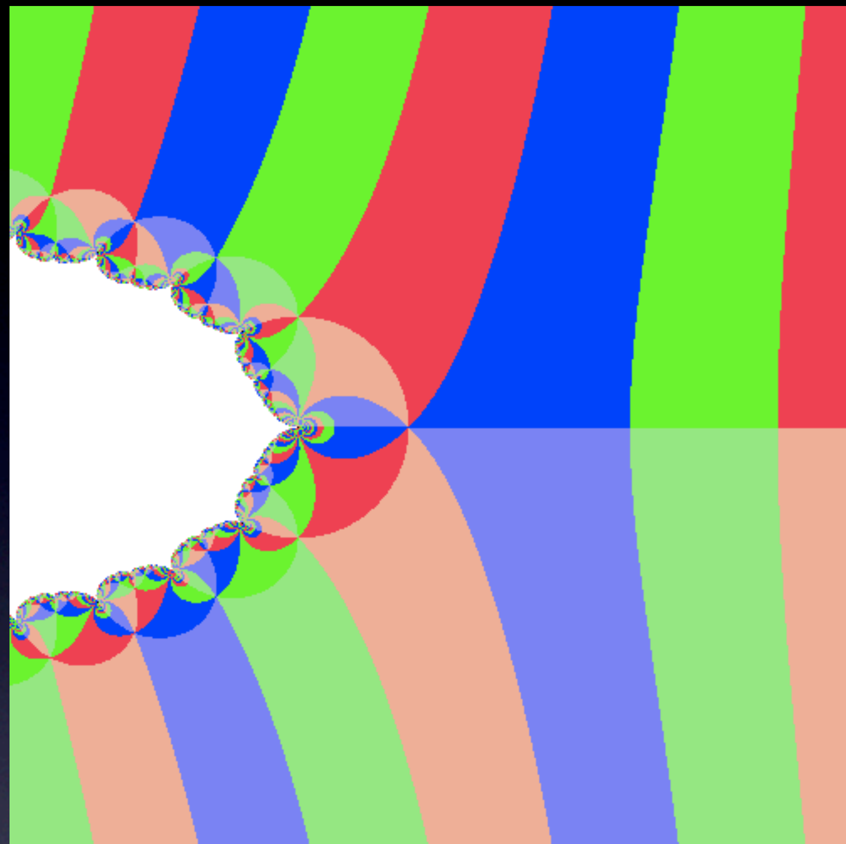

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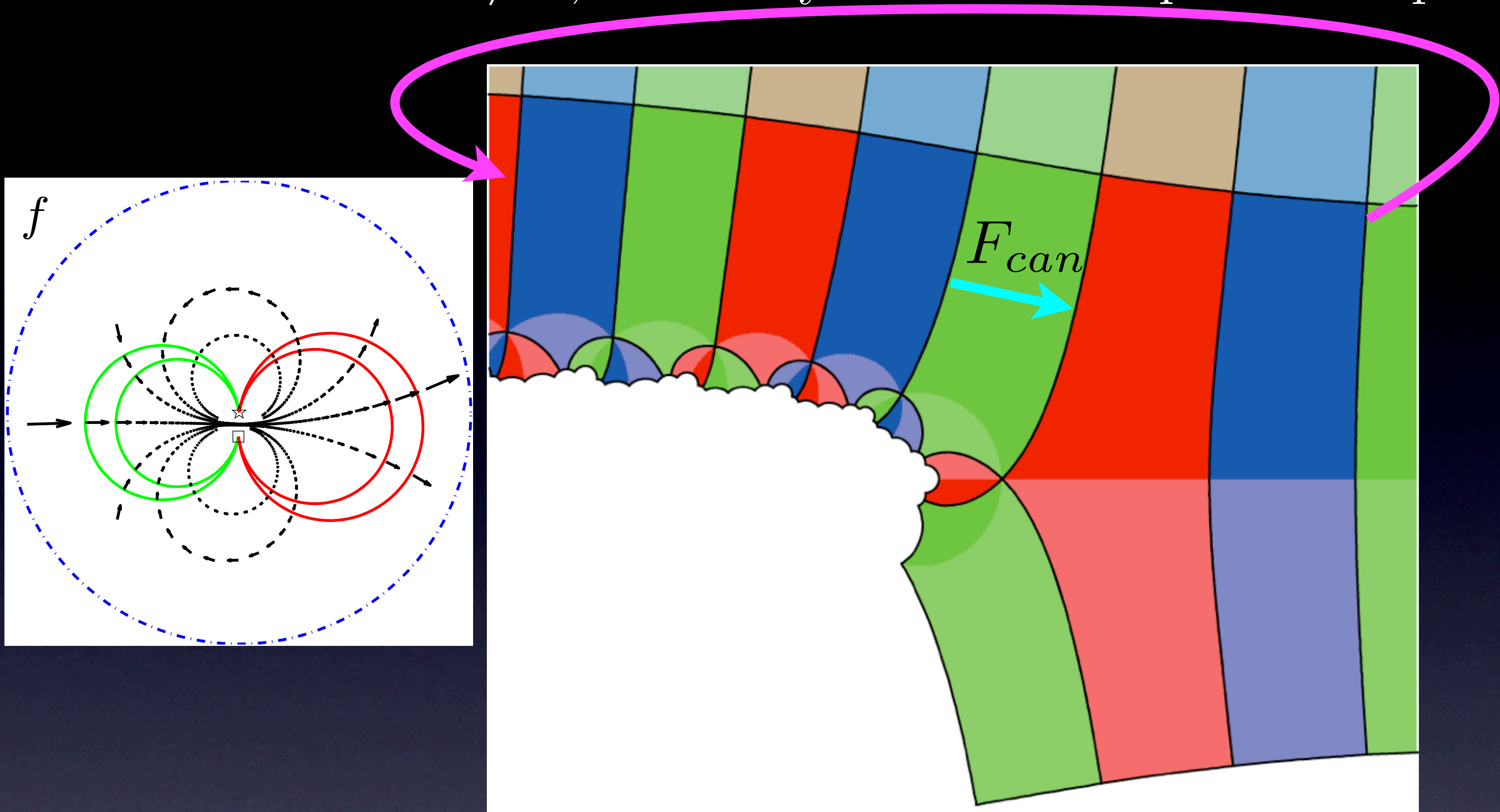

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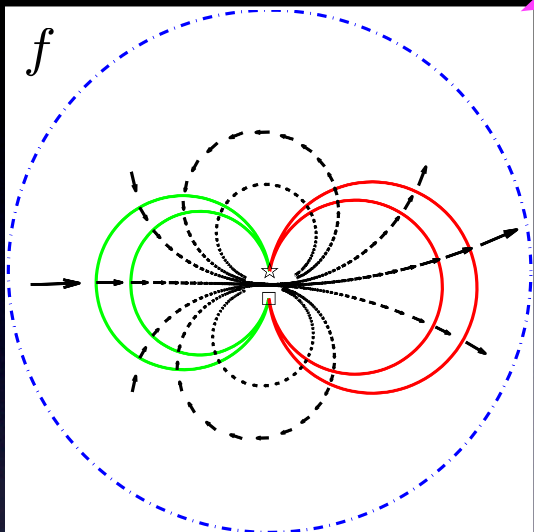

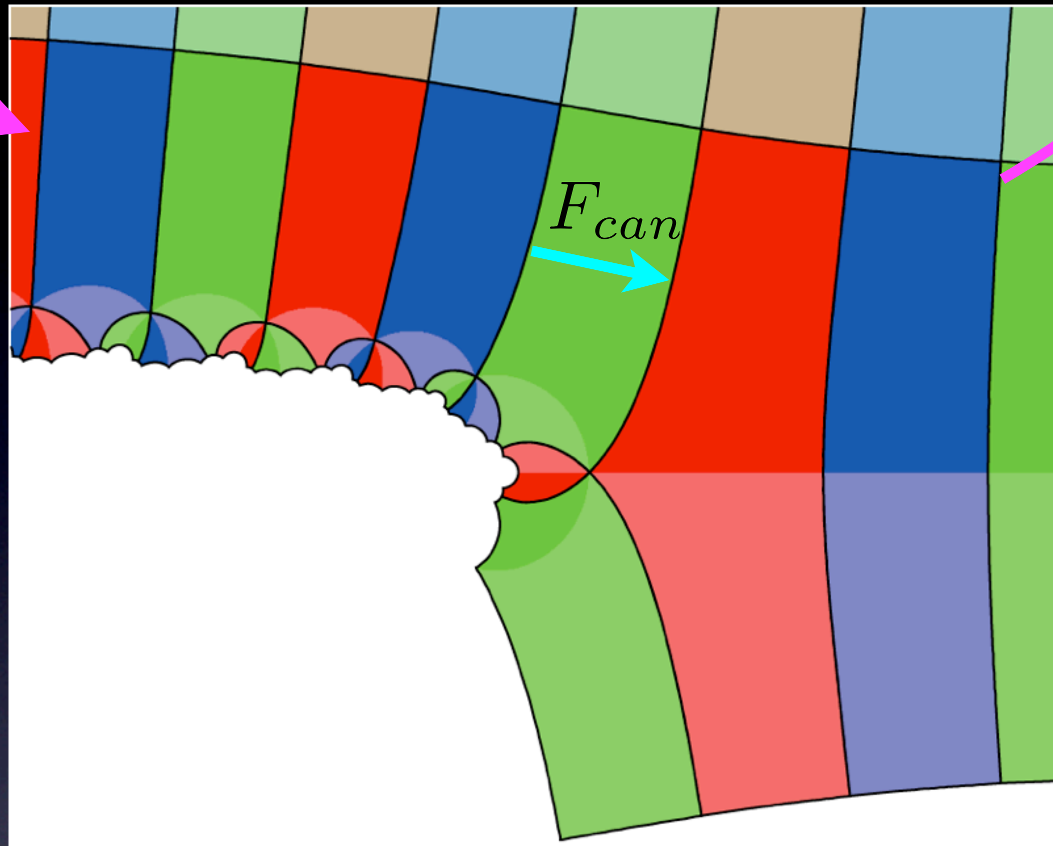

With many estimates, one can show that this much of the pattern is preserved.

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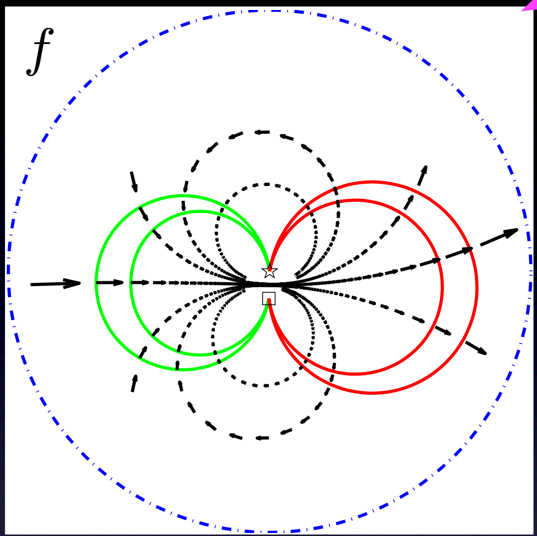

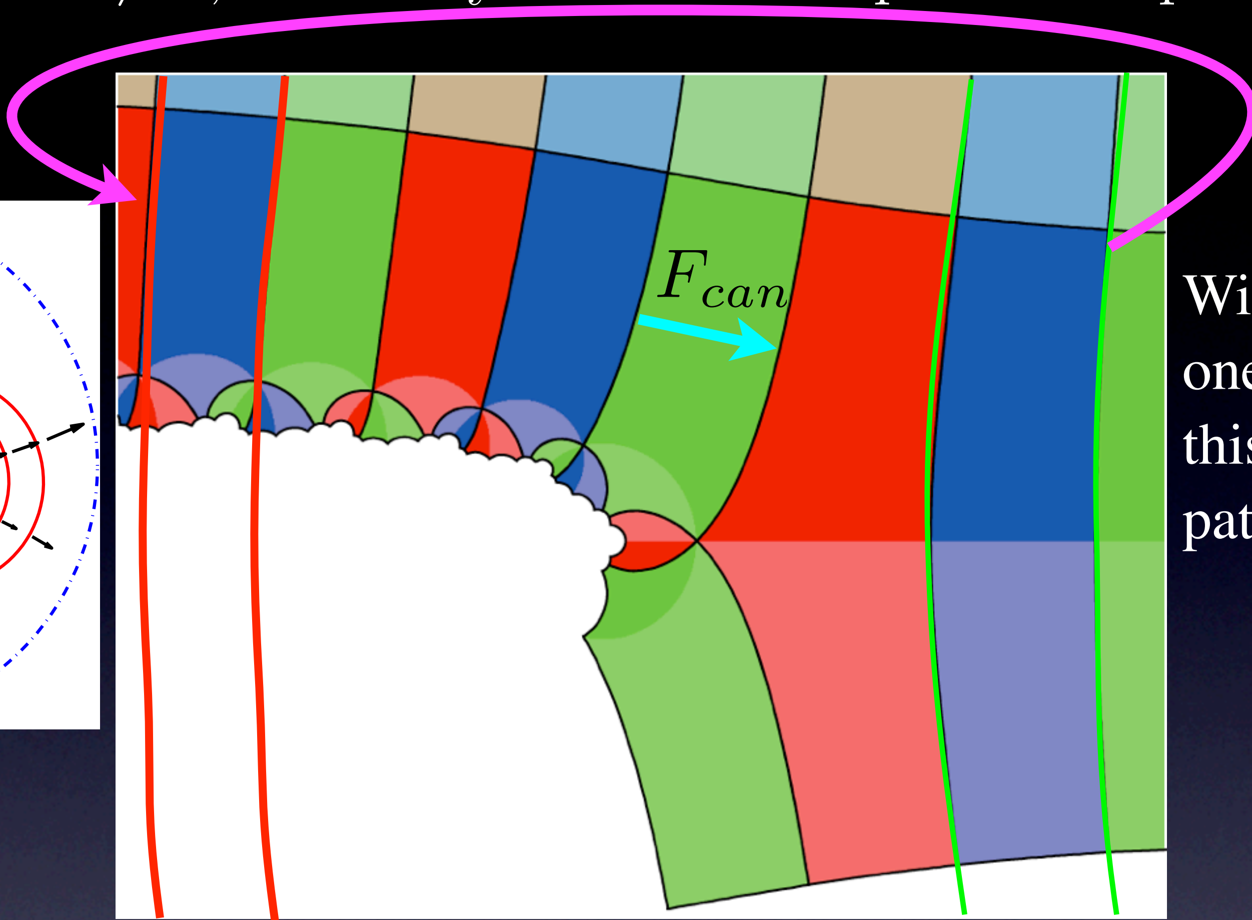

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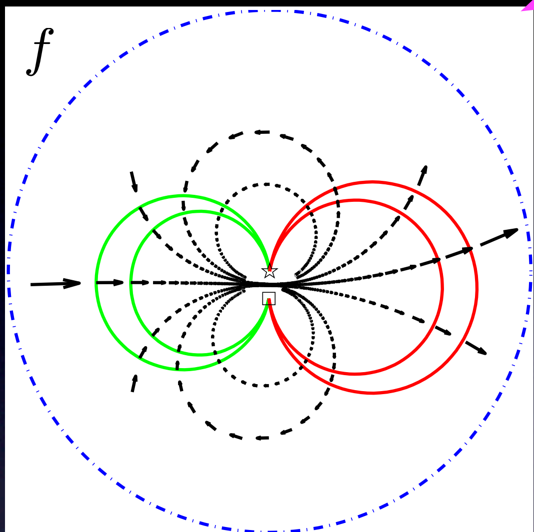

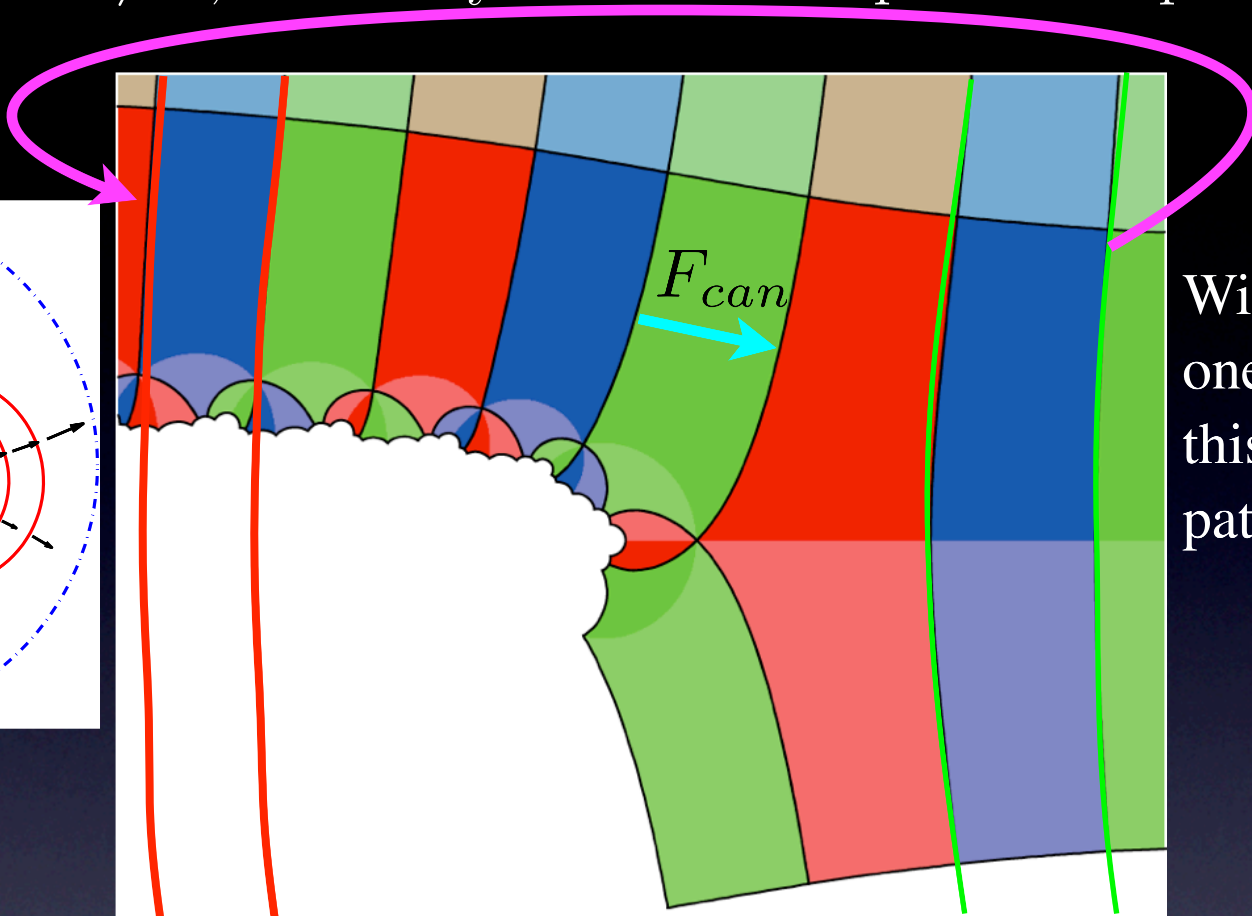

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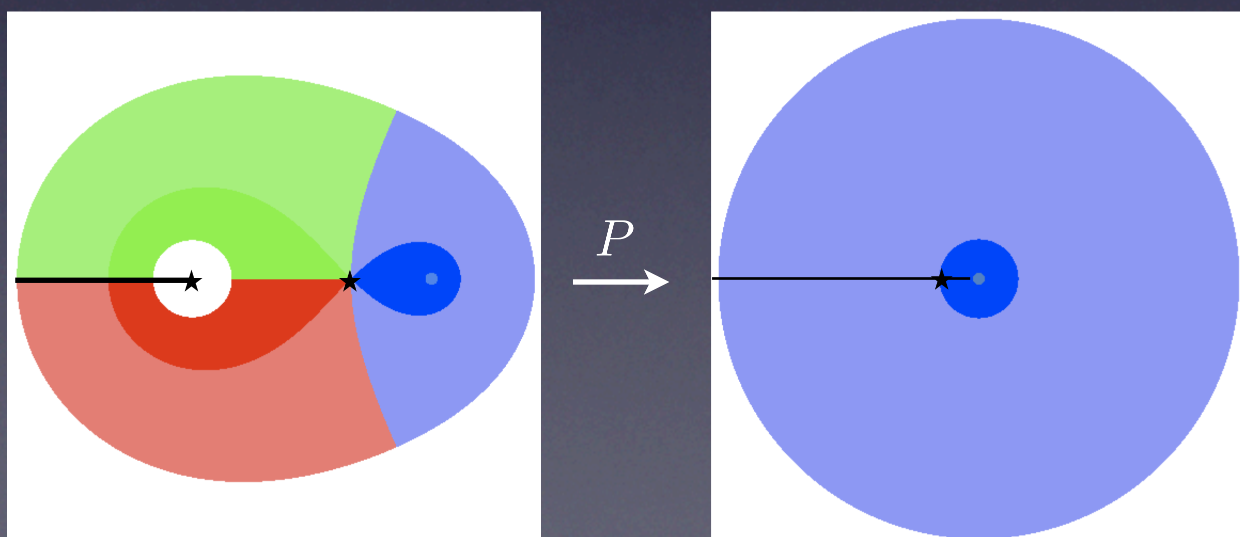
slit



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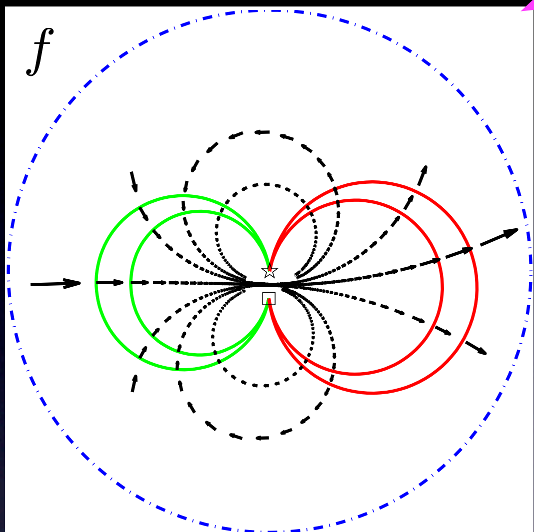

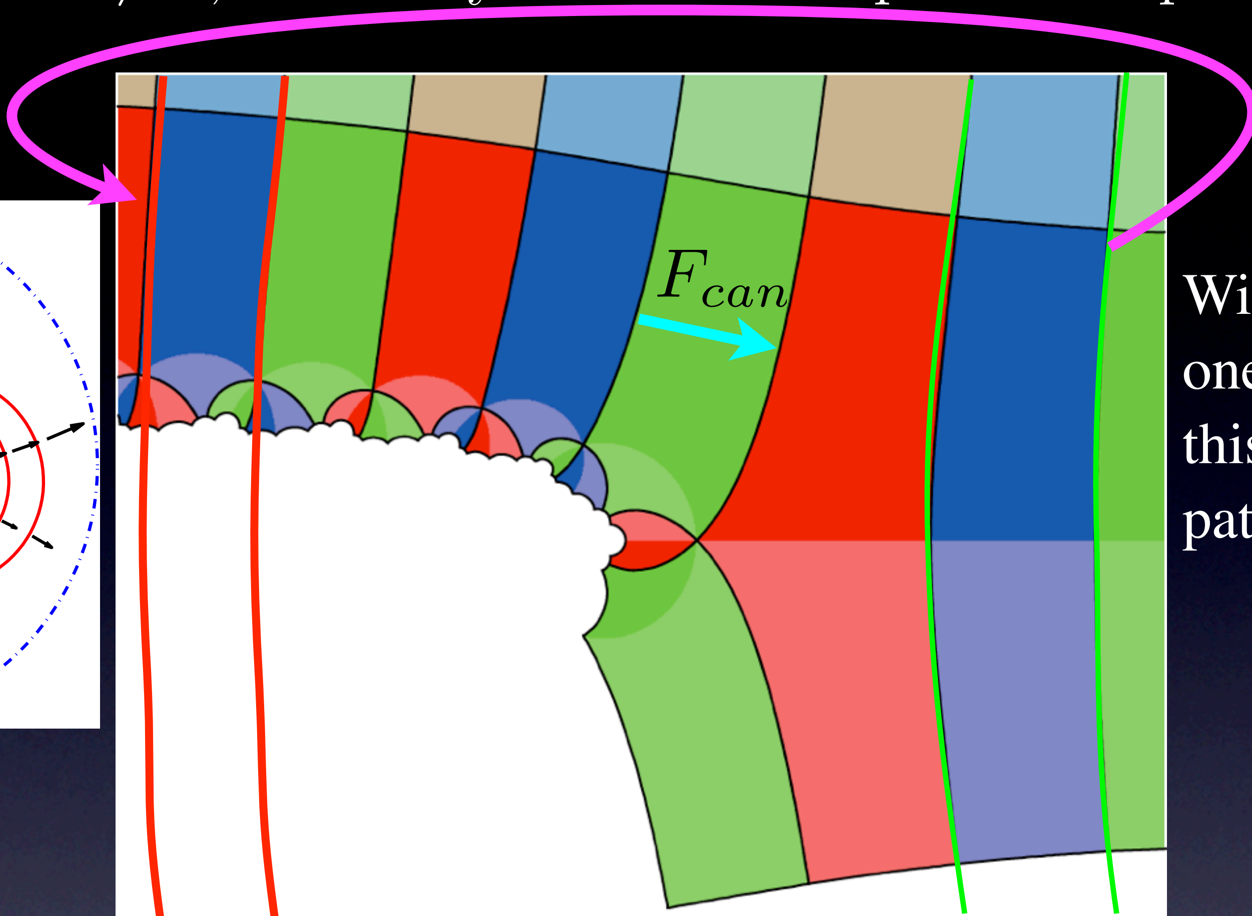

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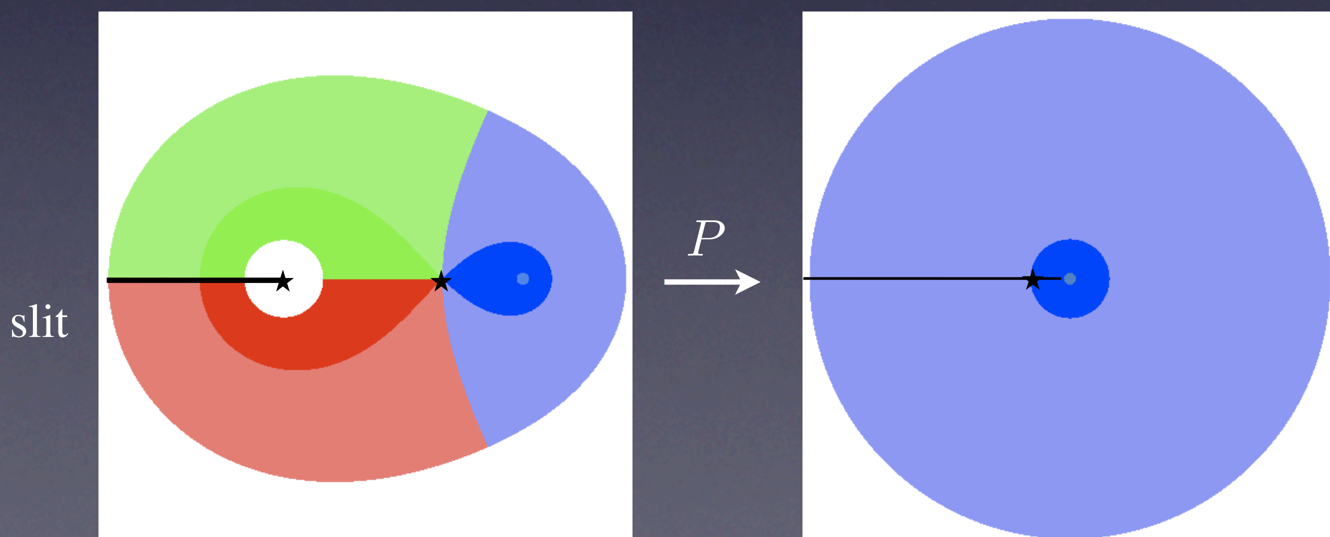

The renormalization has the same covering type

Applications of Inou-S. Theorem

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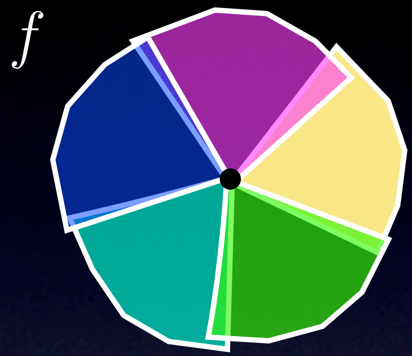

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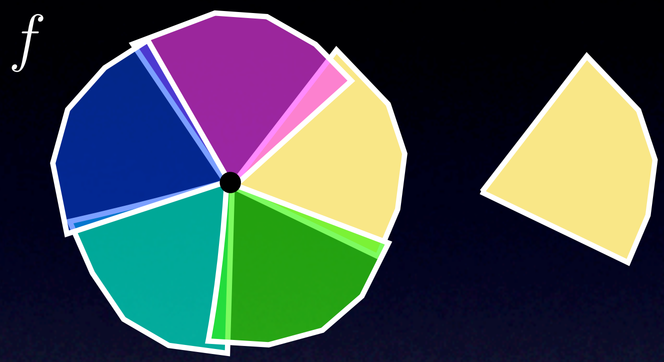

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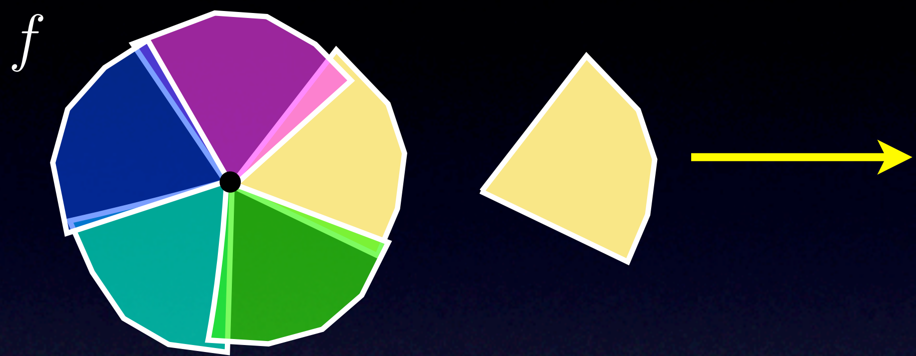

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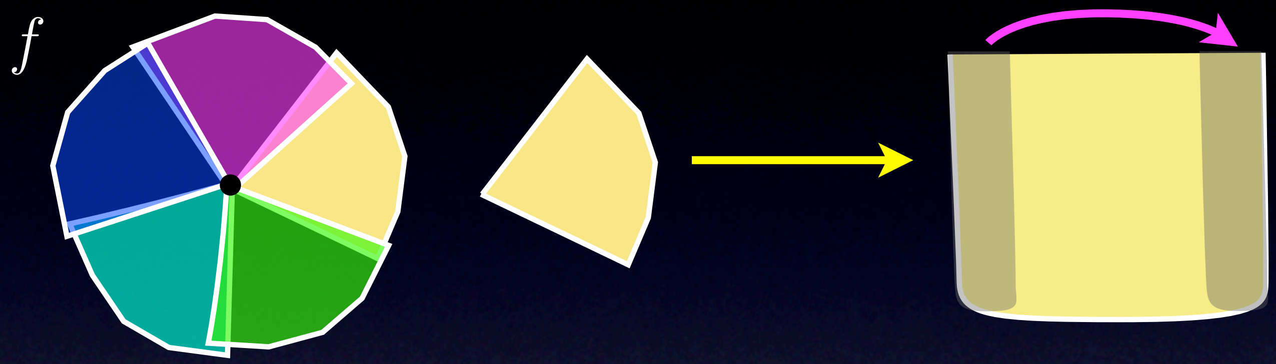

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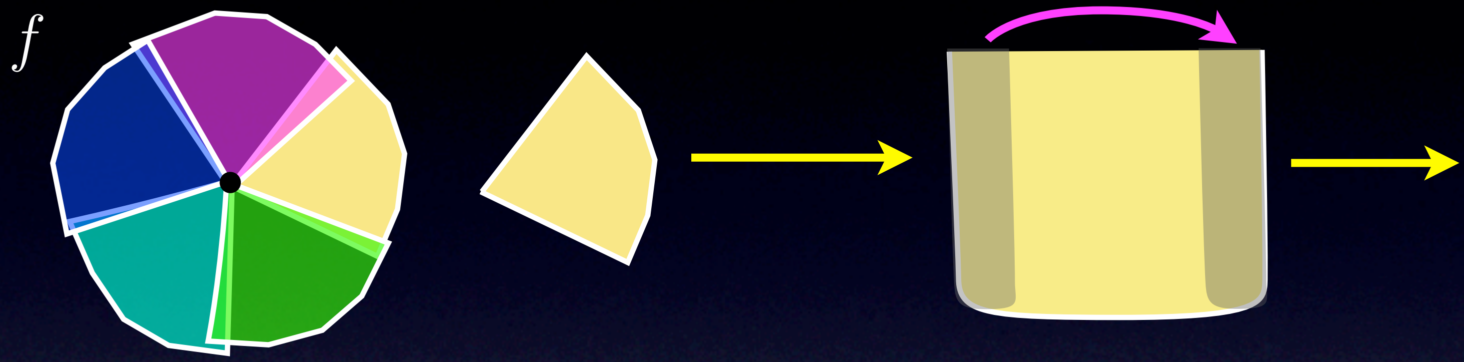

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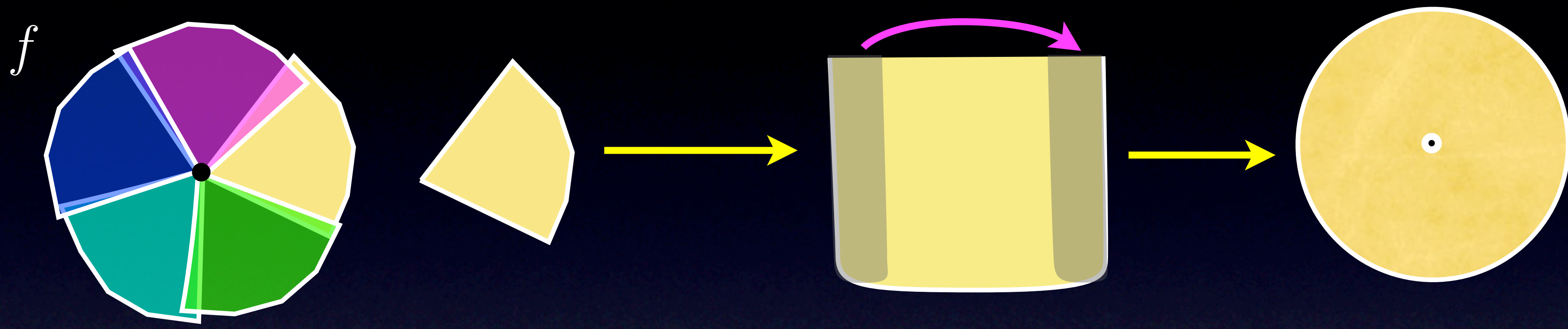

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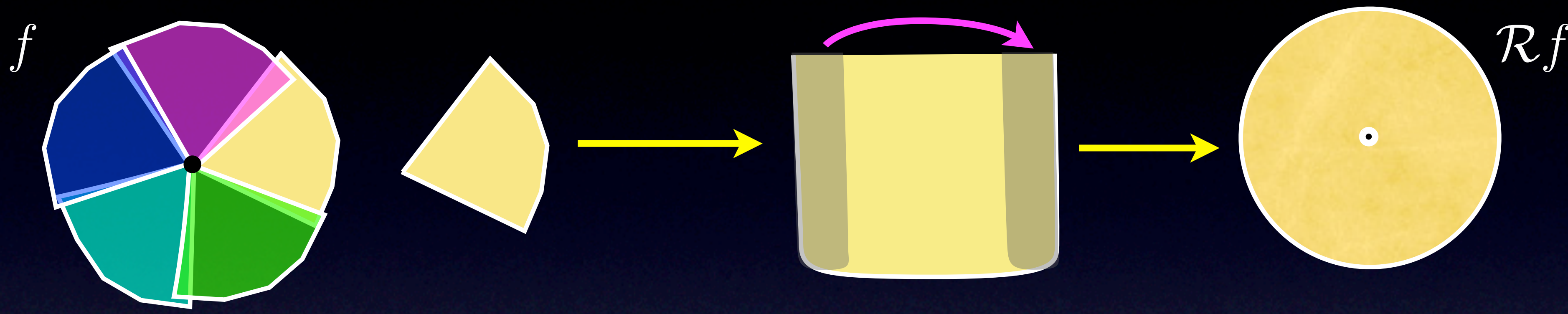

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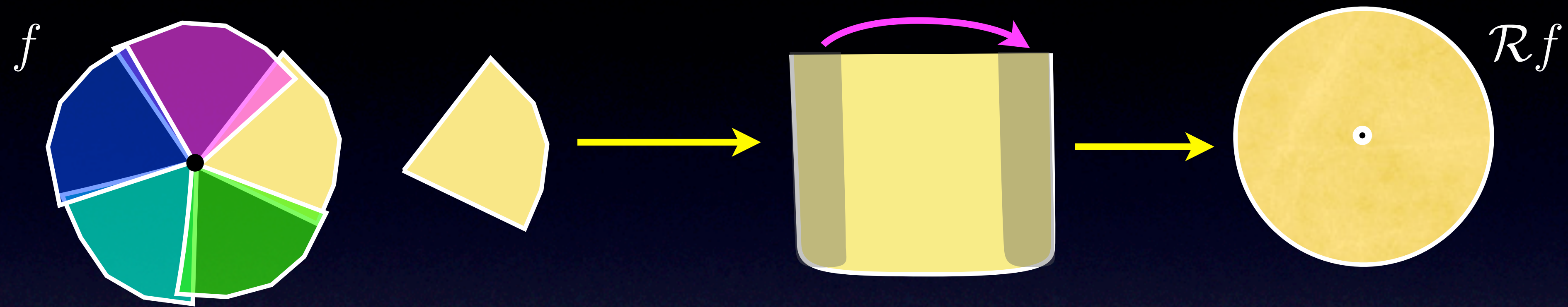

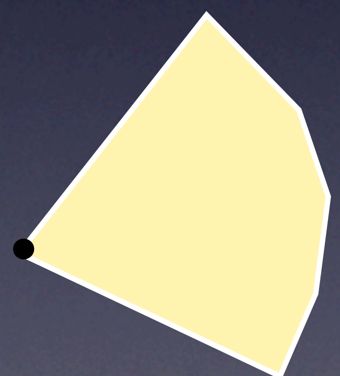

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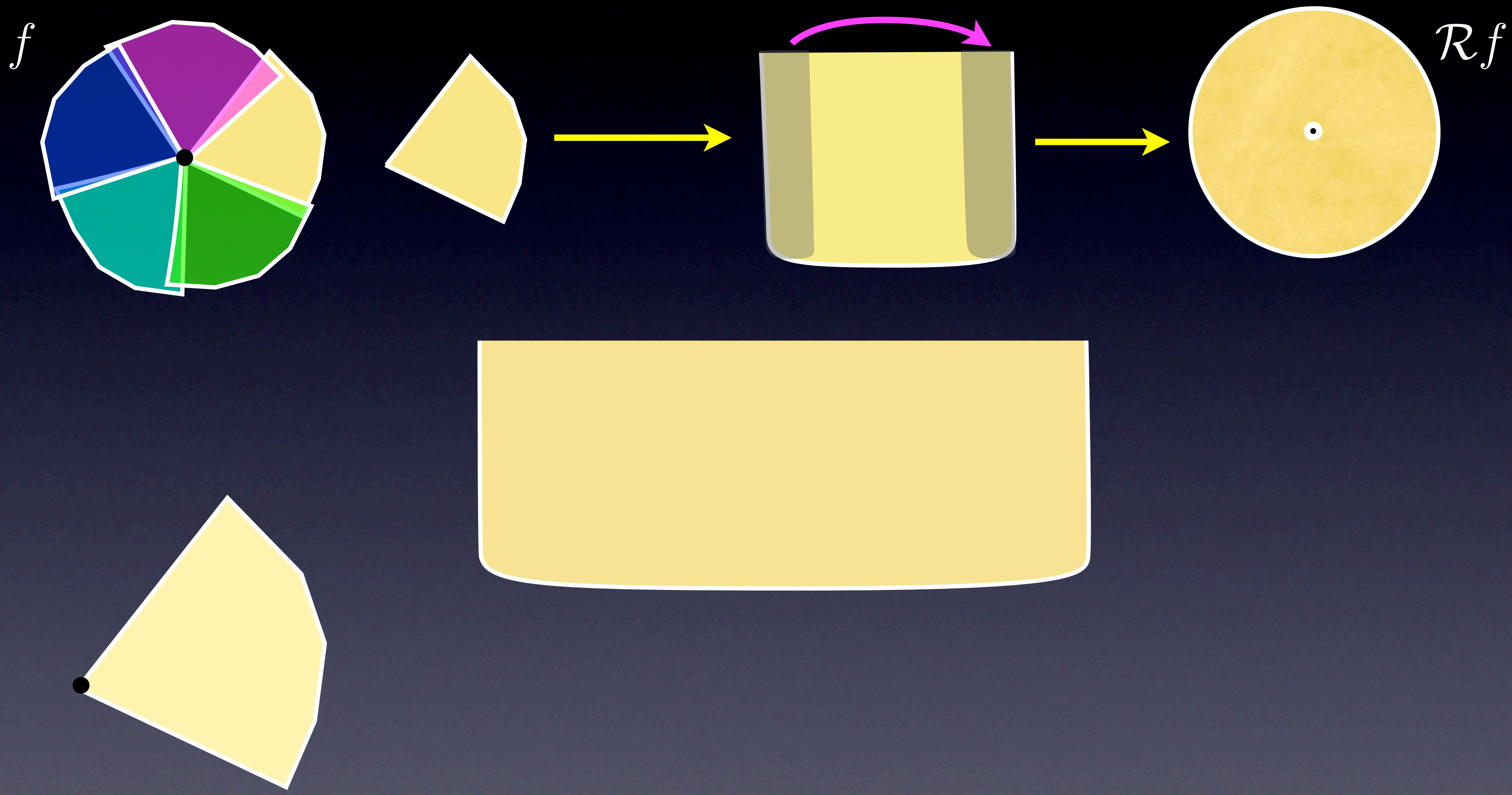

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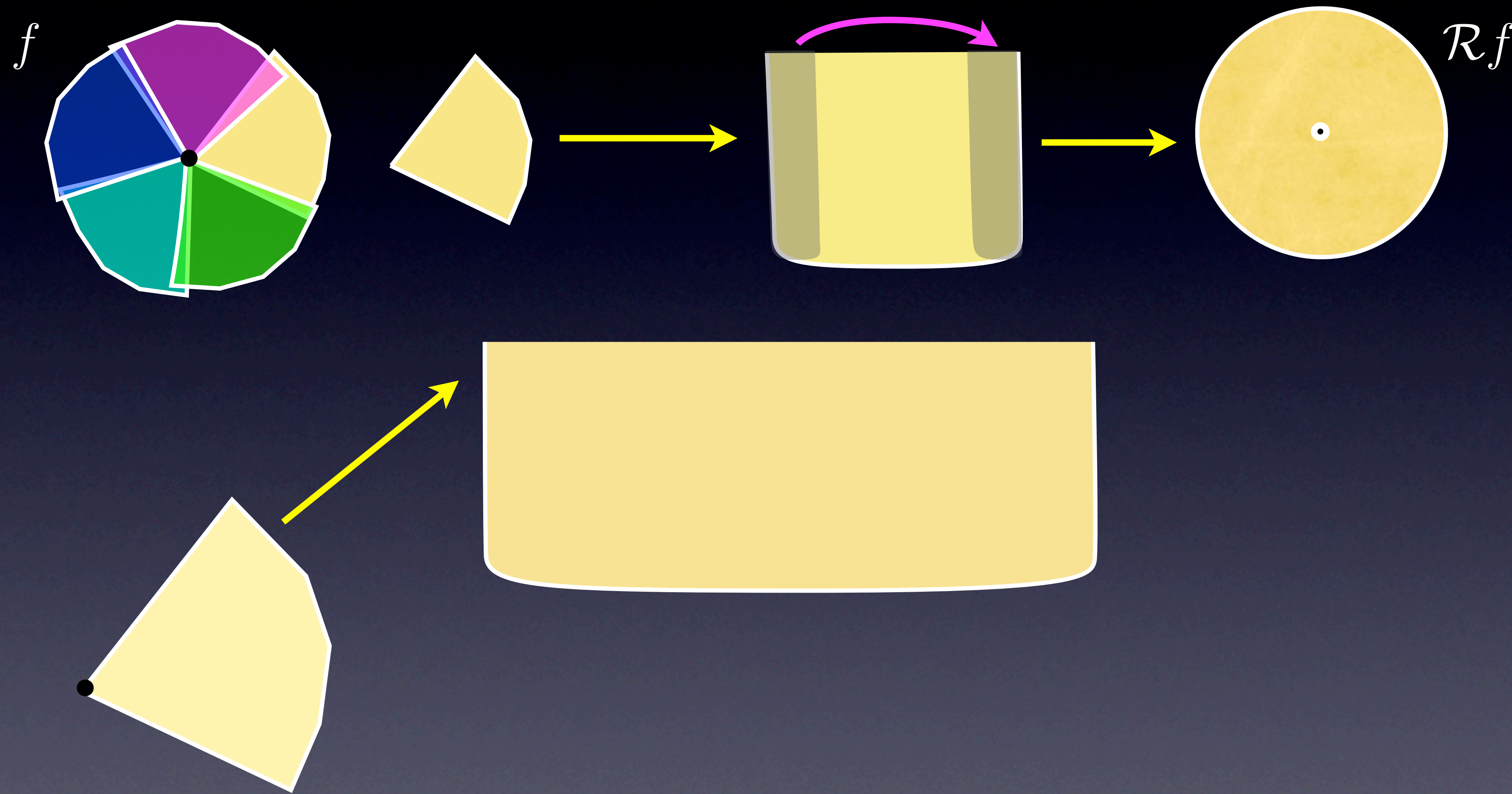

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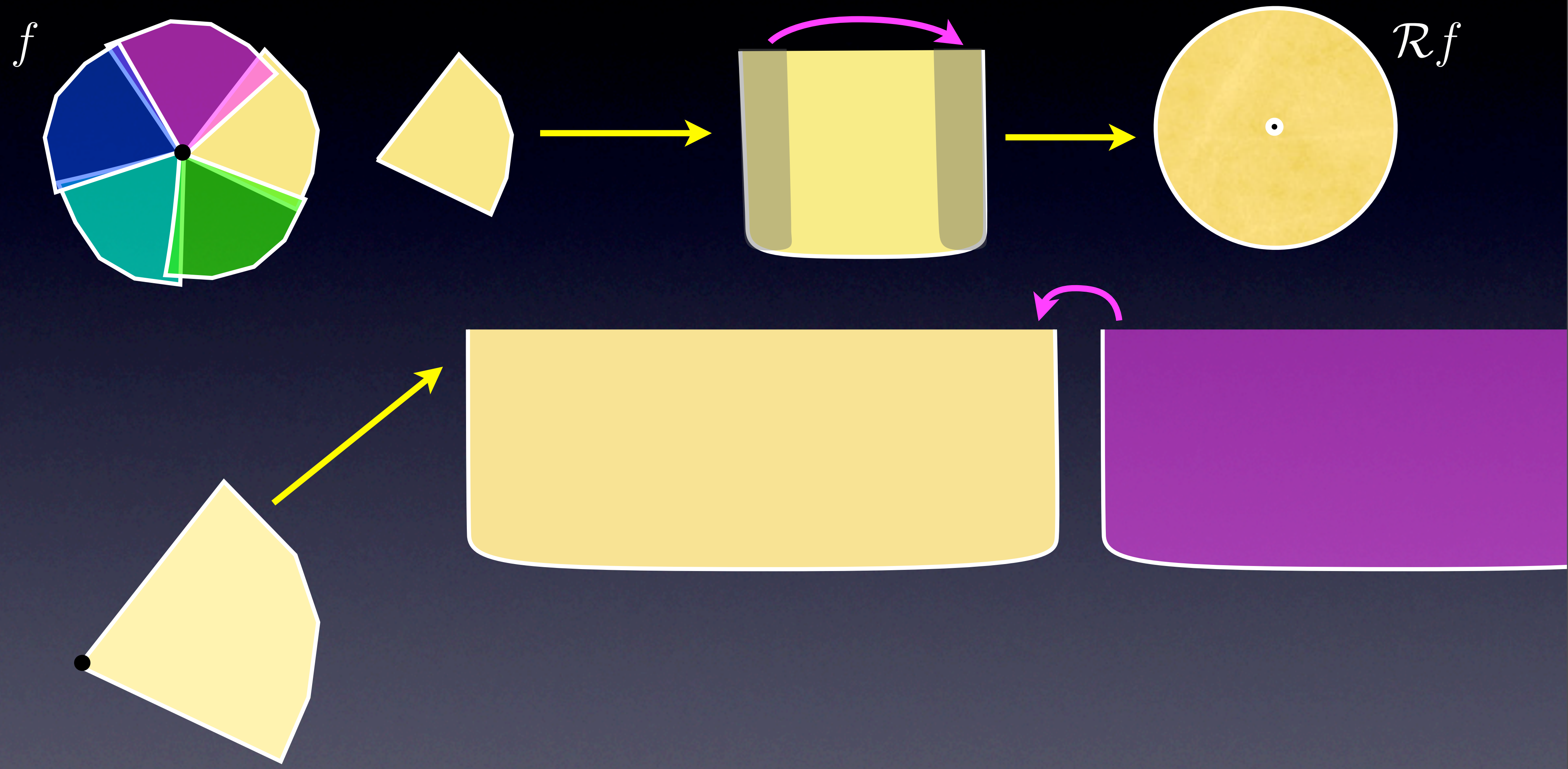

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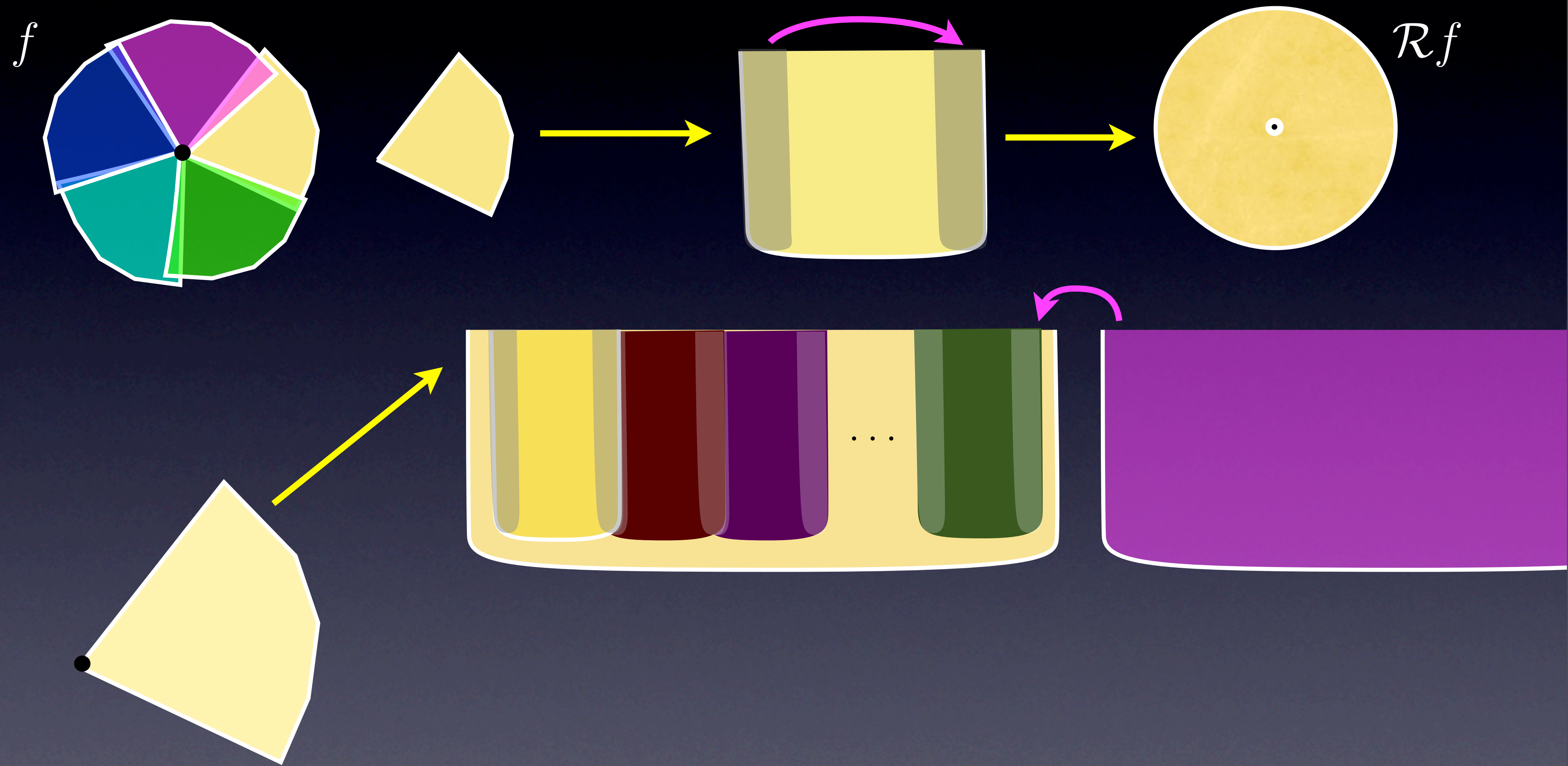

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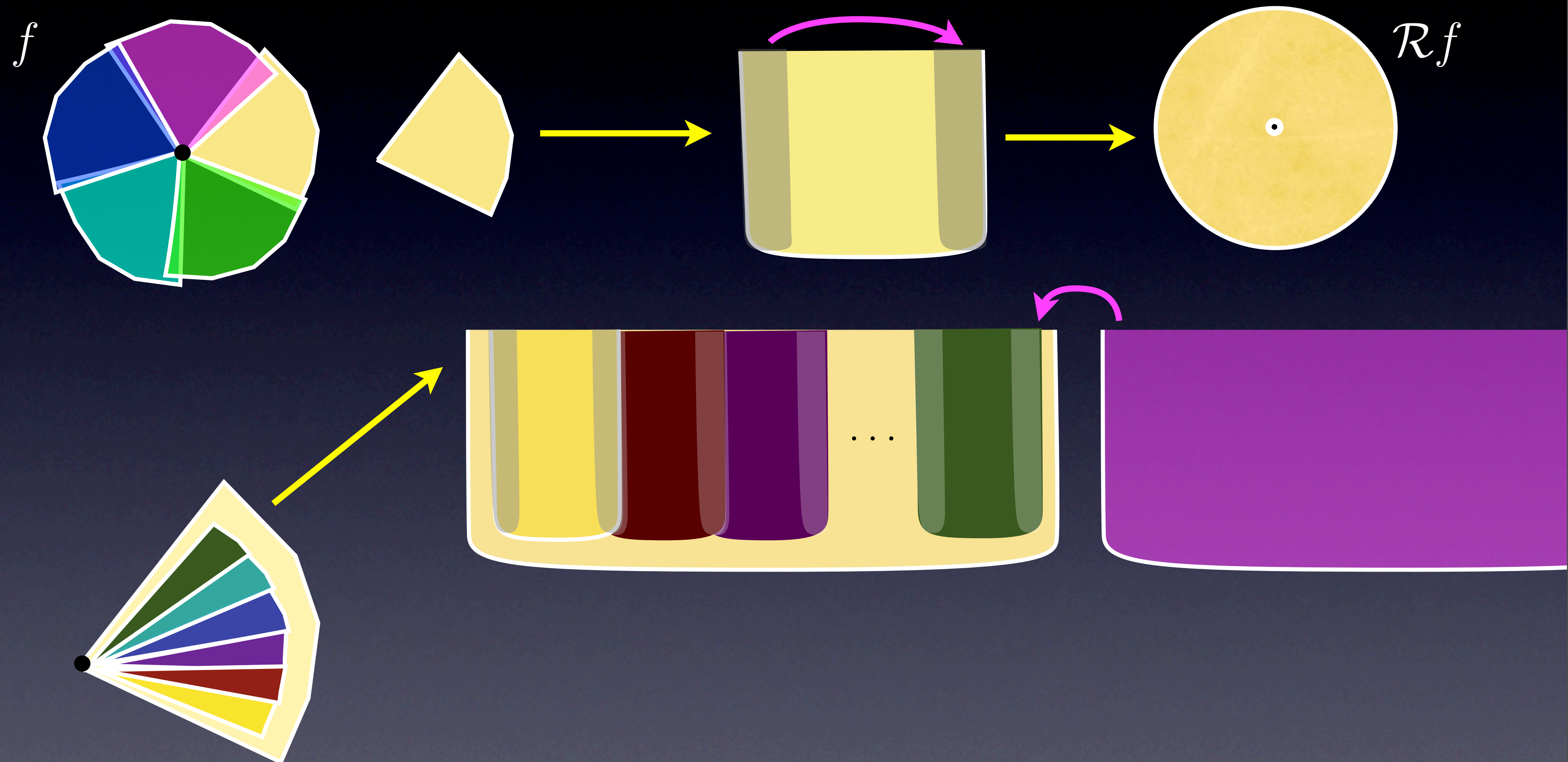

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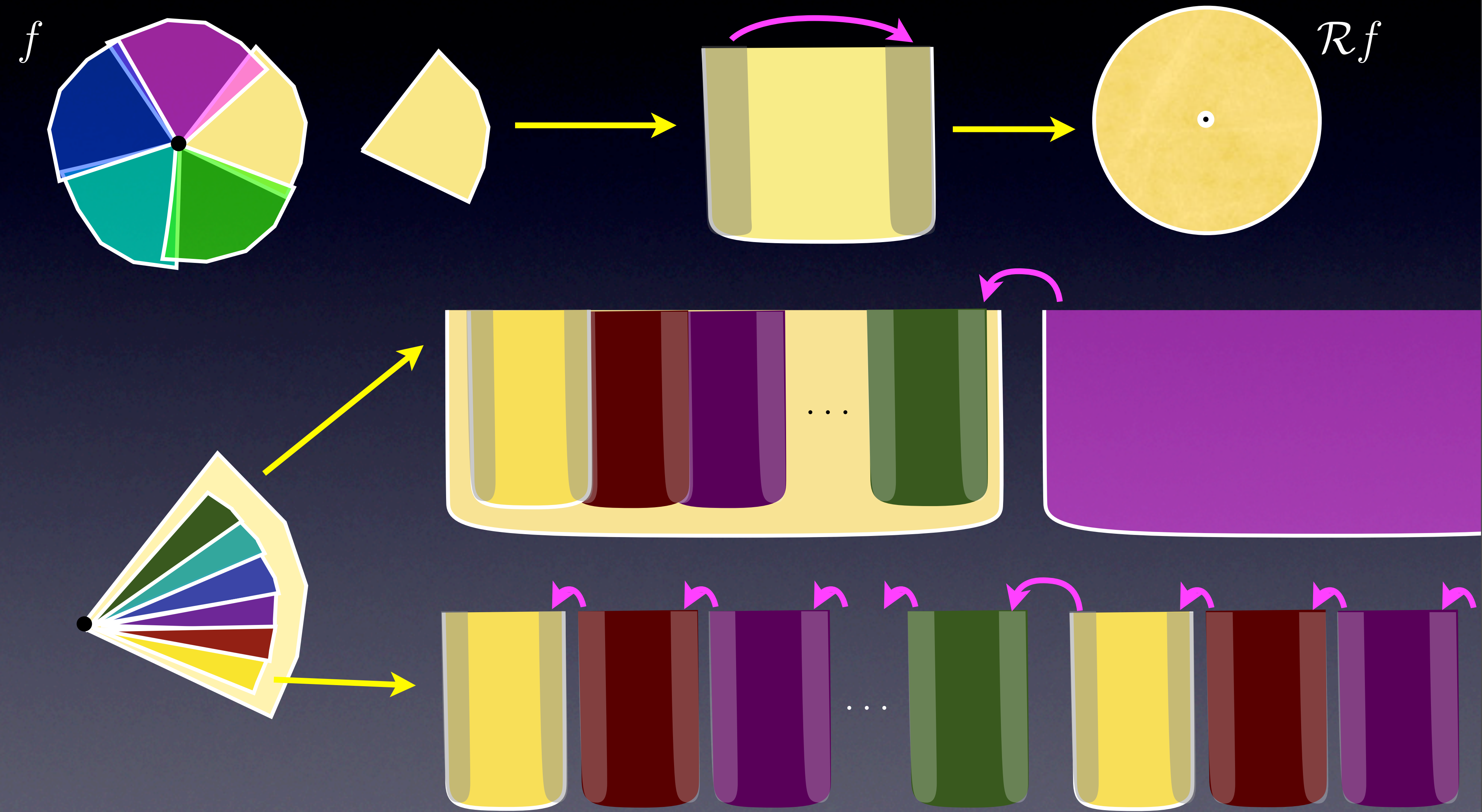

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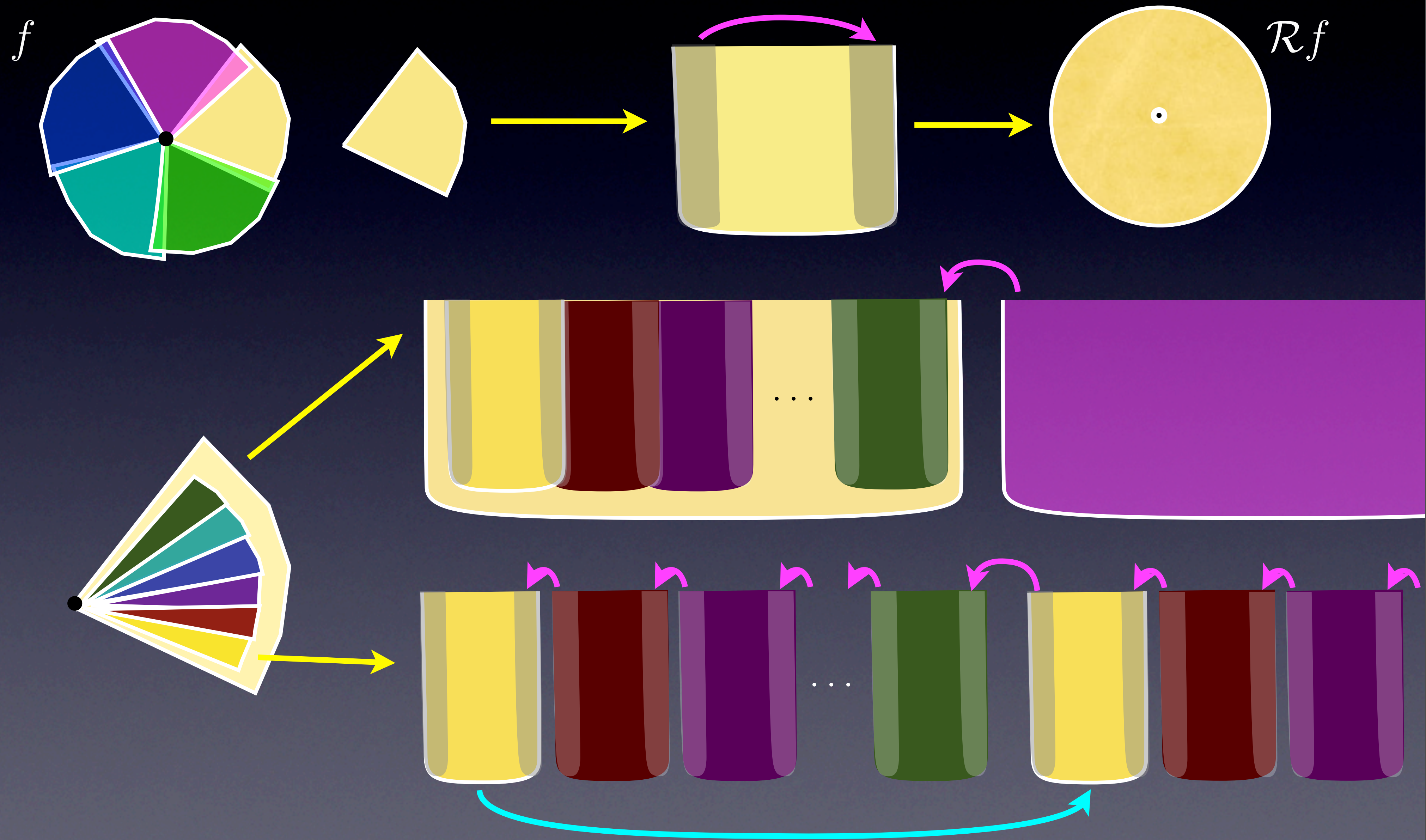

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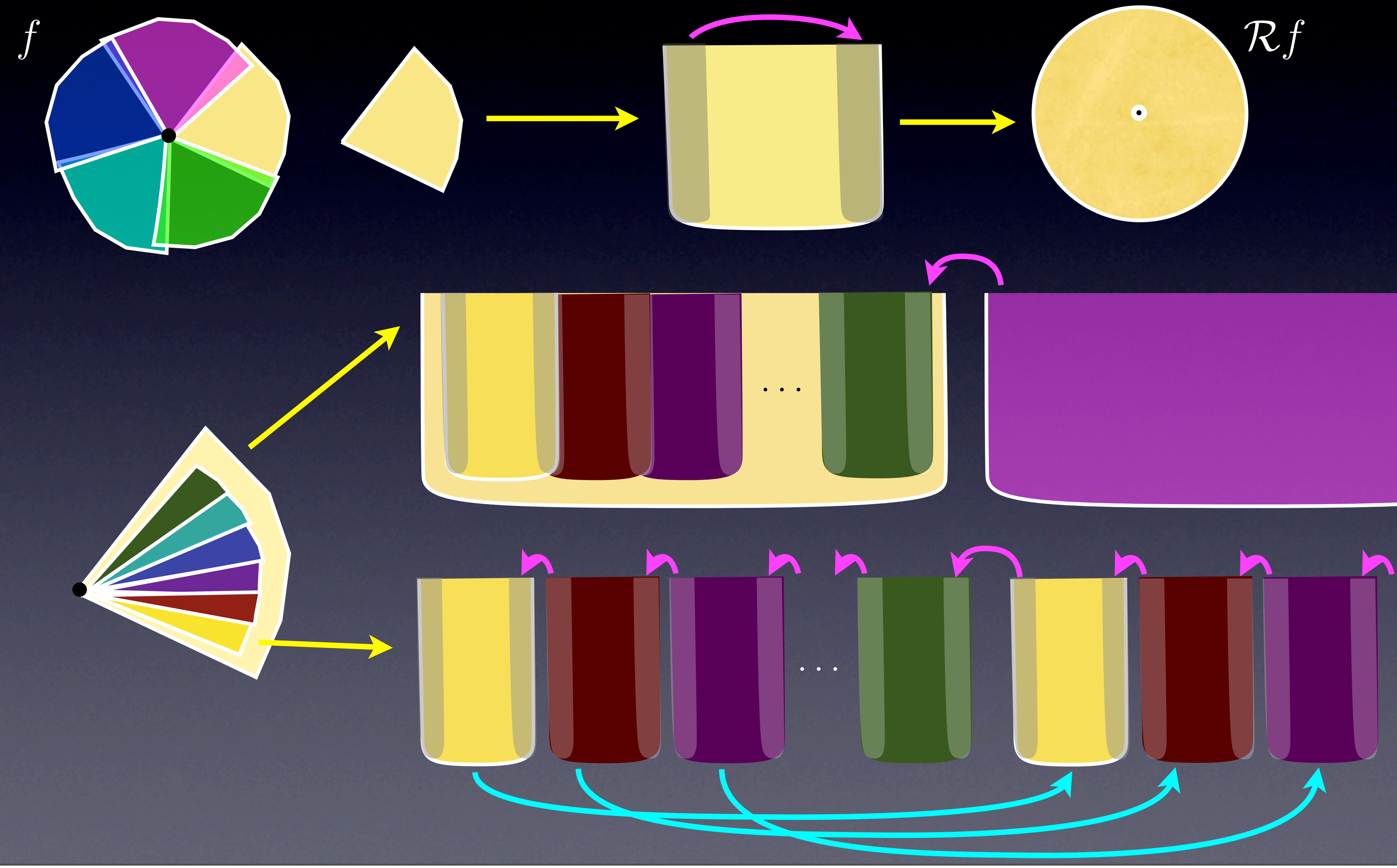

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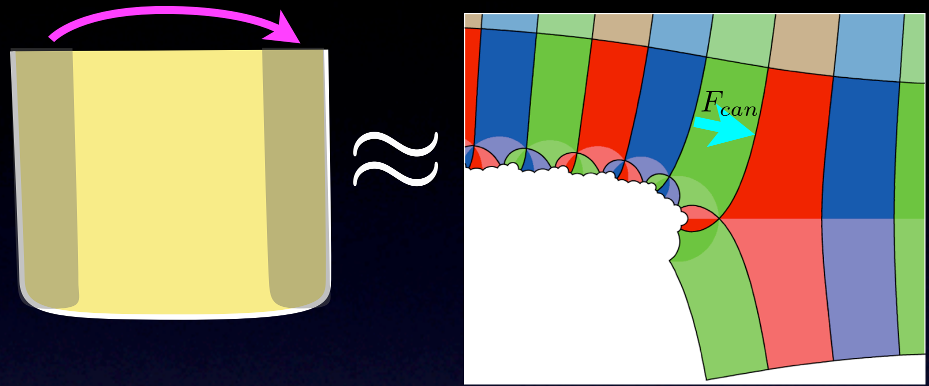

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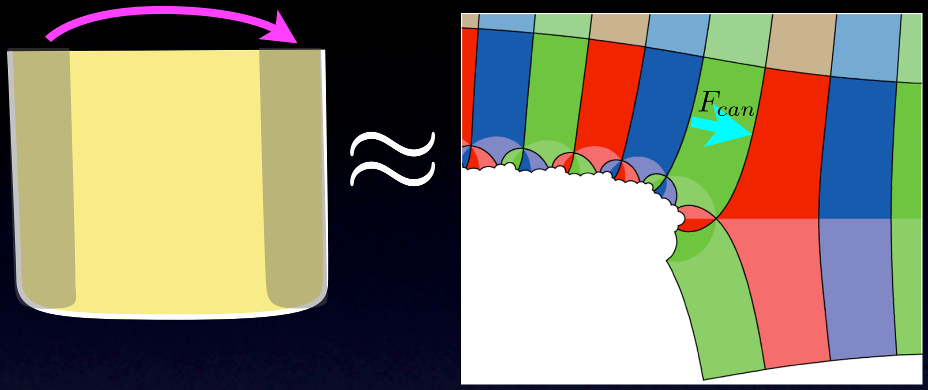

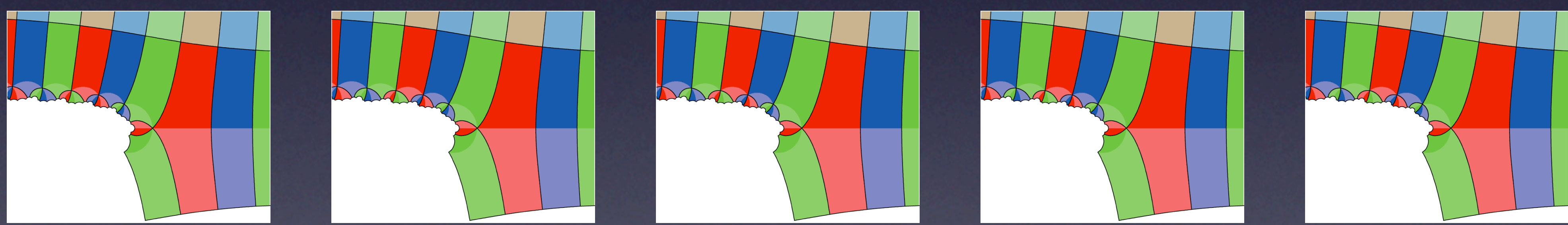

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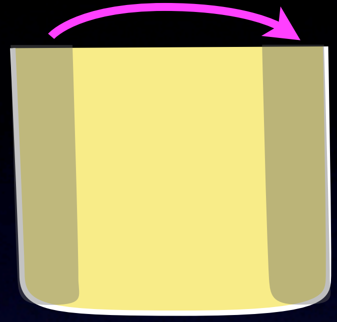
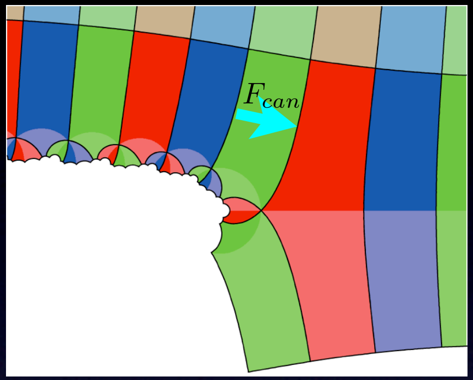
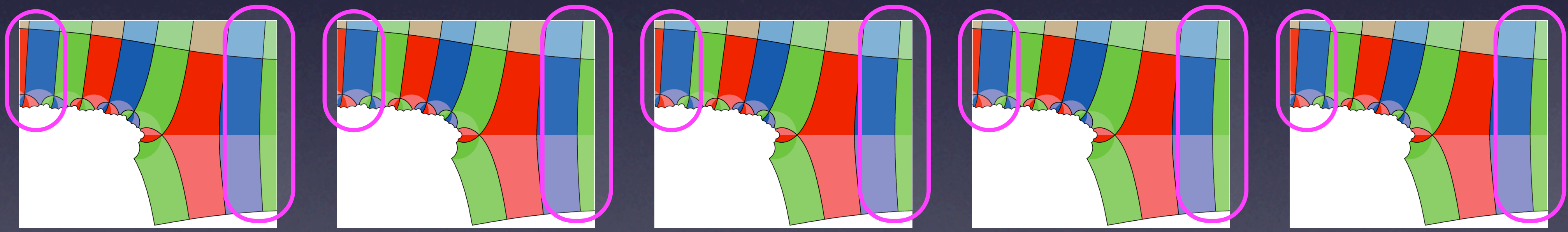


$\approx$





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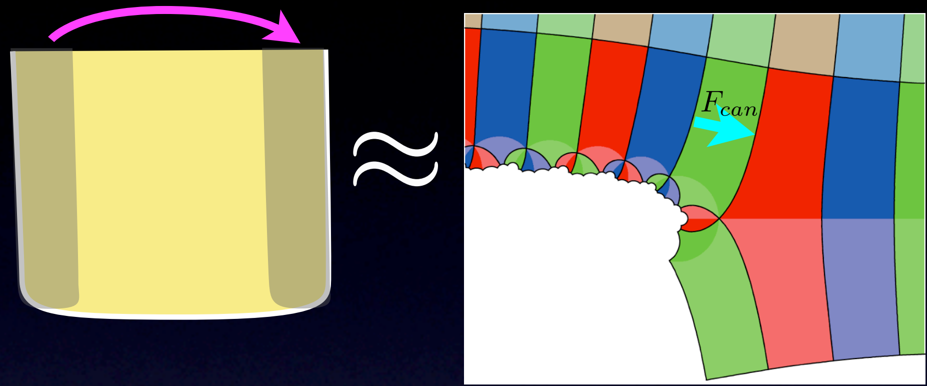

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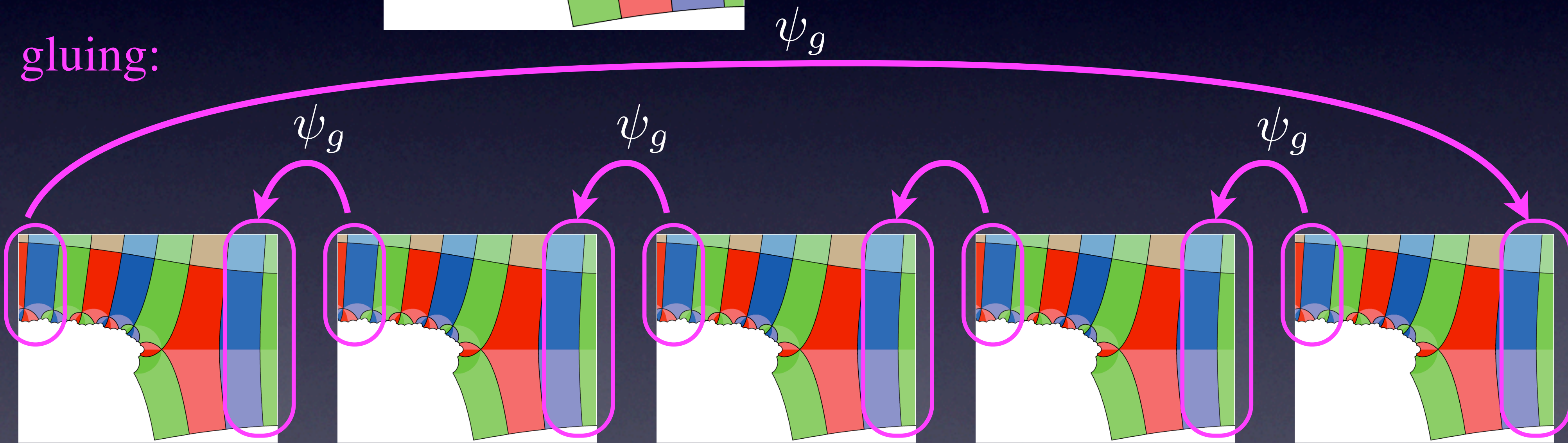

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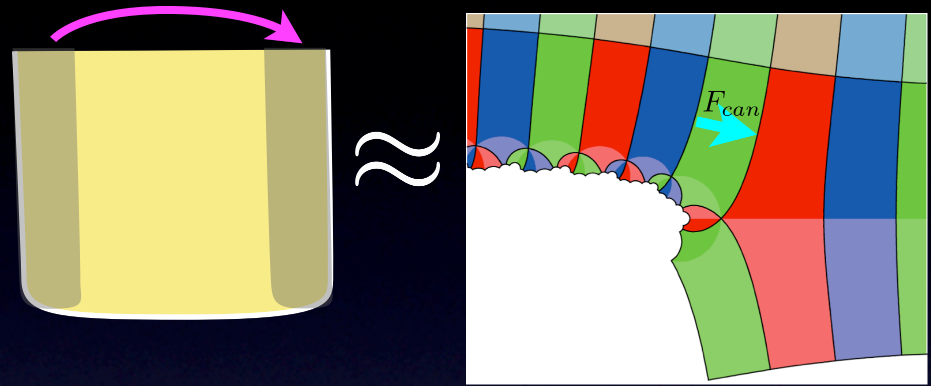

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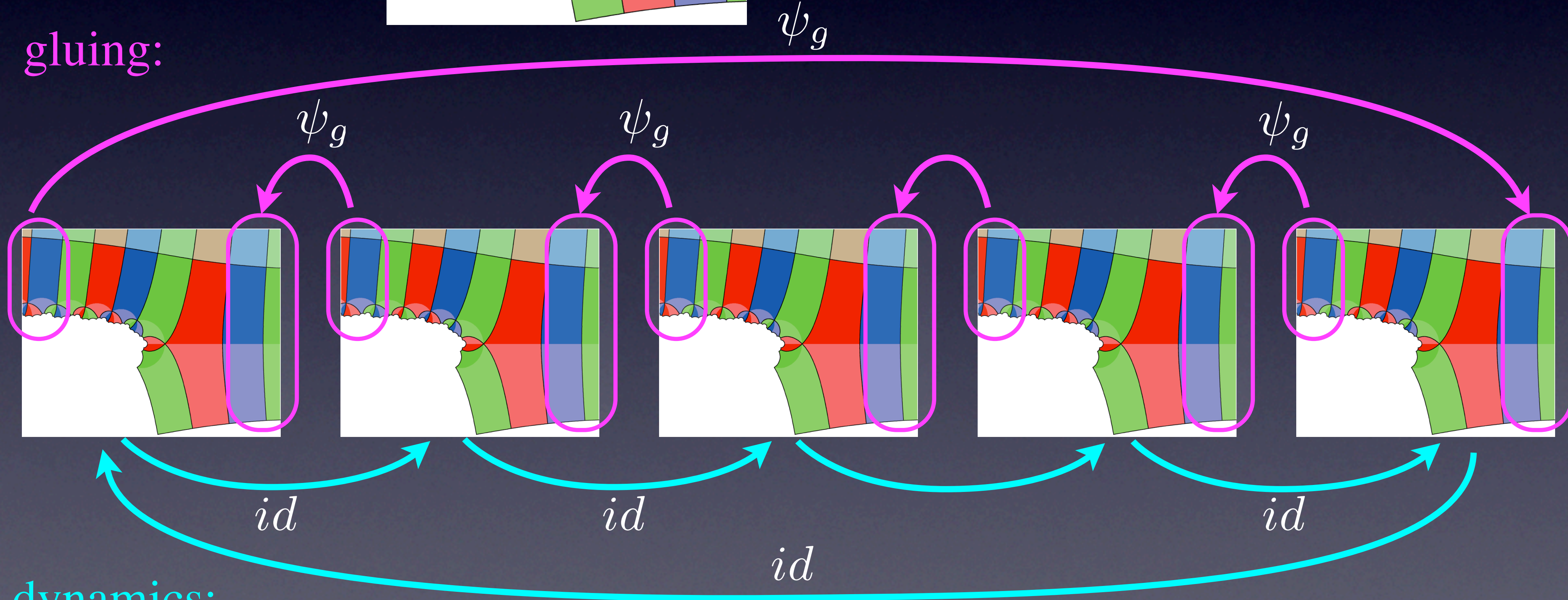

dynamics:

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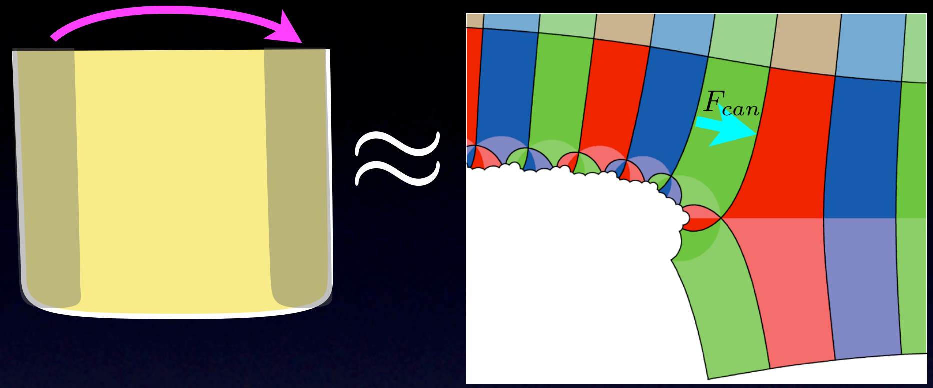

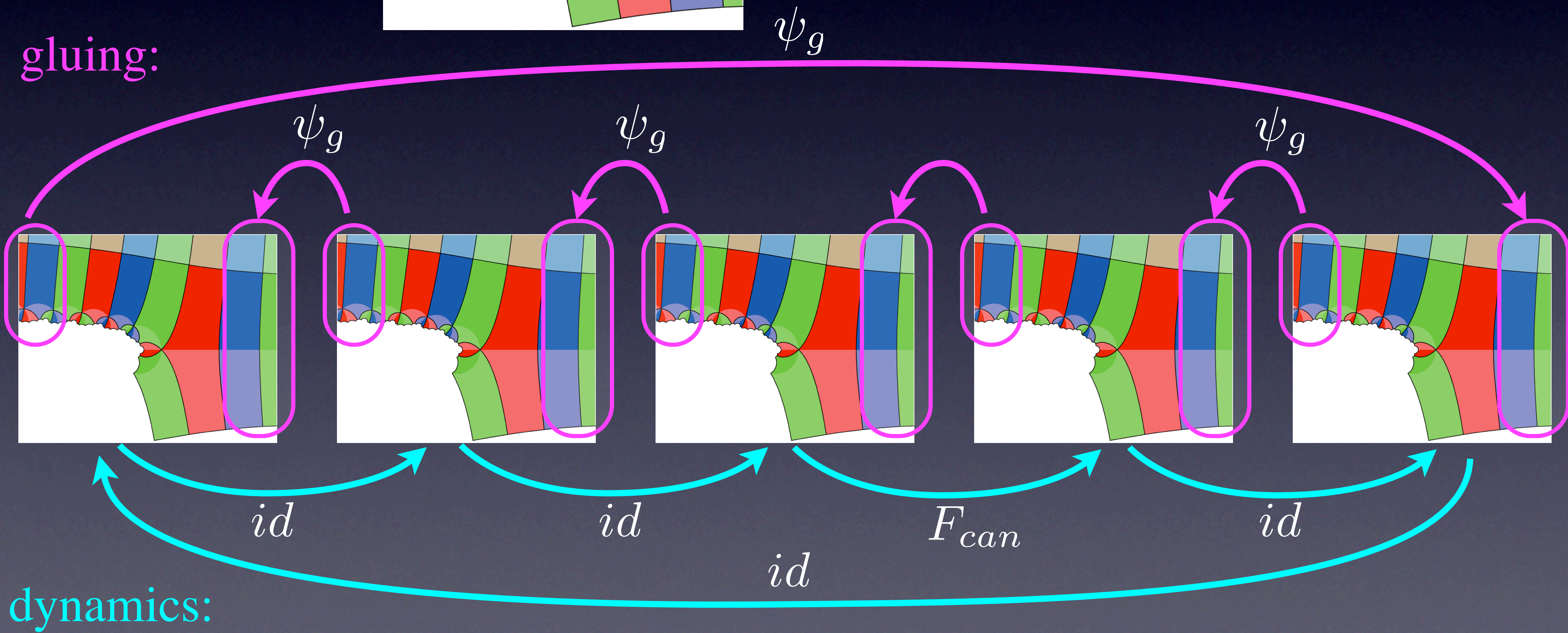

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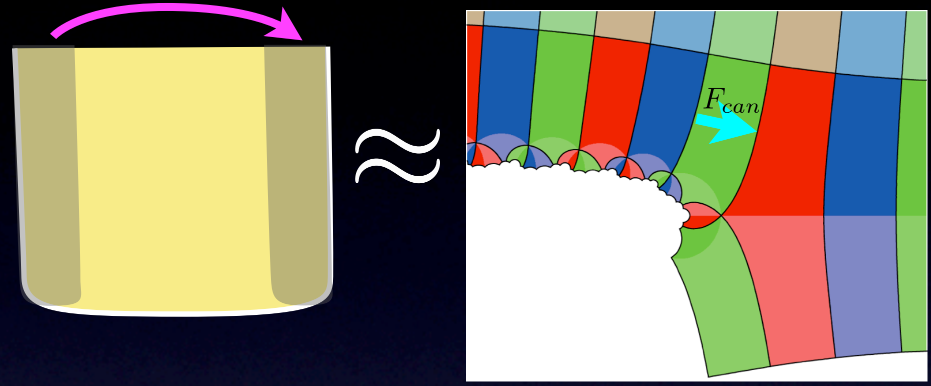

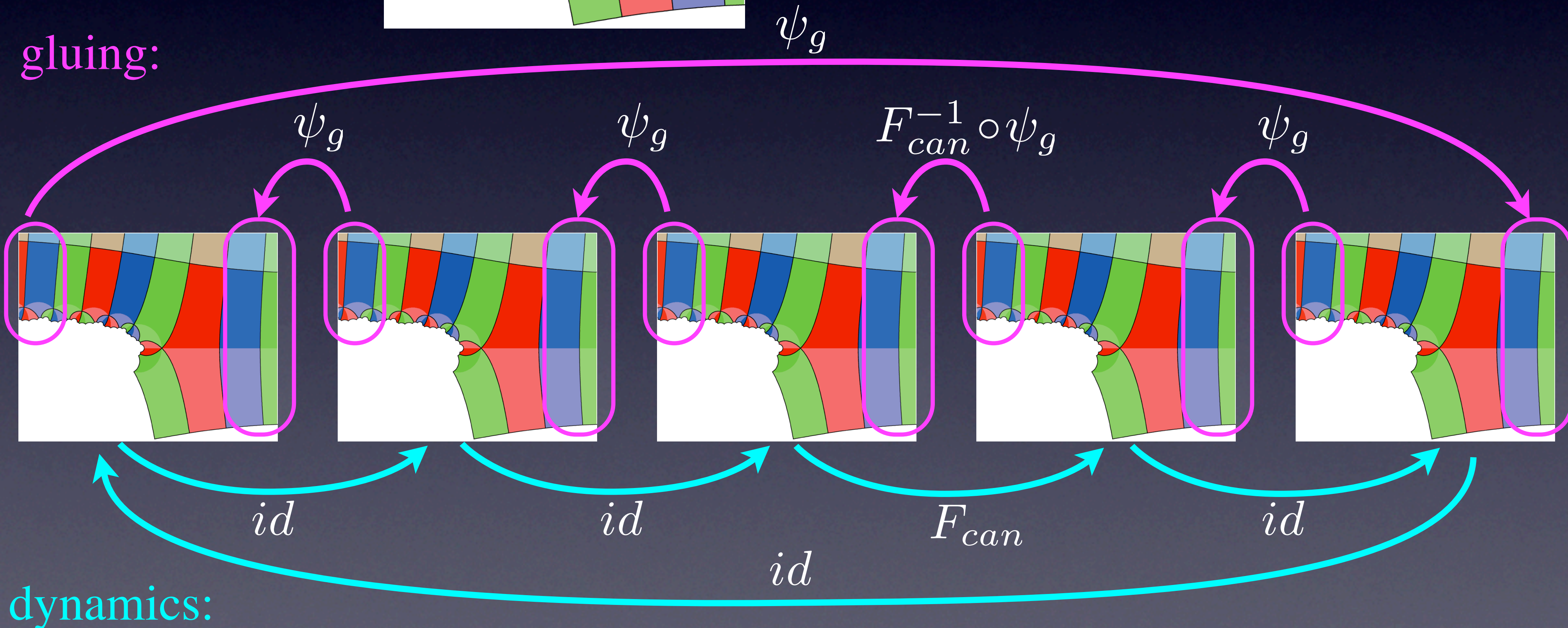

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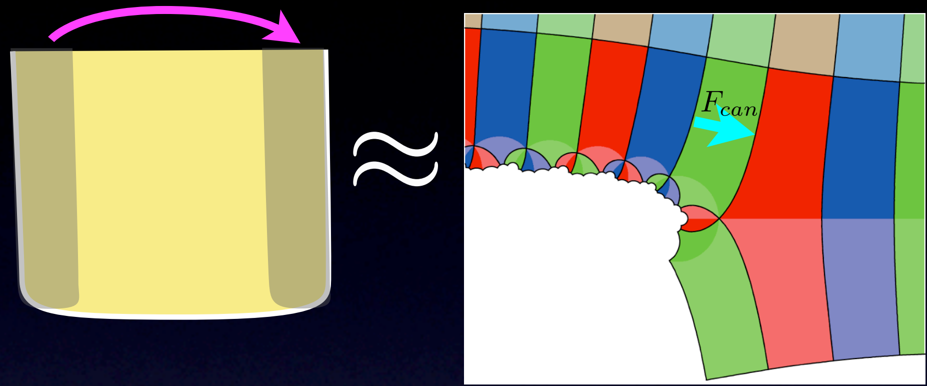
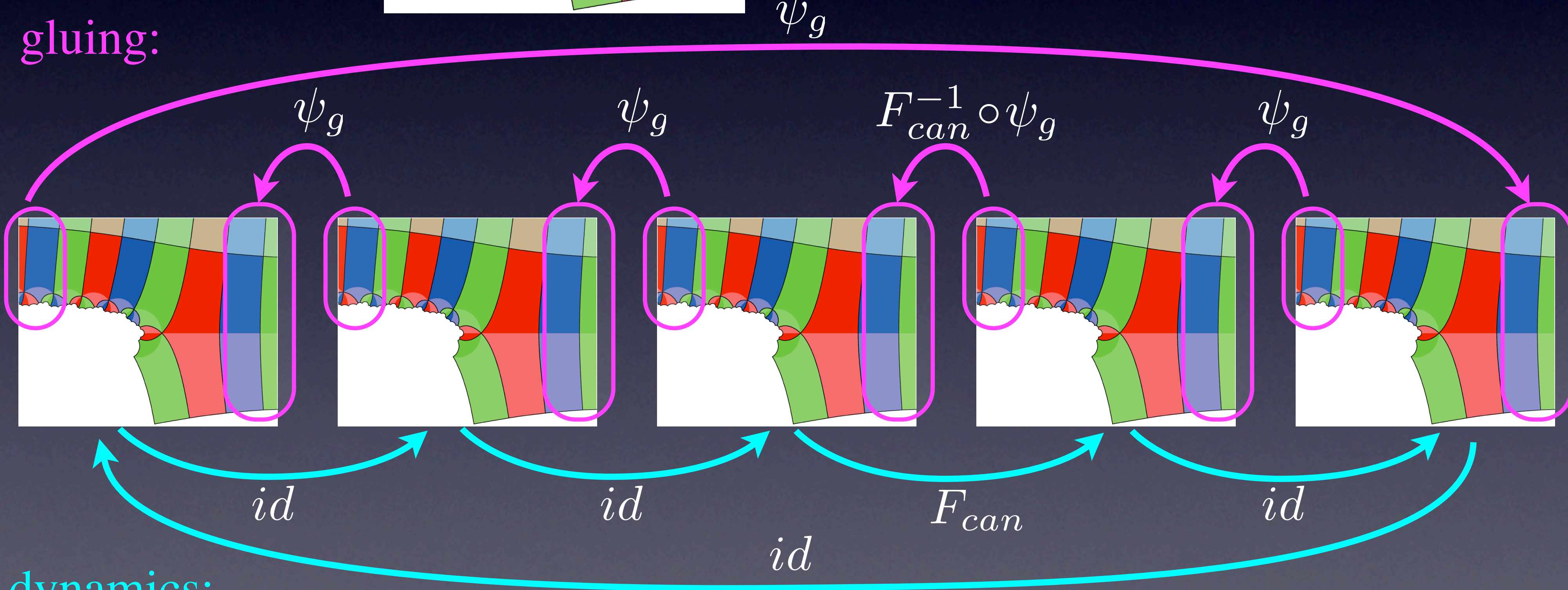


gluing:



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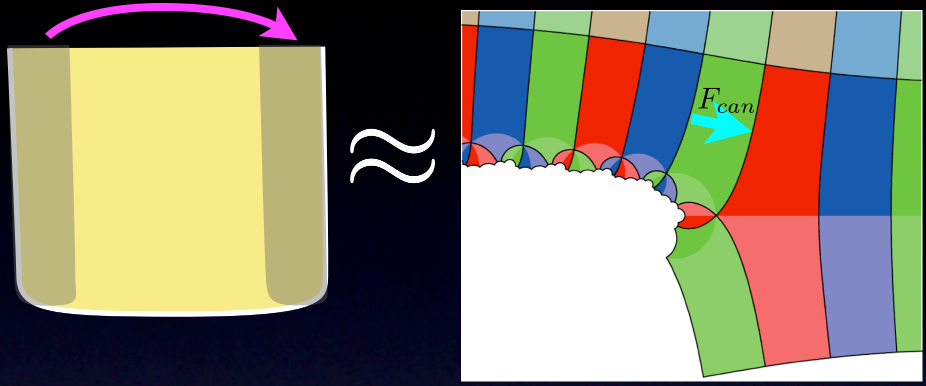

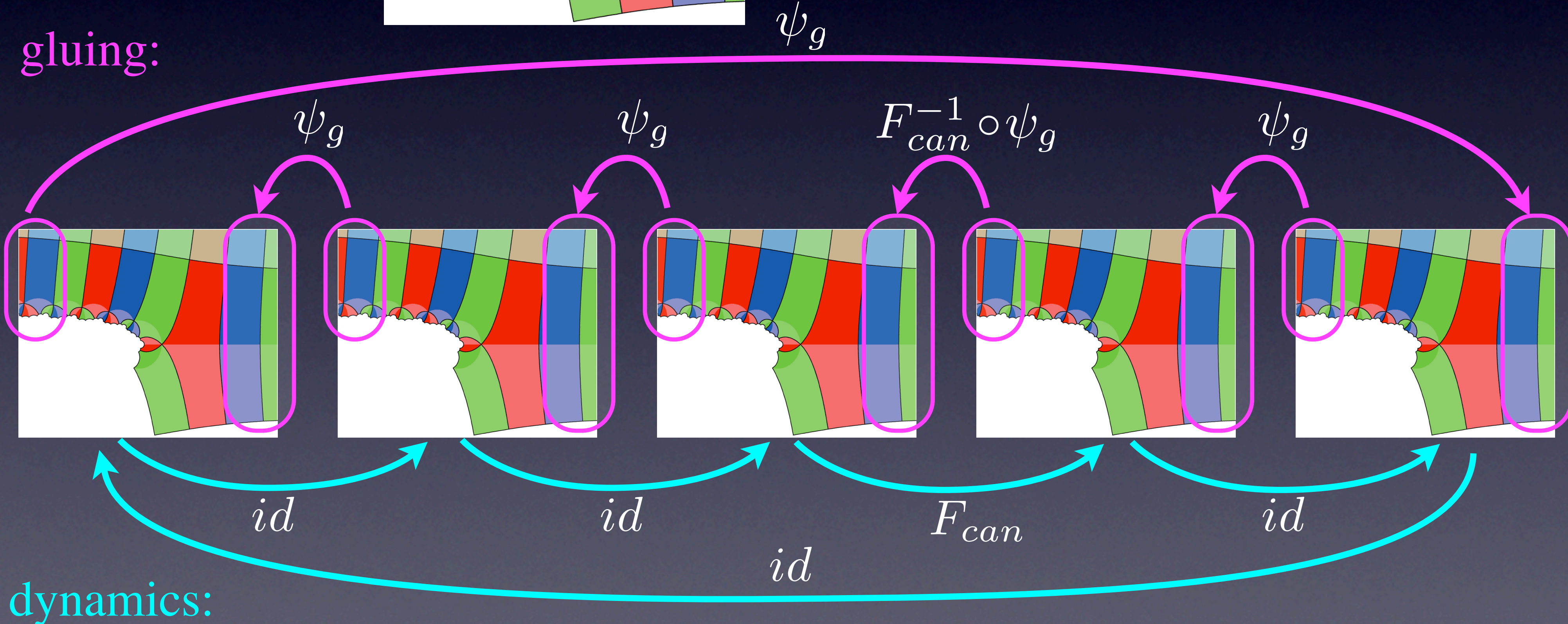

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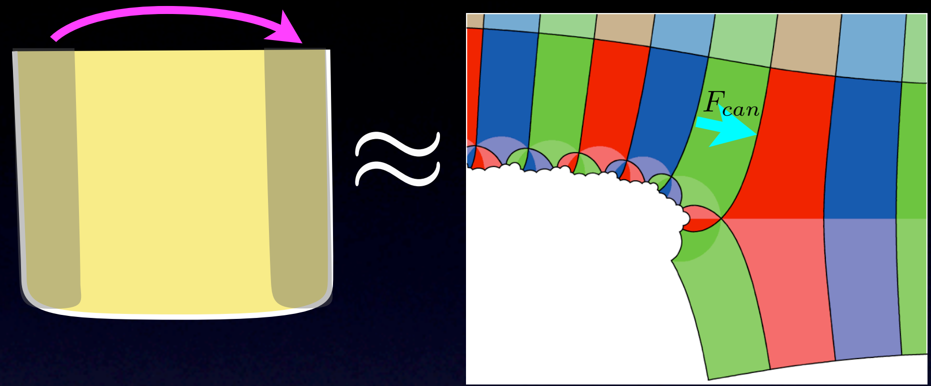

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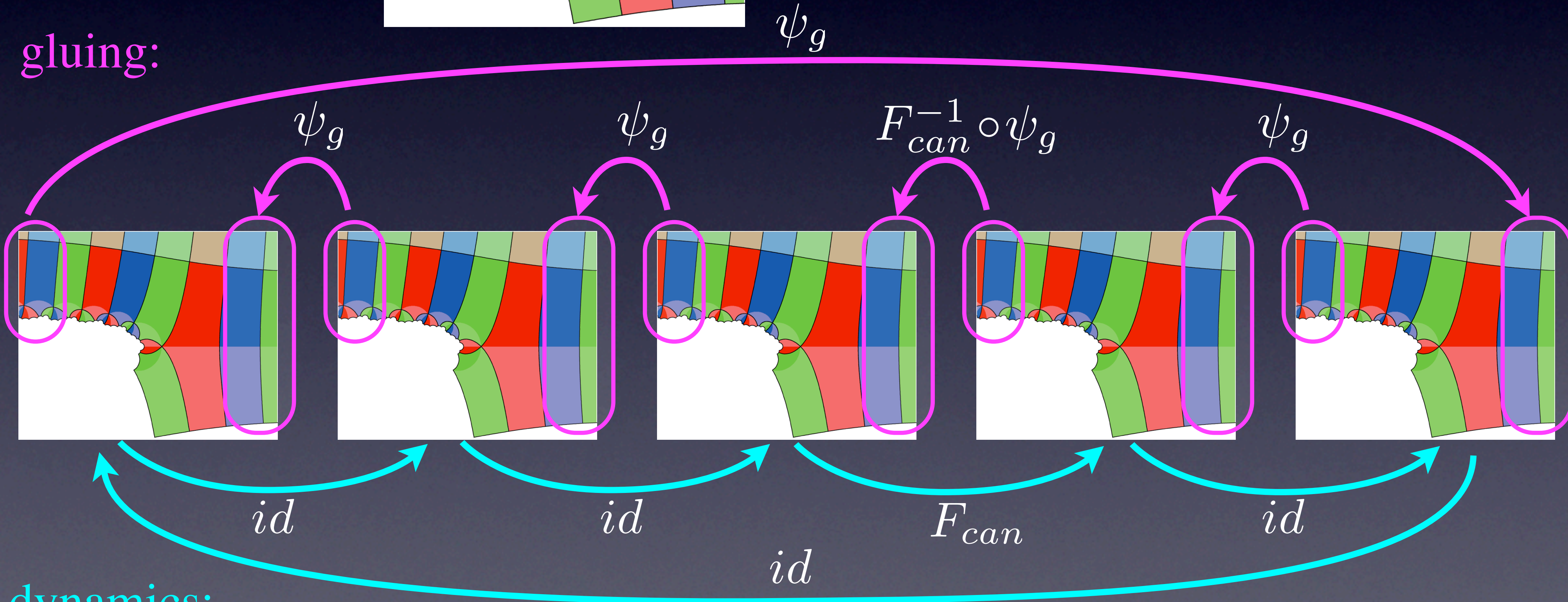

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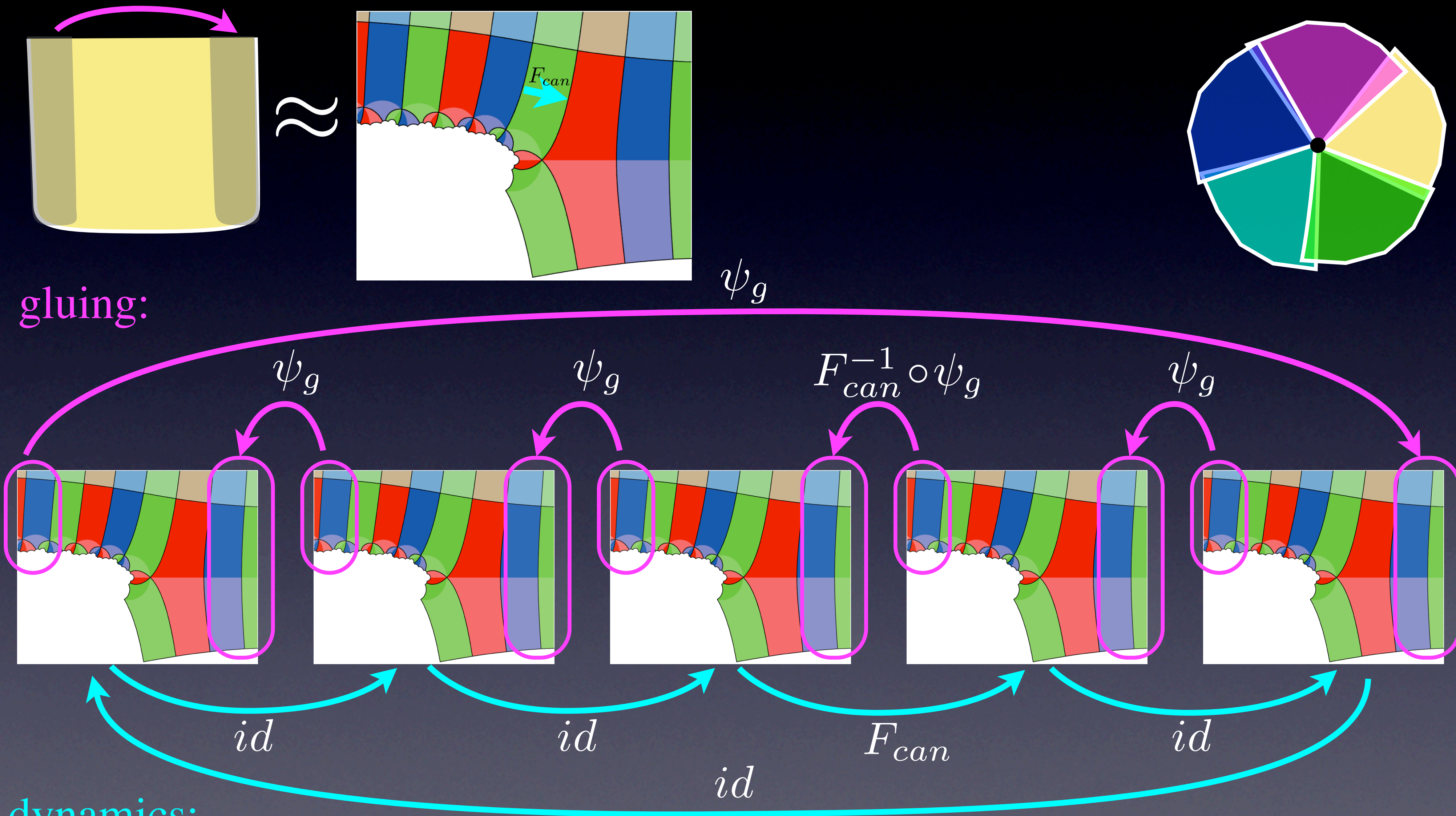

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Construction of successive charts

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f

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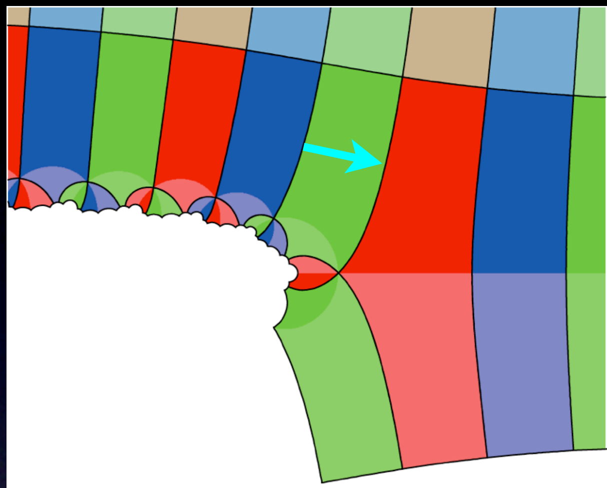
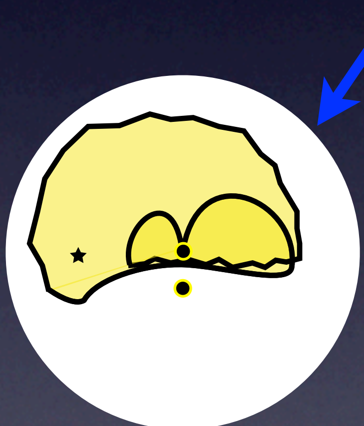
Ω_f





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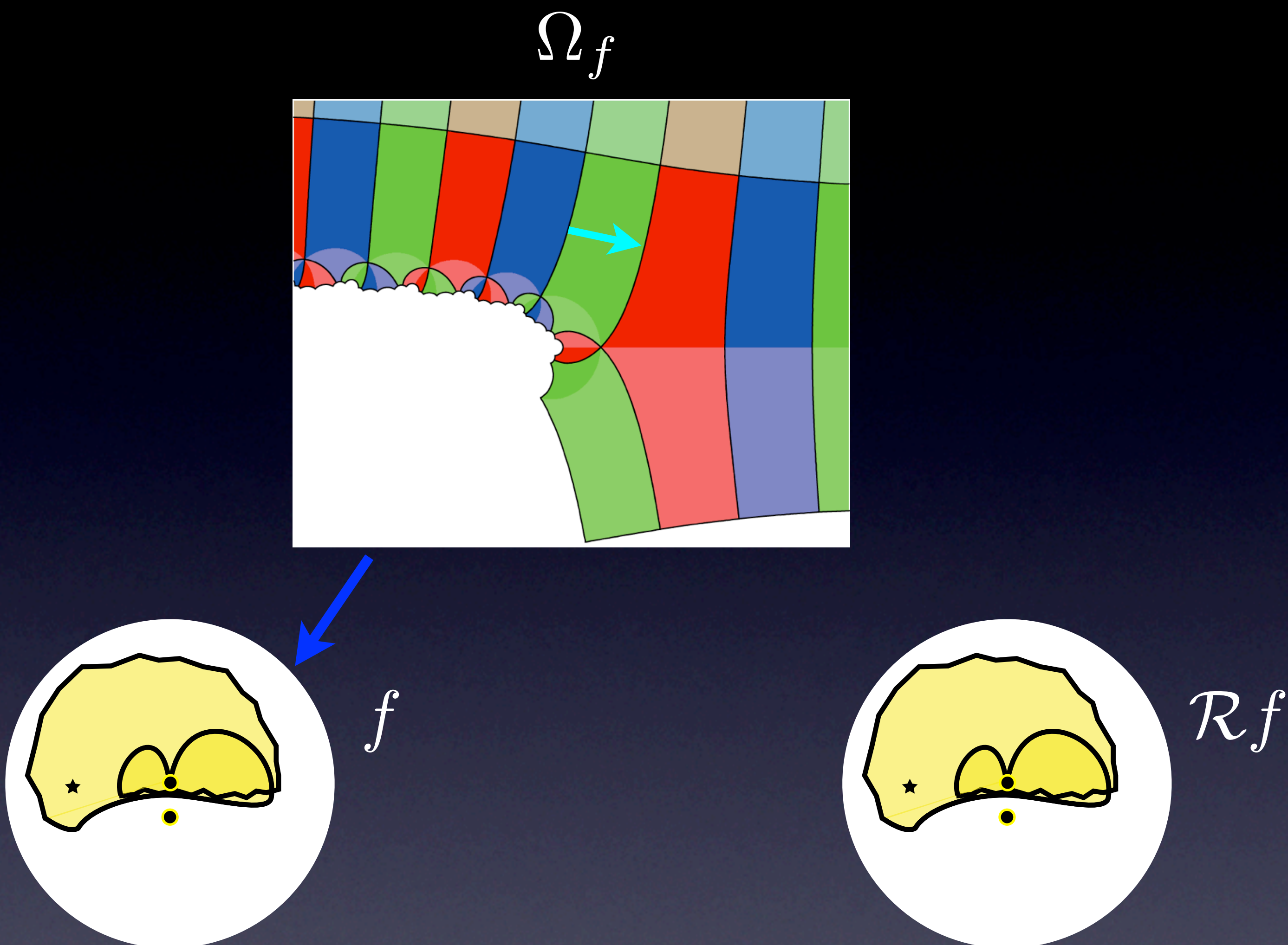

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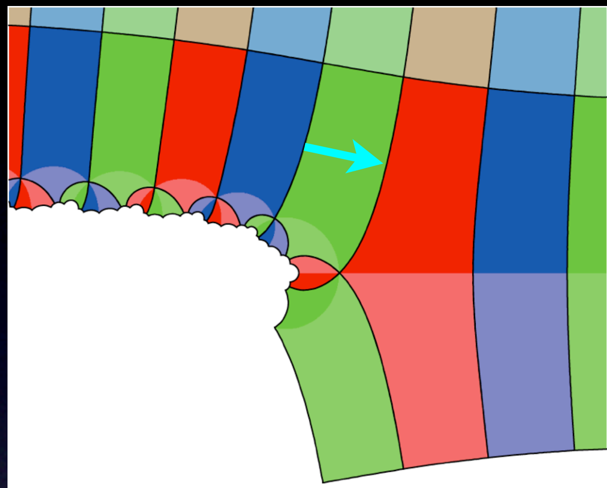

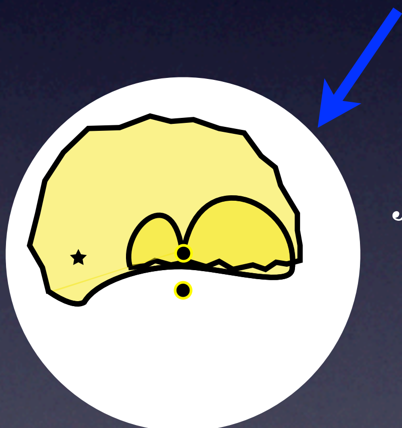

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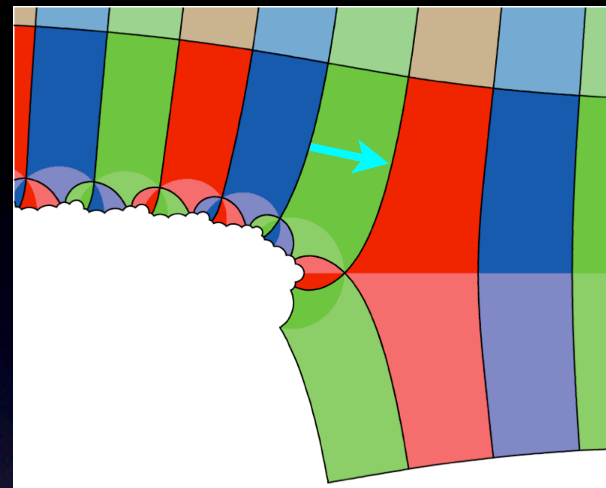
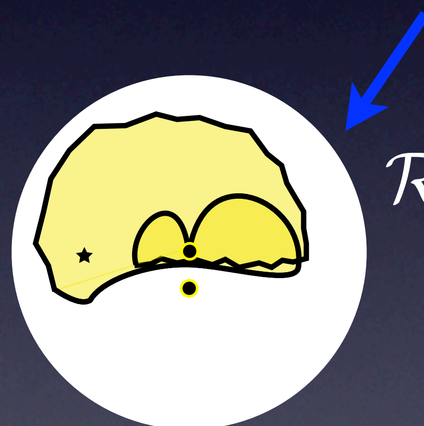
$\Omega_{\mathcal{R}f}$





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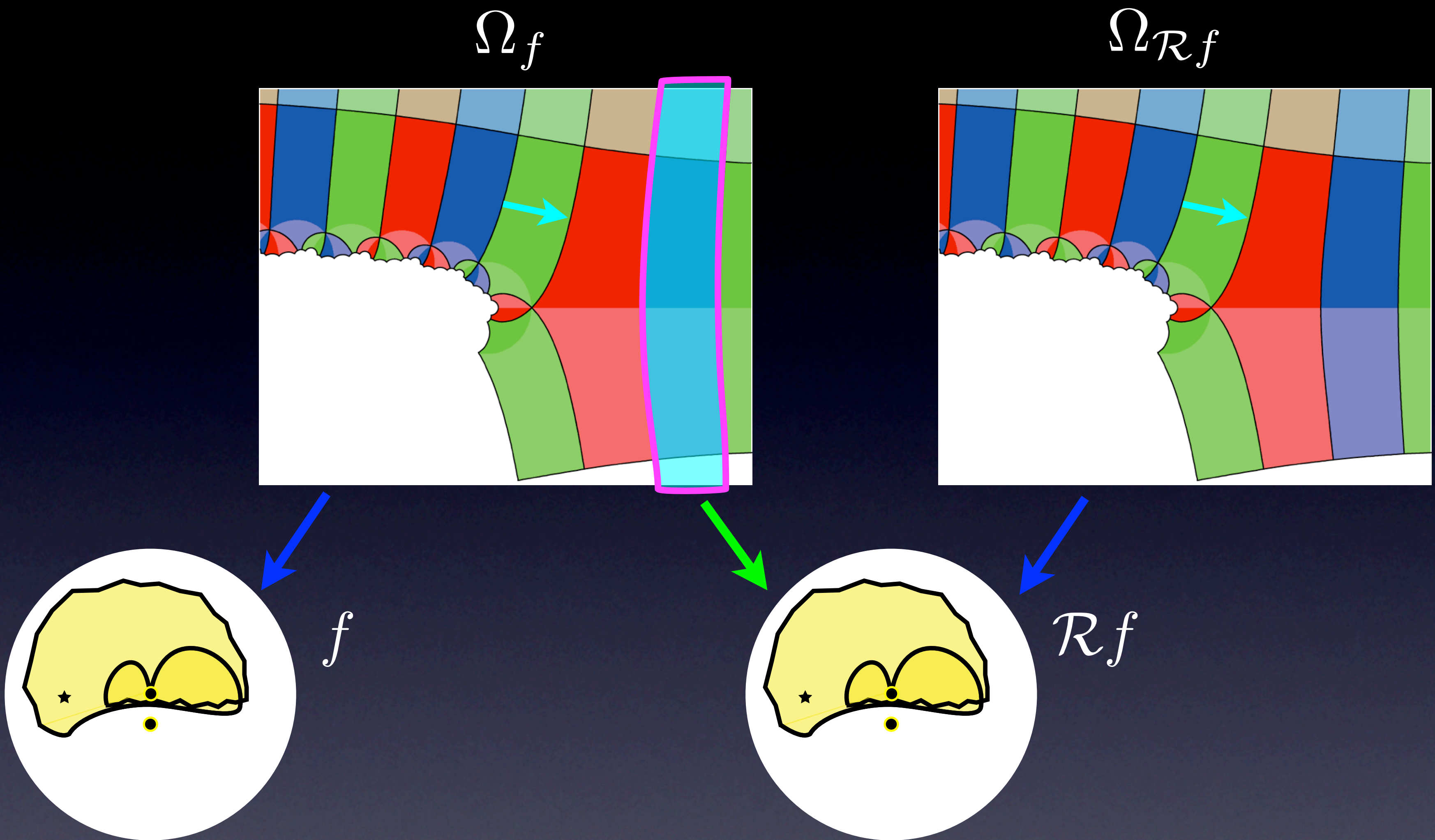

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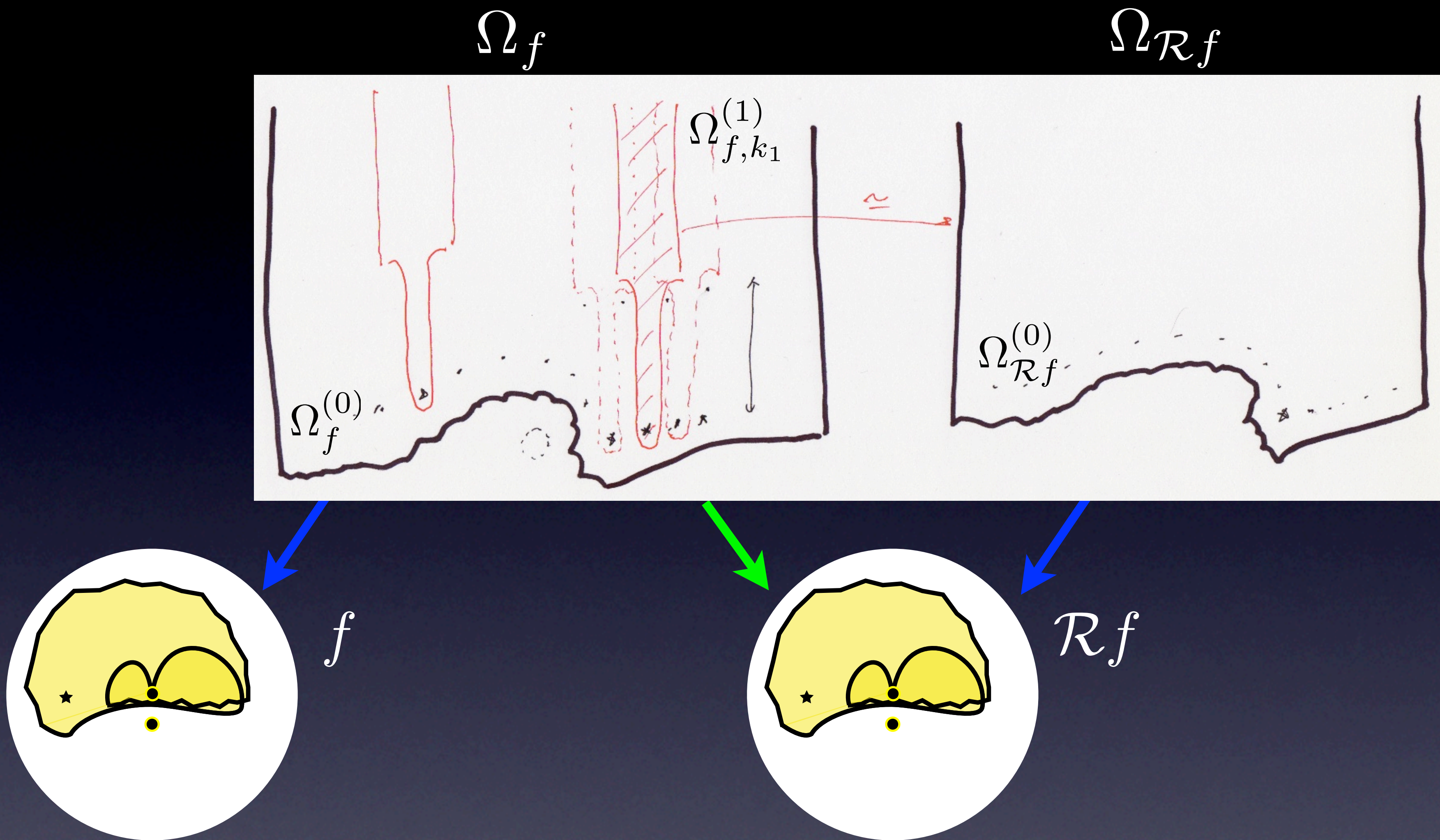

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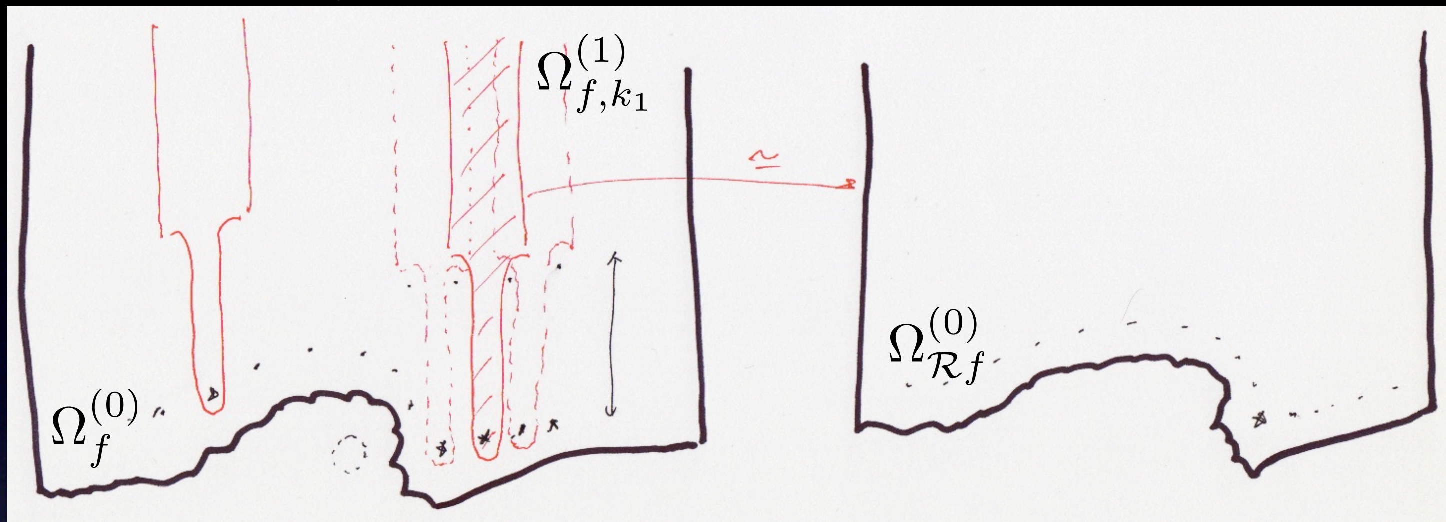

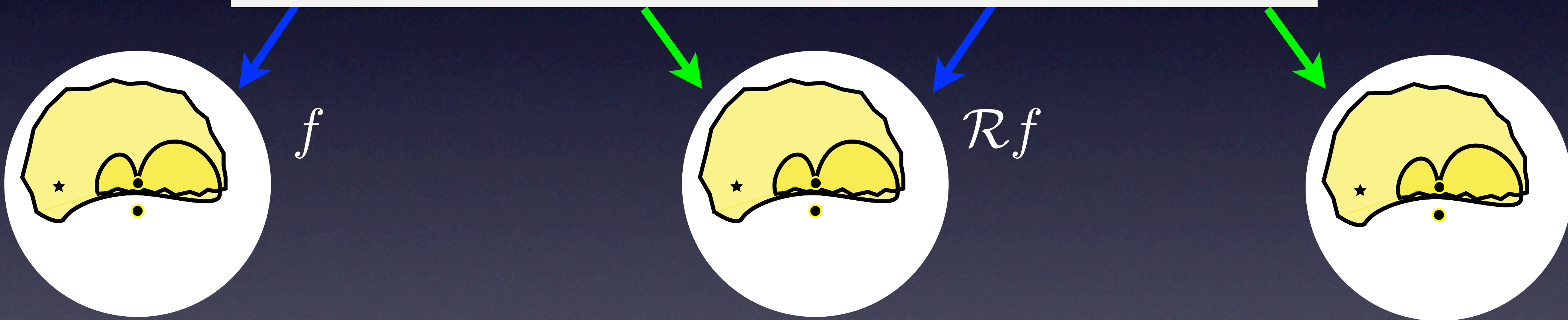

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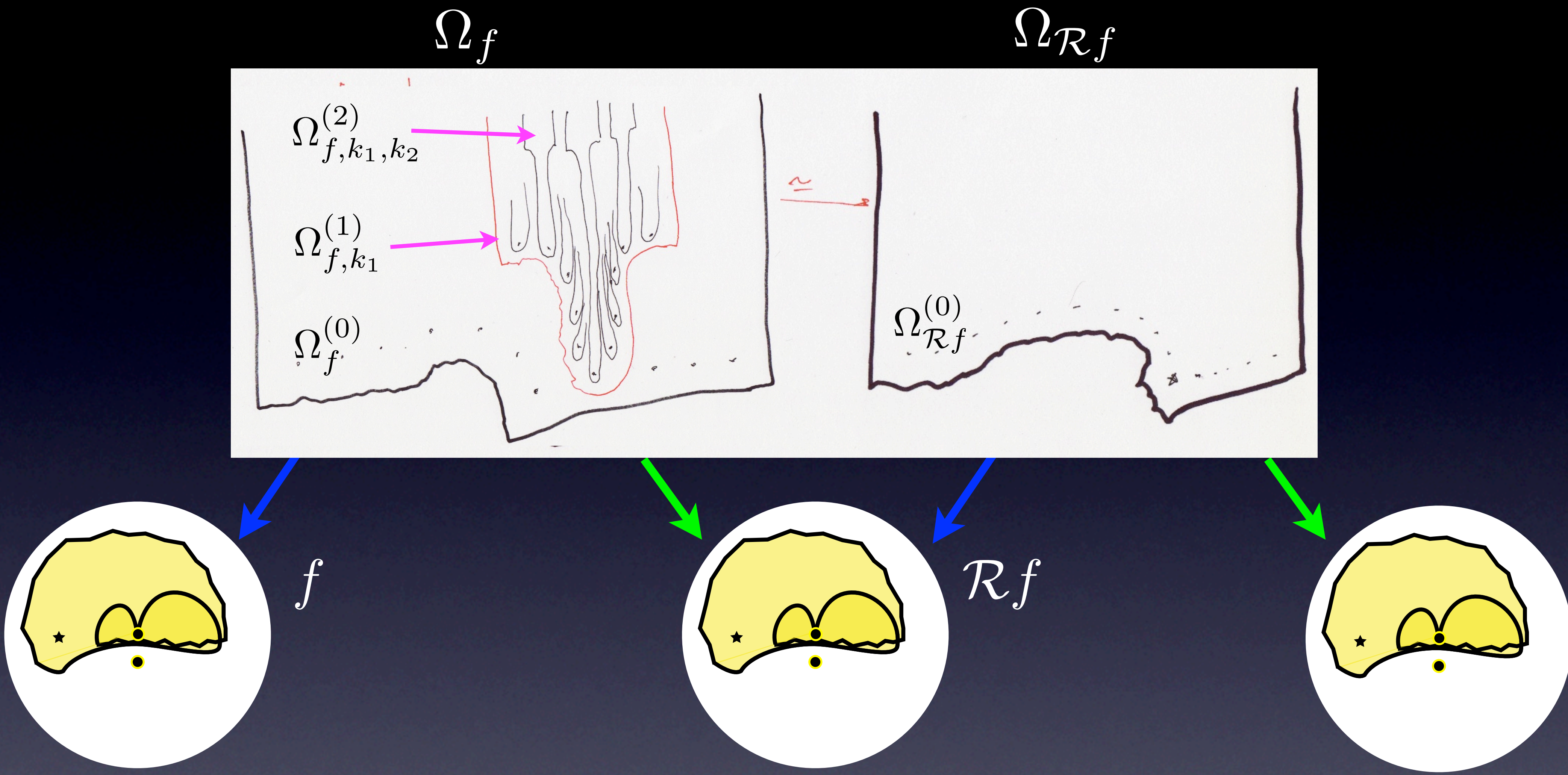

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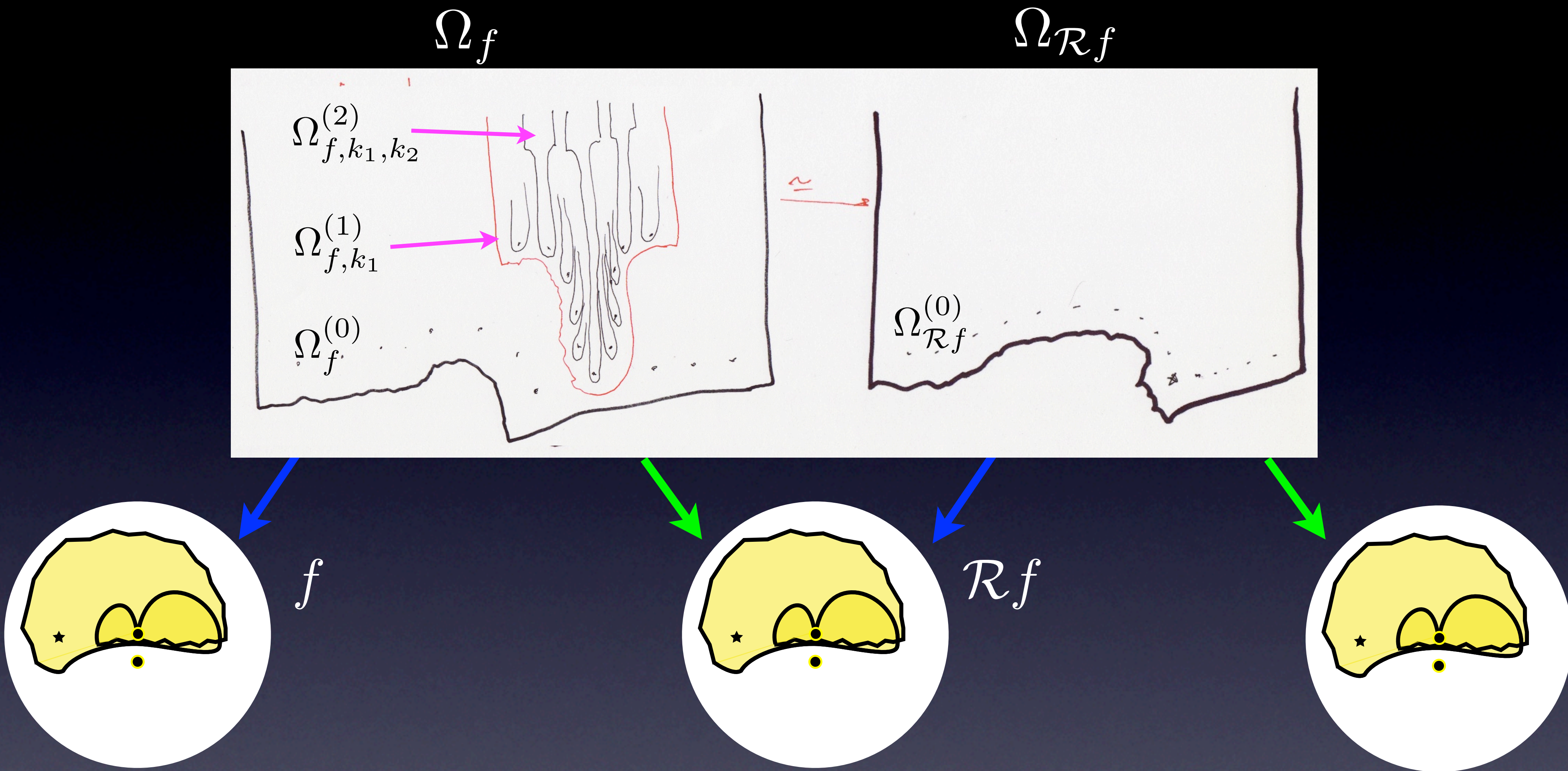

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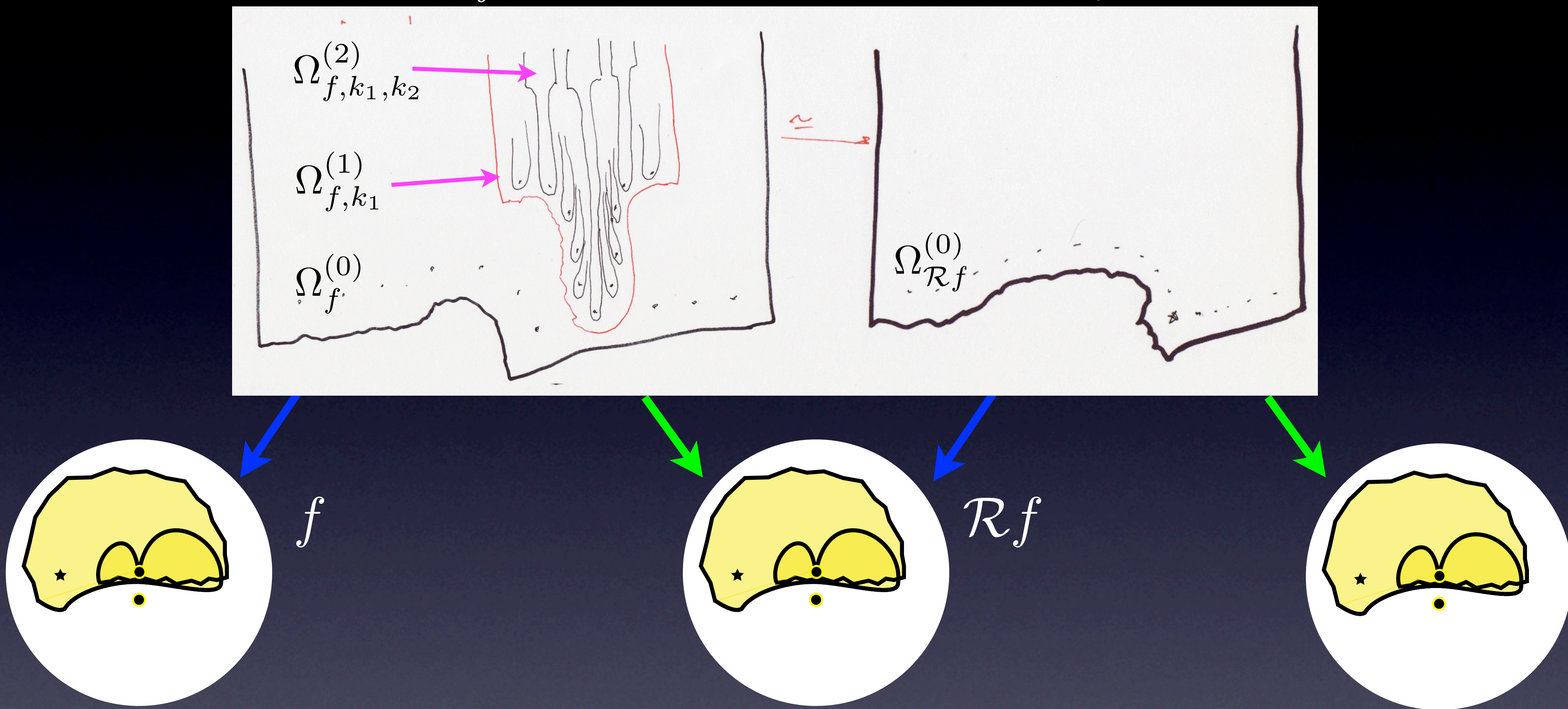

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each $\Omega^{(n)}_{k_1,k_2,\ldots,k_n}$ is isomorphic to truncated checkerboard pattern $\Omega_{\mathcal{R}^n f}$

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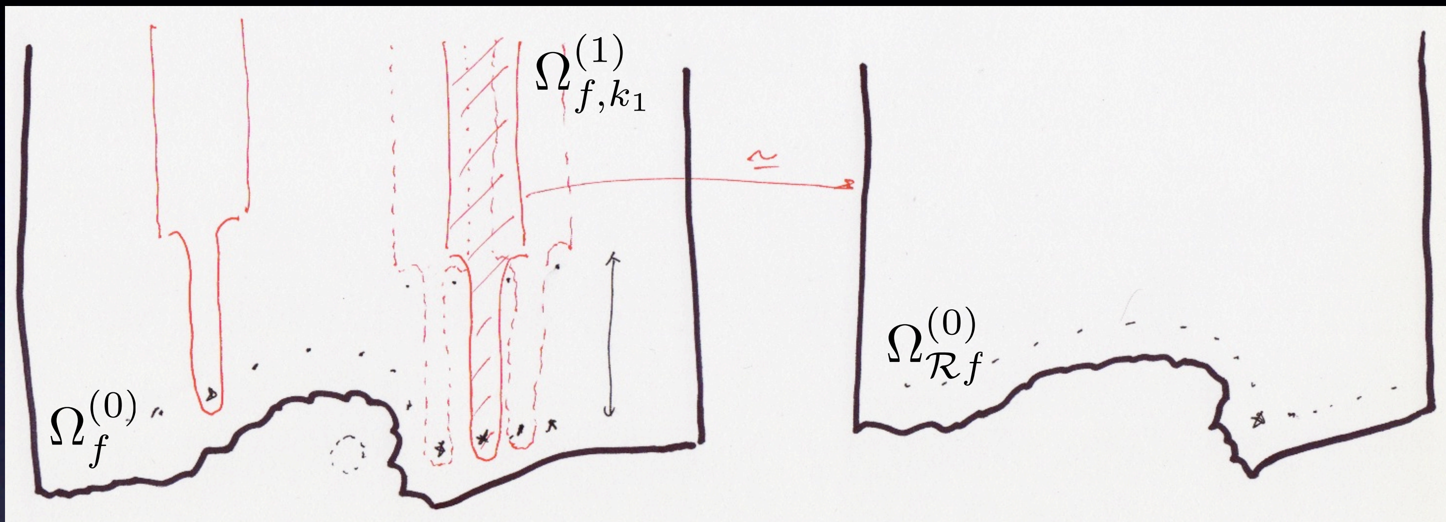

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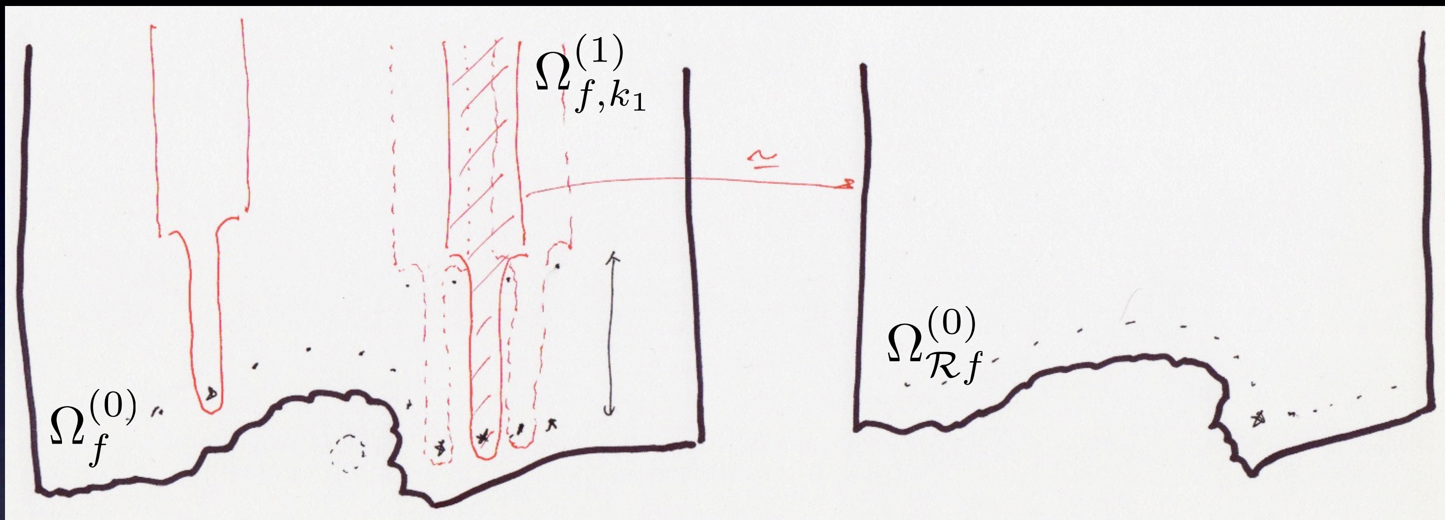

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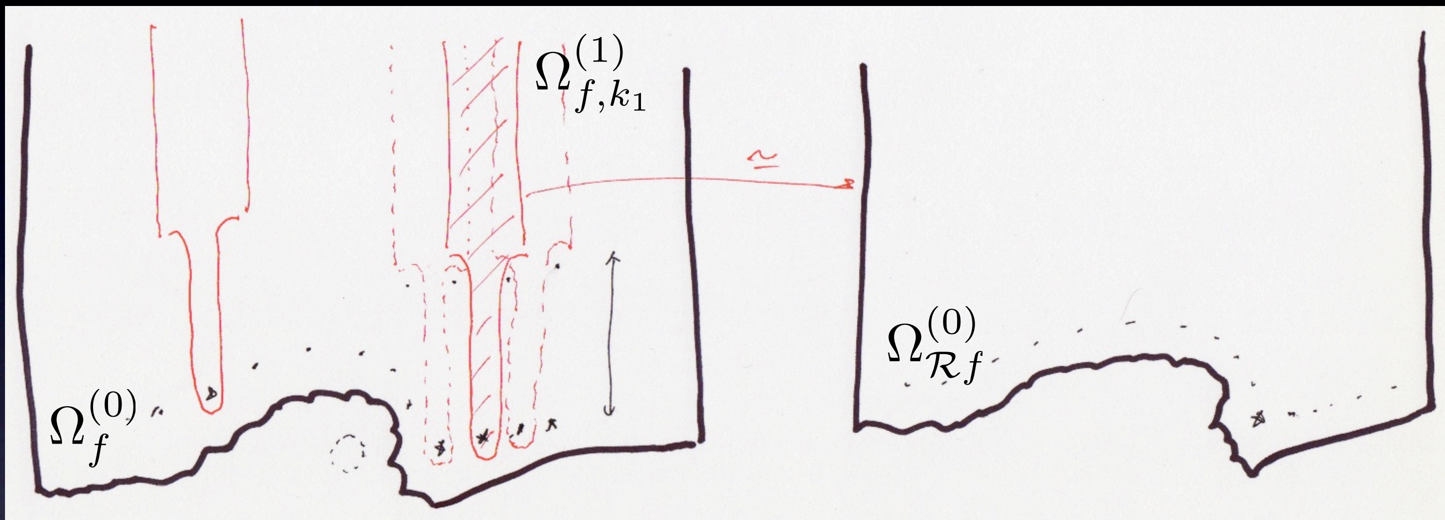

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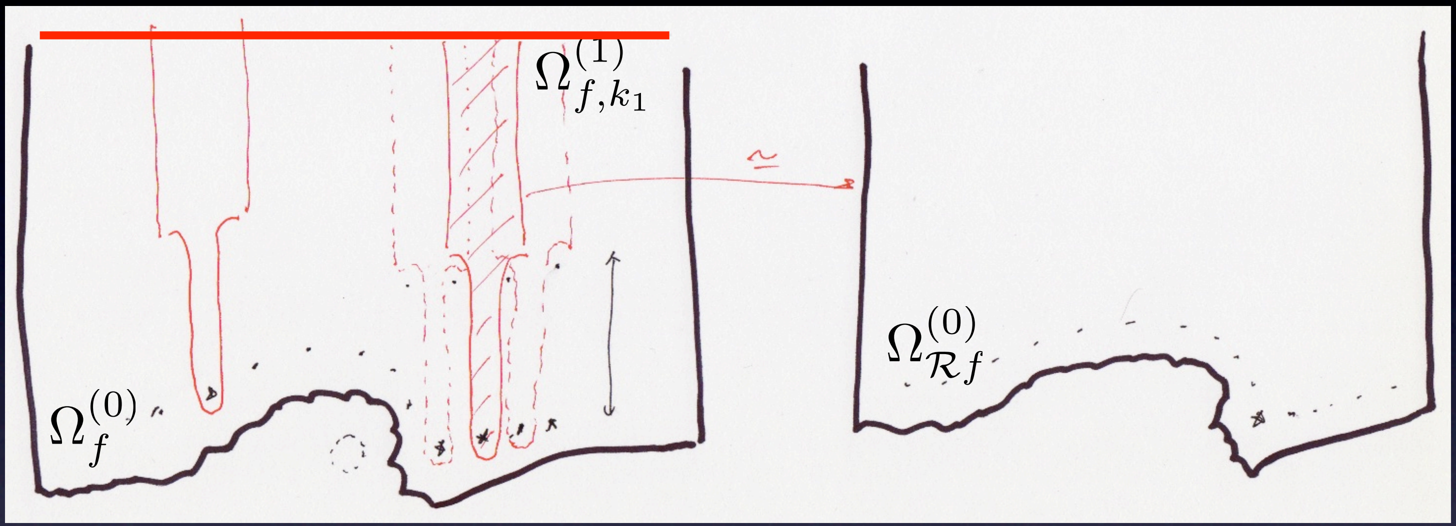

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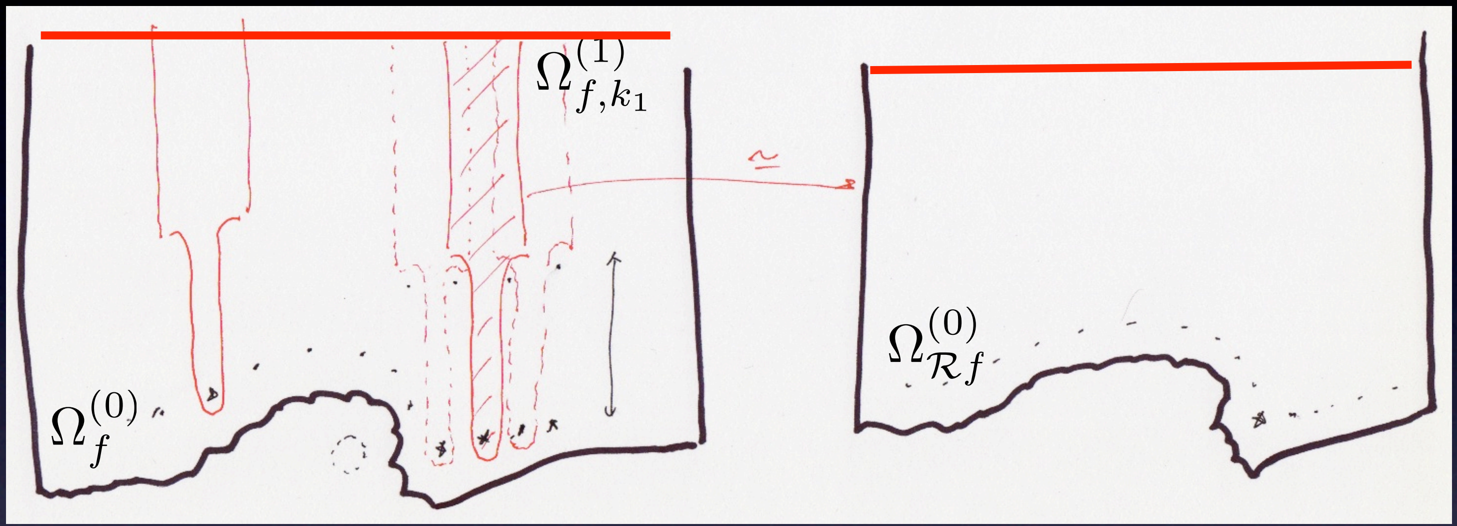

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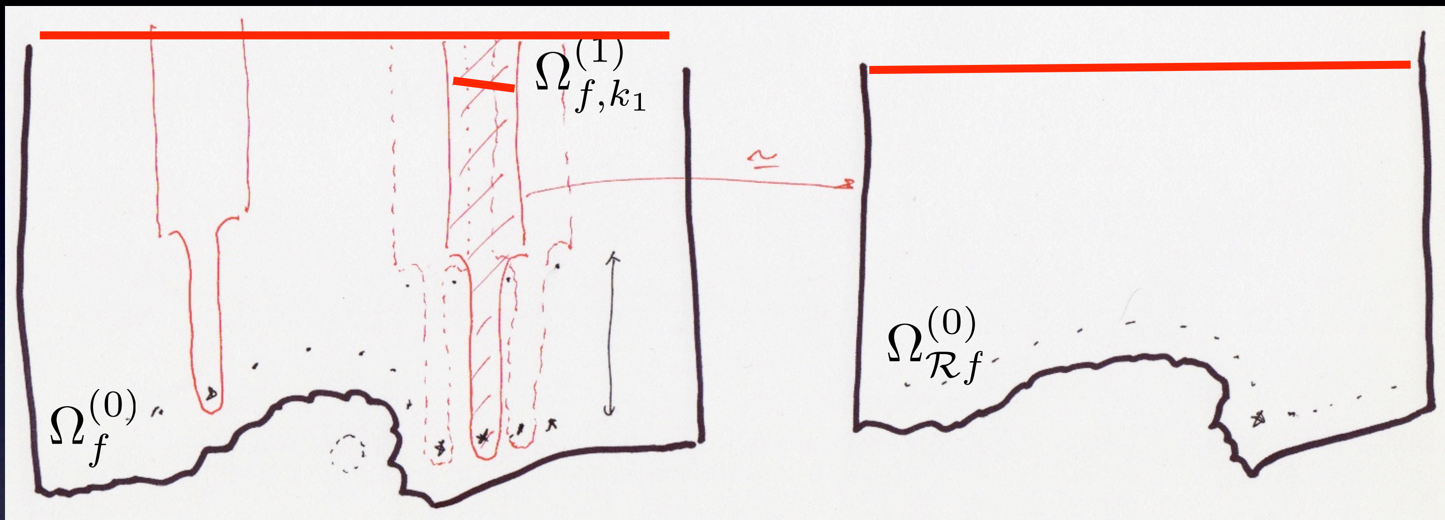

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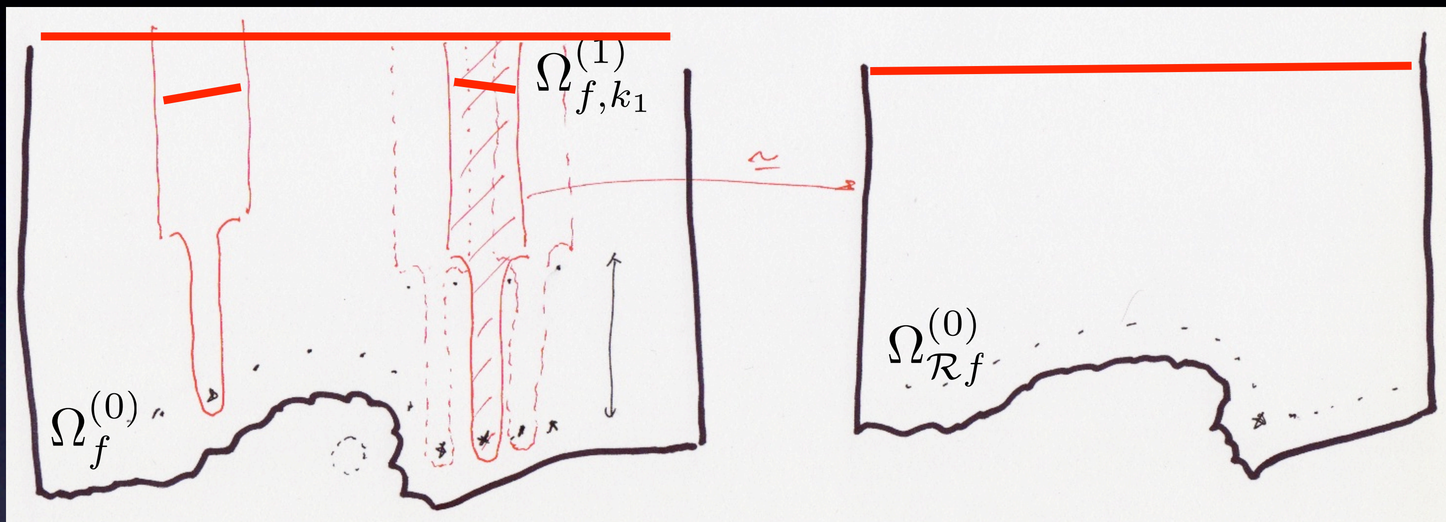

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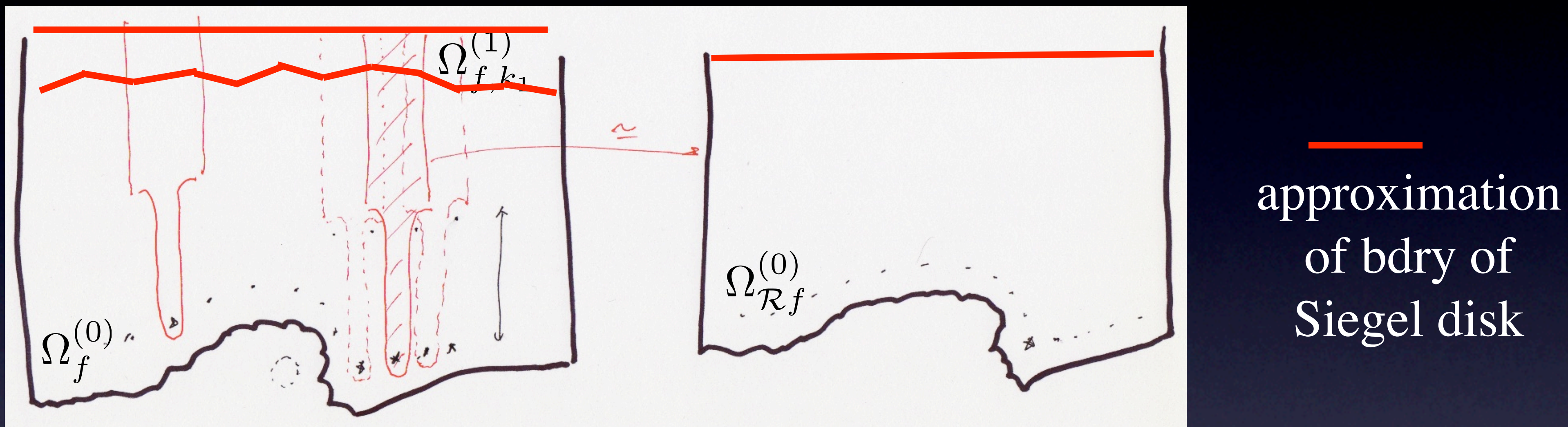

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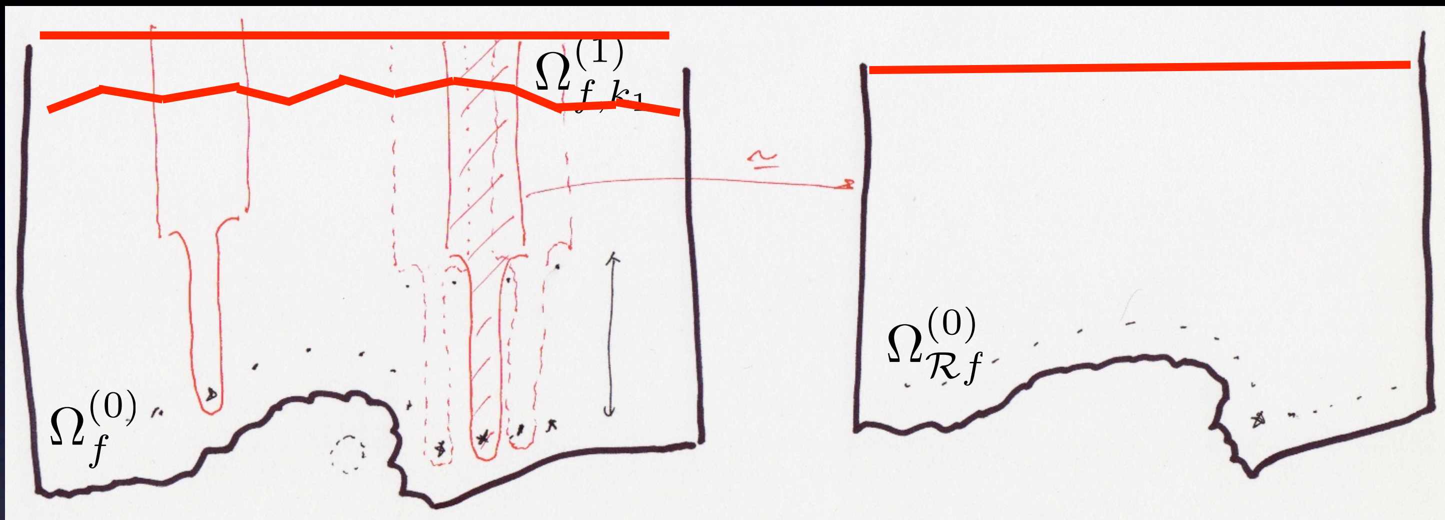

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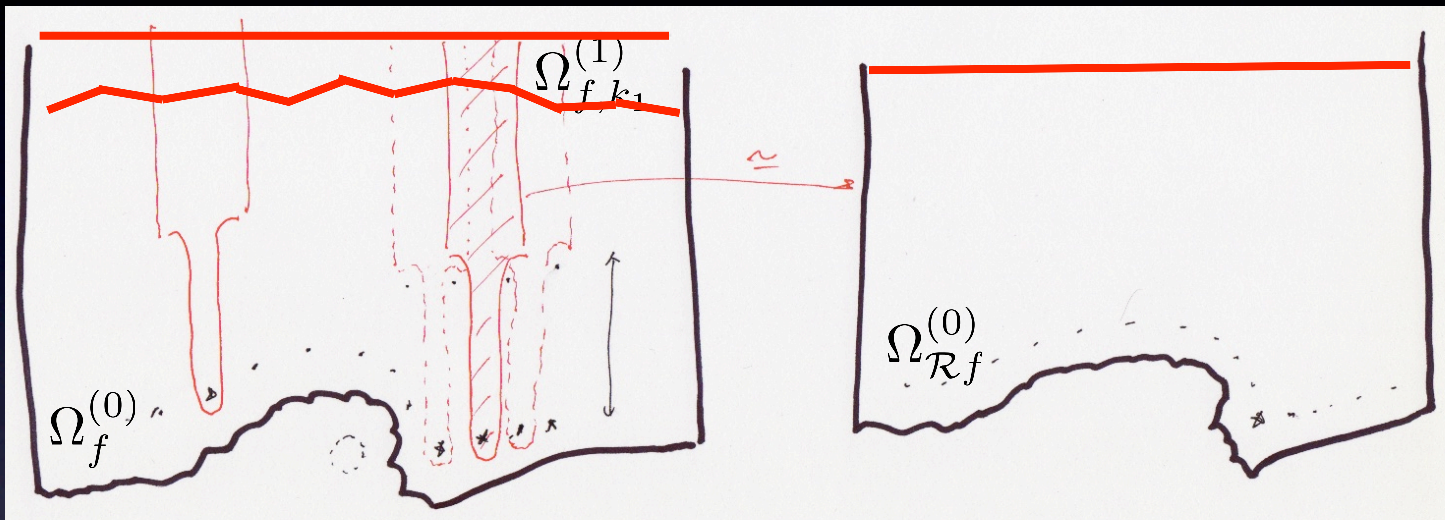

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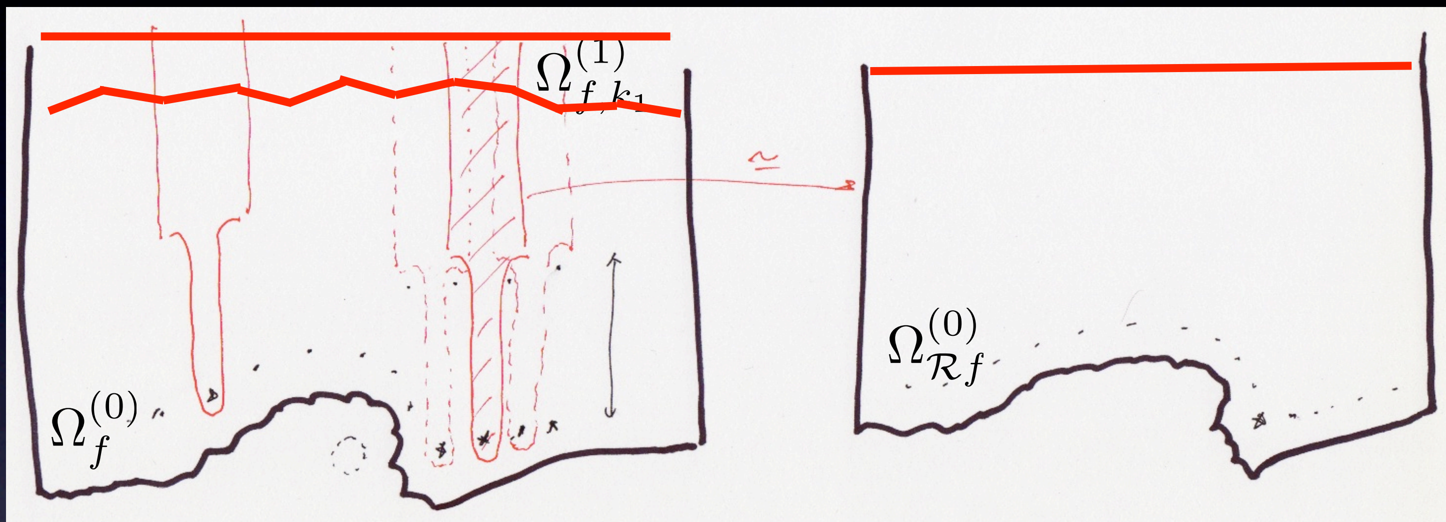

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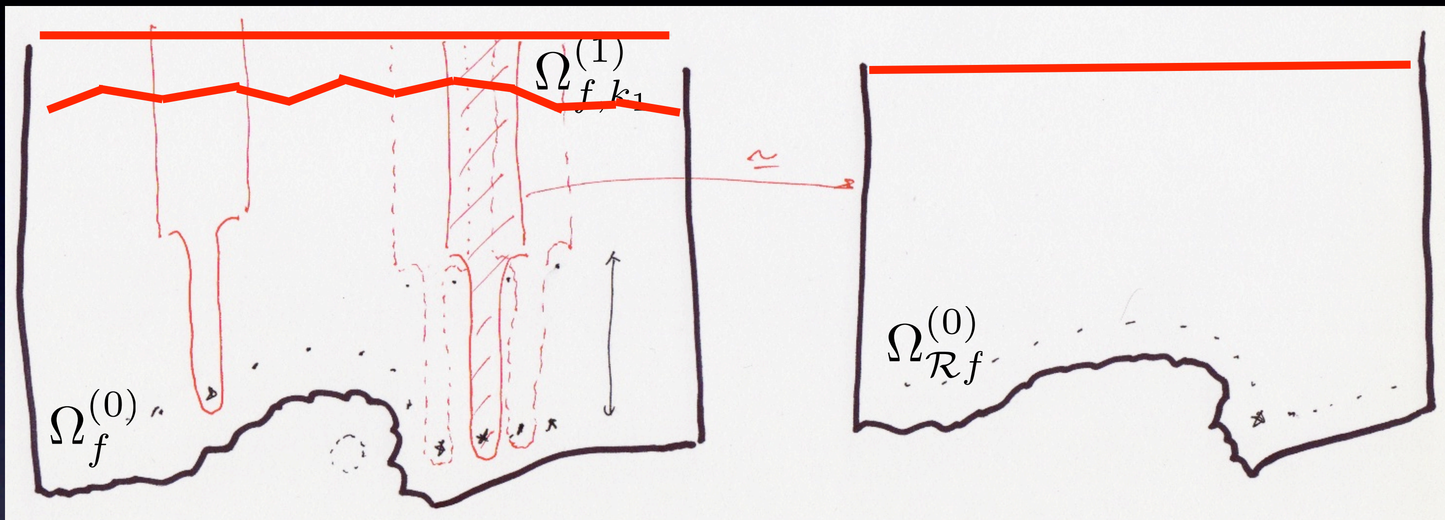

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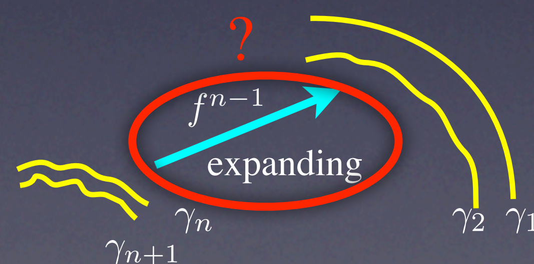

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Happy Birthday, Jack!