The Creation of a Mathematics Education Program at City Tech, CUNY

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‣ The Mathematics Education program at City Tech (CUNY) prepares students to become secondary school mathematics teachers in grades 7-12.

‣ I was the chair of the committee that developed the program, and I have been the director of the program since its inception in the fall of 2012.

- In this talk, I will first examine the challenges in K-12 mathematics education that motivated the program's development.
  - Poor content knowledge and pedagogical content knowledge of K-12 mathematics teachers.
  - Difficulty in finding qualified mathematics teachers, especially in high-needs schools.
  - Lack of precision and cohesion in K-12 mathematics instruction and textbooks.
  - An overreliance on the Trickle-Down-Theory of mathematics teacher preparation.

‣ I will then describe the program, which is housed in the Department of Mathematics at City Tech, that we created to address these challenges.

  ‣ A very brief history of the program,
  ‣ fundamental characteristics of the program,
  ‣ structure of the curriculum (i.e., courses in the program),
  ‣ specific examples of courses or parts of courses, and how they reflect our overall philosophy and goals.

CITY TECH AND ITS STUDENT POPULATION

- New York City College of Technology, "City Tech," is the designated senior college of technology within the City University of New York (CUNY).
- Located in downtown Brooklyn.
- The 2013 student enrollment was $17,374$, of whom 36% attended part-time.

CITY TECH AND ITS STUDENT POPULATION

- 31% of students self-identified as Black (non-Hispanic), 35.6% as Hispanic, 20.6% as Asian/Pacific Islander, 11.1% as White, 0.5% as Native American, and 1.2% as Other.
- 61% reported a household income of less than $30,000.
- 80% of incoming first-year students received need-based financial aid.
- 67% are the first in their families to attend college.

CITY TECH AND ITS STUDENT POPULATION

‣ The student body reported 138 different countries of origin.

‣ 25% of students reported working 20 or more hours per week.

‣ The college is a federally designated Hispanic Serving Institution (HSI).

The creation of our program was a reaction to interrelated challenges that we saw in K-12 mathematics education.

‣ Poor content knowledge and pedagogical content knowledge of K-12 mathematics teachers.

‣ Difficulty in finding qualified mathematics teachers, especially in high-needs schools.

‣ Lack of precision and cohesion in K-12 mathematics instruction and textbooks.

‣ An overreliance on the Trickle-Down-Theory of mathematics teacher preparation.

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

- A study of the National Science Foundation (NSF, 2006) found that between 23 and 29 percent of middle school and high school mathematics and science teachers did not have a college major or even a minor in their teaching field.
- Ingersol (1999) reported that 49 percent of seventh grade U.S. mathematics teachers did not have the equivalent of a minor in mathematics.
- Milbourne (2002) found that out of the 134,000 secondary school mathematics teachers in the U.S., only 86 percent were certified to teach mathematics.

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

- Furthermore, it has proved difficult to find teacher, even with a less-than-ideal math background:
- In a 2009 study, Boyd concluded that even with the creation of the alternative certification route, New York City finds it difficult to recruit sufficient numbers of teachers with substantial math coursework or a math undergraduate major.

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

- ‣ In a 2008 study, 18% of secondary schools reported serious difficulties in filling teaching positions for mathematics. Such staffing difficulties tended to occur at high-poverty, high-minority public schools [MET2].

- ‣ Several other studies echo the findings of MET2: The problem of recruiting effective teachers appears to be more acute in schools serving high poverty student populations [Boyd et al., 2008; Boyd et al., 2006; Hunushek, 2004].

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

- ‣ Most would agree that you should "know what you teach."
- ‣ A number of studies have examined the relationship between students' mathematical achievement and teachers' mathematical knowledge.
- ‣ Results of such studies depend very much on how one measures knowledge of mathematics.
    - ‣ Number and type of math courses taken.
    - ‣ Test (university math, "mathematics for teaching")
    - ‣ Pedagogical content knowledge

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

- ‣ E. G. Begle was among the first to examine this relationship.
- ‣ Begle (1972) administered multiple-choice tests to 308 first-year high school algebra teachers and their students.
- ‣ He found "little empirical evidence to substantiate any claim that, for example, training in mathematics for mathematics teachers will have payoff in increased mathematics achievement for their students."

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

- ‣ It is significant to note, that the measures Begle administered to teachers' consisted of abstract algebra (groups, rings, and fields), and the real number system.
- ‣ It didn't address direct connections to the 7-12 classroom–mathematical knowledge for teaching–and did not include any instructional effectiveness measurement component.

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

- Studies that broadened their measure of teacher's knowledge to include content knowledge and pedagogical content knowledge seem to have more positive results.
- A 2005 study from U. of Michigan concluded that that teachers' mathematical knowledge (CK and PCK) was significantly related to student achievement gains in both first and third grades [Hill, Rowan, Ball, 2005].

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

- ‣ A 2009 meta-analysis of professional development studies found that those in which teachers focused, for a sustained period, on examining mathematics underlying the curriculum and how to teach it were associated with improved student acheivement [Ingersoll, 1999].
- ‣ Mathematics content knowledge is necessary but not sufficient preparation for a 7-12 math teacher. Pedagogical content knowledge.

POOR CONTENT KNOWLEDGE & DIFFICULTY IN FINDING QUALIFIED TEACHERS

The National Research Council Study *Preparing Teachers (2010)* concludes:

" Current research and professional consensus correspond in suggesting that all mathematics teachers ... rely on: mathematical knowledge for teaching, that is, knowledge not just of the content they are responsible for teaching, but also of the broader mathematical context for that knowledge and the connections between the material they teach and other important mathematics content."

LACK OF PRECISION AND COHESION

- ‣ A lack of precision, cohesion, and continuity in K-12 mathematics in textbooks and classroom instruction.
- ‣ Textbooks: Definitions that are imprecise, nonexistent, or contradictory; disconnected treatment of topic with similar underlying structures (e.g., base-ten notation for whole numbers and decimals).
- ‣ Hung-Hsi Wu refers to the lack of precision; absent or flawed reasoning; and absence of connections within K-12 math instruction–as dictated by common textbooks–as Textbook School Mathematics.

LACK OF PRECISION AND COHESION

Fractions (grades 5-7)
(Hung-Hsi Wu, "Understanding Numbers in Elementary School Math.")

- Lack of a precise definition of fractions. Fractions tend to be presented as at least three things all at once:
    1. Part-Whole: The part-whole interpretation of fractions such as $\frac{2}{3}$ indicates that a whole has been partitioned into three equal parts and two of those parts are being considered.
    2. Quotient: The fraction $\frac{2}{3}$ may also be considered as a quotient, $2 \div 3$.
    3. Ratio: The fraction $\frac{2}{3}$ may also represent a ratio situation, such as there are two dogs for every three cats.

LACK OF PRECISION AND COHESION

Fractions (grades 5-7)
(Hung-Hsi Wu, "Understanding Numbers in Elementary School Math.")

- **Lack of connection to prior learning (e.g., addition).** The typical treatment of addition of fractions involves finding the lcd, and performing some operation involving the numerators.

- How is this related to students prior understanding of addition of whole numbers that involved "combining things"?

- Wu: Define fractions as points on a number line, and the addition of two fractions $\frac{a}{b}$ and $\frac{c}{d}$ as the length of the intervals $[0, \frac{a}{b}]$ and $[0, \frac{c}{d}]$ joined end-to-end. Just like adding whole numbers, adding fraction can now be seen as "combining things."

LACK OF PRECISION AND COHESION

- ‣ The new Common Core State Standards in Mathematics (CCSSM) is promising in terms of precision and coherence.
- ‣ Successful implemtation requires appropriate pre-service math teacher training, professional development for current teachers, and good textbooks.

TRICKLE-DOWN-THEORY OF MATHEMATICS EDUCATION

‣ The Trickle-down-theory holds that future math teachers should be trained only in "real mathematics" and that this knowledge will prepare them for the needs of teaching high school and middle school mathematics.

‣ The Trickle-down-theory is common in teacher preparation programs: Students take pedagogy classes from a faculty of education, and math classes from the math department–the same mathematics classes as math majors.

TRICKLE-DOWN-THEORY OF MATHEMATICS EDUCATION

- For example, a math major's treatment of fractions runs as follows: Let $S = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : y \neq 0\}$. Define an equivalence relation on S by $(x, y) \sim (z, w)$ if $xw = yz$. Denoting the equivalence class of (x, y) in S by $\frac{x}{y}$, we call the set of all $\frac{x}{y}$ the rational numbers $\mathbb{Q}$.

- How could one use this in a grade 5-7 classroom?

MET2 RECOMMENDATIONS

The influential Mathematical Education of Teachers 2, by
CBMS, recommends that future high school teachers take the
following math courses:

- Courses taken by undergraduates in a variety of majors (21
  credits, e.g., Linear algebra, Caluclus I and II)
- Courses intended for mathematics majors (12 credits, e.g.,
  Introduction to proofs, Abstract algebra)
- Courses designed primarily for prospective teachers. High
  school math from an advanced viewpoint (9 credits)

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- A very brief history of the program,
- fundamental characteristics of the program,
- structure of the curriculum (i.e., courses in the program),
- specific examples of courses or parts of courses, and how they reflect our overall philosophy and goals.
- If time permits, we'll look external support for the program (NSF Robert Noyce Grant), and resesrch opportunities for our students.

BRIEF HISTORY OF MATHEMATICS EDUCATION AT CITY TECH

- ‣ Spring 2010: We began to develop the Mathematics Education program. I was the chair of this committee.
- ‣ Fall 2011: We had the outline of the program, including 15 new courses. I wrote and submitted the proposal for a new BSc. in Mathematics Education at City Tech.
- ‣ Spring 2012: The proposal passed the College Council of City Tech, CUNY Central office, and the State.
- ‣ Fall 2012: The Mathematics Education program at City Tech began. I have been the Program Director since then.
- ‣ We're now in the third year of the program. As of Fall 2014, we had 40 students; our first students will graduate this spring.

STRUCTURE OF THE CURRICULUM

‣ The 120-credit BSc. in Mathematics Education prepares students to become teachers of Secondary School Mathematics in grades 7-12. It has has four components:
  ‣ General Education Component (43-45 credits)
  ‣ Mathematics Component (37 credits)
  ‣ Mathematics Pedagogy Component (29 credits)
  ‣ Mathematical Applications Component (9-11 credits)

STRUCTURE OF THE CURRICULUM: MATHEMATICS COMPONENT

- MAT 2675 Multivariable Calculus
- MAT 1476L Maple Calculus Laboratory
- MAT 2580 Linear Algebra
- MAT 2573 Probability and Statistics
- MAT 2630 Numerical Analysis
- MAT 2070 Introduction to Proofs and Logic
- MAT 3020 Number Theory
- MAT 3050 Geometry I
- MAT 3075 Real Analysis
- MAT 3080 Abstract Algebra
- MAT 4050 Geometry II
- MAT 4030 History of Mathematics

STRUCTURE OF THE CURRICULUM: MATHEMATICS COMPONENT

- We don't, at the moment, have a stand-alone course in Secondary mathematics from an advanced perspective.
- Rather, our selection of math courses, and construction of these courses were designed to better connect with the content of high school math.

STRUCTURE OF THE CURRICULUM: MATHEMATICS PEDAGOGY COMPONENT

The pedagogy courses include:

- MEDU 2010 Technology in Mathematics Education
- MEDU 3010 Methods of Teaching 7-9 Mathematics
- MEDU 3020 Methods of Teaching 10-12 School Mathematics
- MEDU 4010 Supervised Student Teaching and Seminar in 7-9 Mathematics
- MEDU 4020 Supervised Student Teaching and Seminar in 10-12 Mathematics
- Considerable integration of mathematics into the methods courses (i.e., examining 7-12 mathematics precisely; clear definitions; precise explanation; sound reasoning; instructional approaches)

STRUCTURE OF THE CURRICULUM: GENERAL EDUCATION COMPONENT

This component includes courses such as,

- MAT 1475 Calculus I
- MAT 1575 Calculus II
- PSY 1101 Introduction to Psychology
- PSY 2501 Child and Adolescent Development
- PSY 3502 Human Learning and Instruction

STRUCTURE OF THE CURRICULUM: MATHEMATICAL APPLICATIONS COMPONENT

In this component, students take 9-11 credits in a range of courses that illustrate the use of mathematics in other disciplines. These course come from,

- ‣ Architecture
- ‣ Physics
- ‣ Computer Systems
- ‣ Applied Mathematics
- ‣ Electrical and Telecommunications Engineering

FUNDAMENTAL CHARACTERISTICS OF THE PROGRAM

A. Mathematics that closely parallels what is taught in the secondary school classroom (mathematics and mathematics pedagogy).

B. Pedagogy courses focused soley on the mathematics 7-12 classroom.

C. Engaging in mathematics in way that is consistenct with CCSSM Standards of Mathematical Practice, and NCTM Process standards.

MATHEMATICS THAT CLOSELY PARALLELS WHAT IS TAUGHT IN THE MIDDLE AND HIGH SCHOOL CLASSROOM

"The young university student [was] confronted with problems that did not suggest...the things with which he had been concerned at school. When, after finishing his course of study, he became a teacher...he was scarcely able to dicern any connection between his task and his university mathematics."

-Felix Klein, "Elementary Mathematics from an Advanced Standpoint."

NCTM PROCESS STANDARDS

1. Problem solving
2. Reasoning and proof
3. Communication
4. Connections
5. Representations

CCSSM STANDARDS FOR MATHEMATICAL PRACTICE

‣ The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

‣ The SMP "describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years."

CCSSM STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

MAT 3080 ABSTRACT ALGEBRA

- Abstract algebra is not only beautiful and interesting, but it is a valuable, perhaps essential, topic for understanding secondary school mathematics. It is intimately related to what is taught in high school.

- "For all undergraduates, but especially for future high school teachers, [an] abstract algebra course can effectively build on familiar algebraic structures encountered in high school and other college mathematics courses. Examples of rings, integral domains, and fields familiar from high school are the most useful for future high school teachers...[G]roup theory woul be closely connected to concrete examples such as isometry groups." (MET)

- Our course emphasizes rings and fields more so than groups.

MAT 3080 ABSTRACT ALGEBRA

Group theory

- We consider connections between group theory and secondary mathematics via symmetry.
- "Instructional programs from prekindergarten through grade 12 should enable all students to–apply transformations and use symmetry to analyze mathematical situations." (NCTM, p. 41).
- CCSSM. G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- CCSSM. G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

MAT 3080 ABSTRACT ALGEBRA

Group theory

- ‣ We examine the group of isometries of $\mathbb{R}^n$.
    - ‣ The group of isometries of $\mathbb{R}^2$, the Euclidean group $E(2) \cong O(2) \ltimes \mathbb{R}^2$.
    - ‣ Euclidean group $E(3) \cong O(3) \ltimes \mathbb{R}^3$.
- ‣ We examine symmetry groups. A symmetry group in $\mathbb{R}^n$ is a subgroup of the group of isometries on $\mathbb{R}^n$ that fixes a set of points $X \subset \mathbb{R}^n$.
- ‣ Symmetry groups of regular polygons (dihedral groups) and polyhedra.

MAT 3080 ABSTRACT ALGEBRA

$\mathbb{C}$ and $\mathbb{R}[x]$

- The connection between $\mathbb{C}$ and $\mathbb{R}[x]$ provides another link between abstract algebra and secondary mathematics.
- We construct $\mathbb{C}$ as a quotient of $\mathbb{R}[x]$. ($\mathbb{C} \cong \mathbb{R}[x]/(x^2 + 1)$)
- We examine splitting fields for polynomials constructed in this manner.
- (The splitting field of $p(x)$ over $\mathbb{F}$ is constructed via a sequence of fields $F = K_0, K_1, \cdots, K_m = K$ such that K_i is an extension of K_{i-1} containing a new root of $p(x)$.)

MAT 3075 Real Analysis

- The rational numbers $\mathbb{Q}$ take center stage in pre-college math, with the real numbers $\mathbb{R}$ playing a less central, but important role.
- In 7-12 math it is (usually) an unstated assumption that the real numbers exist and satisfy the same properties of operations as the rational numbers.
- MET2: "Teachers need to know how to prove what is unstated in high school in order to avoid false simplifications and to be able to answer questions from students seeking further understanding."

MAT 3075 REAL ANALYSIS

‣ MAT 3075 Real Analysis begins with a construction of the reals, a proof that they satisfy the properties of operations and the Completeness Axiom.

‣ The course includes a proof of the Intermediate Value Theorem. This links to the graphical method of solving equations studied in 7-12 mathematics.

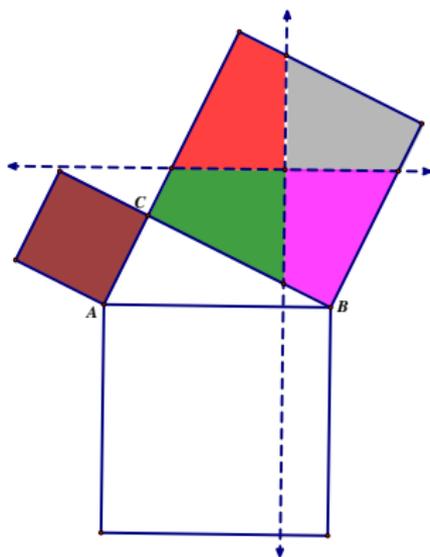# MEDU 2010 TECHNOLOGY IN MATH EDUCATION

‣ In this course we explore how technology can encourage and enhance students' study and understanding of mathematics.

‣ dynamic geometry software, computer algebra systems, graphing calculators, (statistical software).

‣ "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhance students' learning." (NCTM Technology Principle)

MEDU 2010 TECHNOLOGY IN MATH EDUCATION

- We engage in detailed, interactive mathematical investigations, illustrating how technology may be integrated into the classroom.
- Some of these investigations are lead by students.
- E.g., we used Geometer's Sketchpad to explore the Pythagorean Theorem and its converse. We also investigated symmetry in regular polygons using GSP.
- E.g., used GSP to explore similarity and congruence using translations, rotations, reflections, and dilations–consistent with the treatment in the CCSSM.
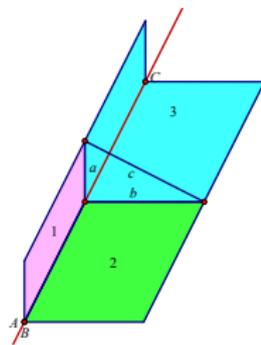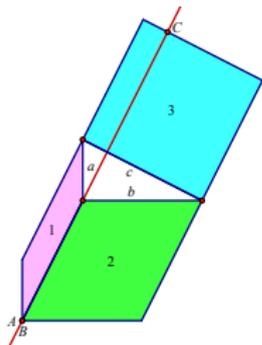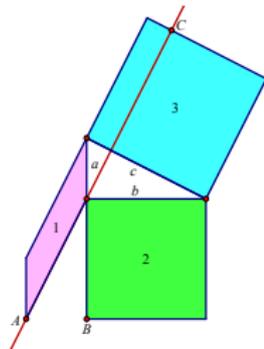
MEDU 2010 TECHNOLOGY IN MATH EDUCATION

Dissection demonstration of the Pythagorean Theorem
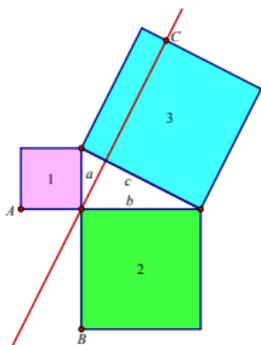with Geometer's Sketchpad

MEDU 2010 TECHNOLOGY IN MATH EDUCATION

Dissection demonstration of the Pythagorean Theorem
with Geometer's Sketchpad
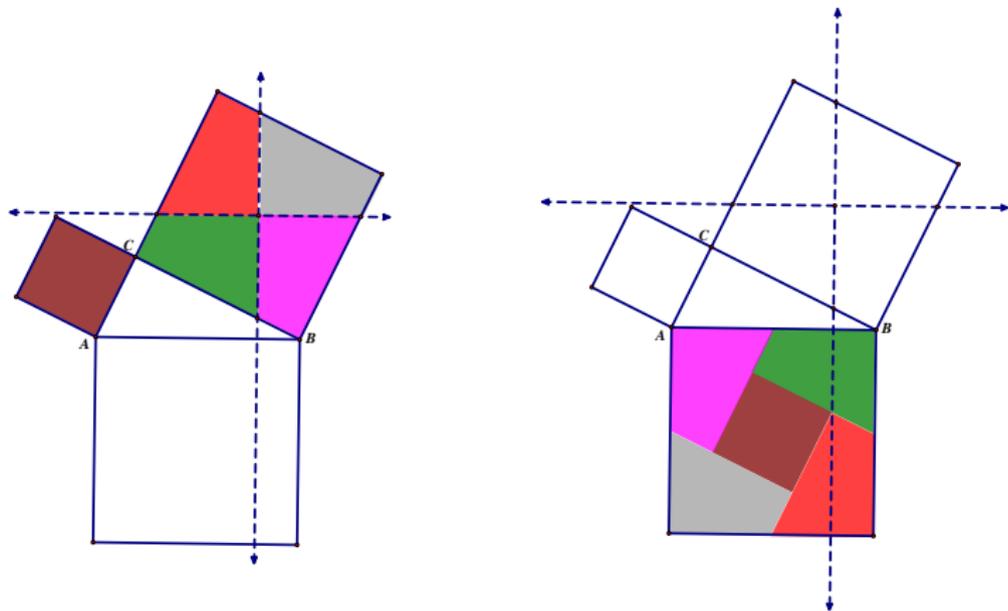
MEDU 2010 TECHNOLOGY IN MATH EDUCATION

Visual demonstration of the Pythagorean Theorem
with Geometer's Sketchpad

MAT 3090 MATHEMATICS FOR SECONDARY SCHOOL TEACHERS

- ‣ The selection of math courses in our program, and the construction of these courses were designed to better connect with the content of high school math.
- ‣ We don't, at the moment, have a stand-alone course in Secondary mathematics from an advanced perspective.
- ‣ I'm currently developing such a course.

MAT 3090 MATHEMATICS FOR SECONDARY SCHOOL TEACHERS

- Examine the mathematics curriculum from an advanced perspective. Emphasize precision; clear and correct definitions; sound reasoning with and an eye to the 7-12 classroom.
- Build interconnections among topics within 7-12 mathematics. View mathematics as a coherent whole.
- Examine connections between pre-college and college mathematics.

MAT 3090 MATHEMATICS FOR SECONDARY SCHOOL TEACHERS

‣ Possible textbooks:

‣ E. G. Bremigan, R. J. Bremigan, J. D. Lorch. *Mathematics for Secondary School Teachers.* MAA, 2011.

‣ Z. Usiskin, A. Peressini, E. A. Marchisotto, D. Stanley. *Mathematics for High School Teachers: An Advanced Perspective.* Pearson, 2003.

EXTERNAL SUPPORT: NSF ROBERT NOYCE GRANT

‣ Title: NOYCE Explorers, Scholars, Teachers (NEST): Fostering the Creation of Exceptional Mathematics and Technology Teachers in New York City.

‣ I'm Co-PI, wrote approx. half of the proposal.

‣ Amount: $1,418,976.

‣ Funding for student teaching placement.

‣ Summer workshops for math education students.

‣ Research experiences for math education students (MET2 recommendation).

‣ Scholarships