How do teachers expect students to represent mathematical work?

A study of teachers' recognition of routine ways that proofs are presented and checked in high school geometry

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Motivation

The aim of my research is to investigate how people develop discipline-specific communication skills.

Mathematical experts use a range of semiotic resources that include speaking, writing, drawing, and gesturing when communicating about mathematics.

What opportunities are available to mathematical novices to develop proficiency with these modes of communication?

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Overview of study

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Part I: Analyzed video episodes of geometry classrooms to identify routine ways that students communicate when doing proofs.

Part II: Conducted an experiment to gauge the extent to which the routines identified in Part I are recognized by secondary mathematics teachers.

Research questions

How do students communicate when doing proofs in geometry?

To what extent do secondary teachers recognize routine (i.e., normative) ways that students communicate when doing proofs?

Background (I)

The TIMSS study[1] identified mathematics *teaching scripts* that describe what regularly happens in classrooms.

- general descriptions of the activity that takes place in classrooms
- valid from an external observer's perspective
- distilled from representative samples of classroom videos

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How could we describe specific teaching routines without analyzing a representative sample of classroom videos?

→**use multimedia questionnaires**

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First part of study

How do students communicate when doing proofs in geometry?

Theoretical Framework (I): Instructional Situations

Instructional situations (Herbst & Chazan, 2012) facilitate the exchange of work (produced by students) for knowledge claims (from the teacher).

- The research conducted for this study focuses on the situation of *doing proofs in geometry* (Herbst & Brach, 2006).

The exchange of work-for-knowledge claims is an exchange of semiotic resources.

Theoretical Framework (II): Semiotic Norms

A *norm* is not only something that regularly happens, but is also something that is *expected* (by participants in situations) to happen (Garfinkel, 1963; Herbst & Chazan, 2003)

- Norms are shared, social, and generally unnoticed *unless they are breached.*

- By *semiotic norm,* I refer to the expected ways of using semiotic resources to exchange work in instructional situations.

Conjecturing Semiotic Norms

- I reviewed video episodes of geometry classrooms doing proofs; episodes were recorded from the classes of 3 geometry teachers.

- Across the episodes, I observed recurring ways that semiotic resources were used during the activities of *presenting* and *checking* proofs.

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The Sequence Norm

When students present proofs at the board, students *can* act as *transcribers*:

- the order in which the parts of a proof are transcribed need not be mathematically coherent

- students can reproduce the different parts of the argument in whatever order makes the transcription convenient

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The Details Norm

For a proof to be considered acceptable, specific details (but not others) need to be included in the written statements that make up the proof:

- Students are expected to include statements that unpack explicit givens.

- Students are *not* expected to include statements that are tacitly warranted by diagrams.

Second part of study

To what extent do secondary teachers recognize routine (i.e., normative) ways that students communicate when doing proofs?

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Method

- The method of investigation was a *virtual breaching experiment with control.*

- Variant of classic breaching experiments (Garfinkel, 1963; Rafalovich, 2006)
  - Breaches are virtual: participants view representations of situations (rather than experience breaches directly)
  - Participants are randomly assigned to treatment or control conditions

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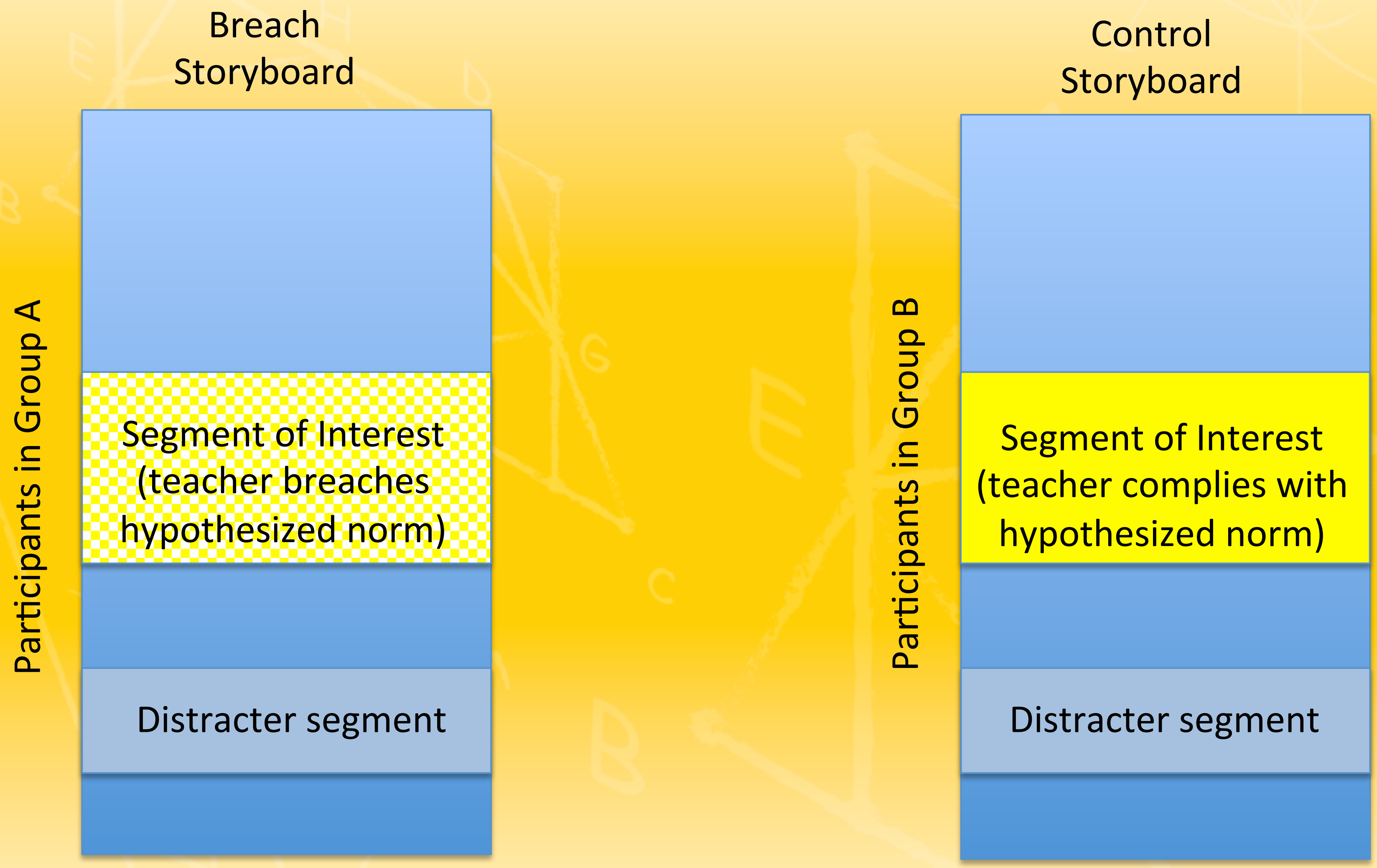
Method (II)

I used a customizable graphic language (Herbst, Chen, Chieu, & Weiss, 2012) to create storyboard probes in breach/control pairs

- Storyboards were created in breach/control pairs to represent episodes of geometry instruction

- Breach/control pairs were identical *except* during a 3-5 frame segment during which a teacher breaches or does not breach the norm.

Method (II)


Breach Storyboard
Participants in Group A
Segment of Interest (teacher breaches hypothesized norm)
Distracter segment
Control Storyboard
Participants in Group B
Segment of Interest (teacher complies with hypothesized norm)
Distracter segment

Developing storyboards

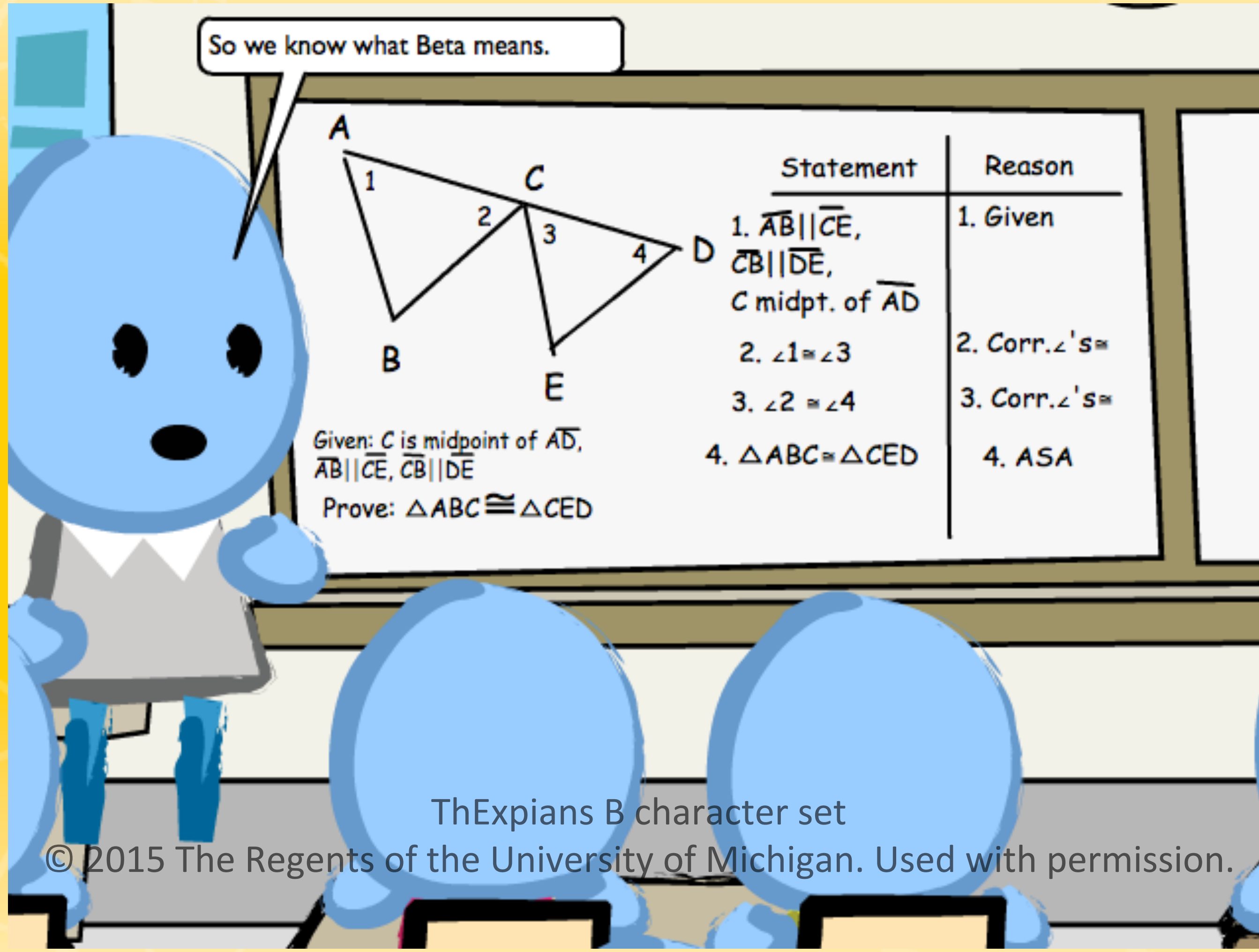
- Cyclical use of records of practice[2]
- 4 storyboards were developed to target the *details* norm.
- The storyboards breached the *details* norm in distinct ways

less details storyboards: teacher accepts a proof that omits a hypothesized-to-be-required detail

more details storyboards: teacher insists on details that are hypothesized to be excessive

2. Jacobs, Kawanaka, & Stigler, 1999

Breach: teacher omits written detail that is hypothesized to be needed



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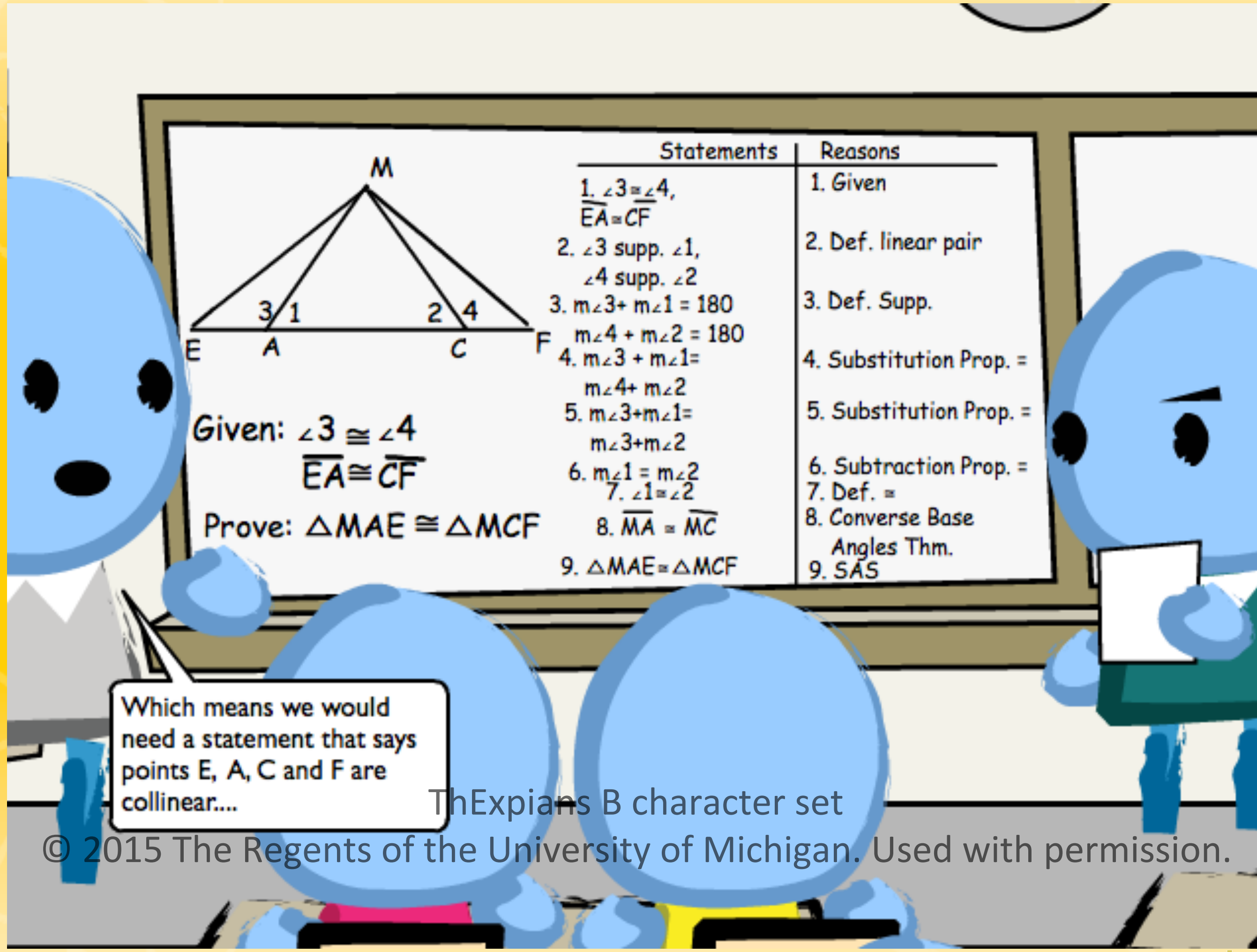

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Design & Data

- 73 secondary mathematics teachers participated in the study.
  - Multimedia survey delivered online
- Planned comparison study; participants randomly assigned to conditions.
  - 5 experiment groups; ~ 15 participants per group.
- Each participant viewed 2 breach storyboards, 2 control storyboards.
- Participants were asked the same set of open- and closed-ended questions:

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Design & Data (II)

- Open-ended question: What did you see happening in this scenario?
- Closed-ended questions
  - *Episode Appropriateness* (EA): How appropriate was the teacher's review of the proof?
  - Two *segment appropriateness* rating questions: How appropriate were the teacher's actions in this segment of the story?
    - *Segment of Interest* (S_IA): The segment where the teacher breaches/does not breach the target norm
    - *Distracter Segment* (S_DA)*:* A segment of the story common across breach/control conditions

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Design & Data (III)

- Participants used the same 6-valued Likert-like format to respond to each appropriateness question:

  1 = Very Inappropriate

  2 = Inappropriate

  3 = Somewhat Inappropriate

  4 = Somewhat Appropriate

  5 = Appropriate

  6 = Very Appropriate

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Hypotheses

Across condition
(different participants)

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II. Segment of interest ratings for breach storyboards ($S_I A_{breach}$) will be less than segment of interest ratings for control storyboards ($S_I A_{control}$)

III. There will be no significant difference in ratings across conditions for the distracter segment rating questions.

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Within group (same participants)

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Within group
(same participants)

IV. For breach storyboards, S_IA ratings will be significantly less than S_DA ratings.

V. For control storyboards, there will be no significant difference between S_IA ratings and S_DA ratings.

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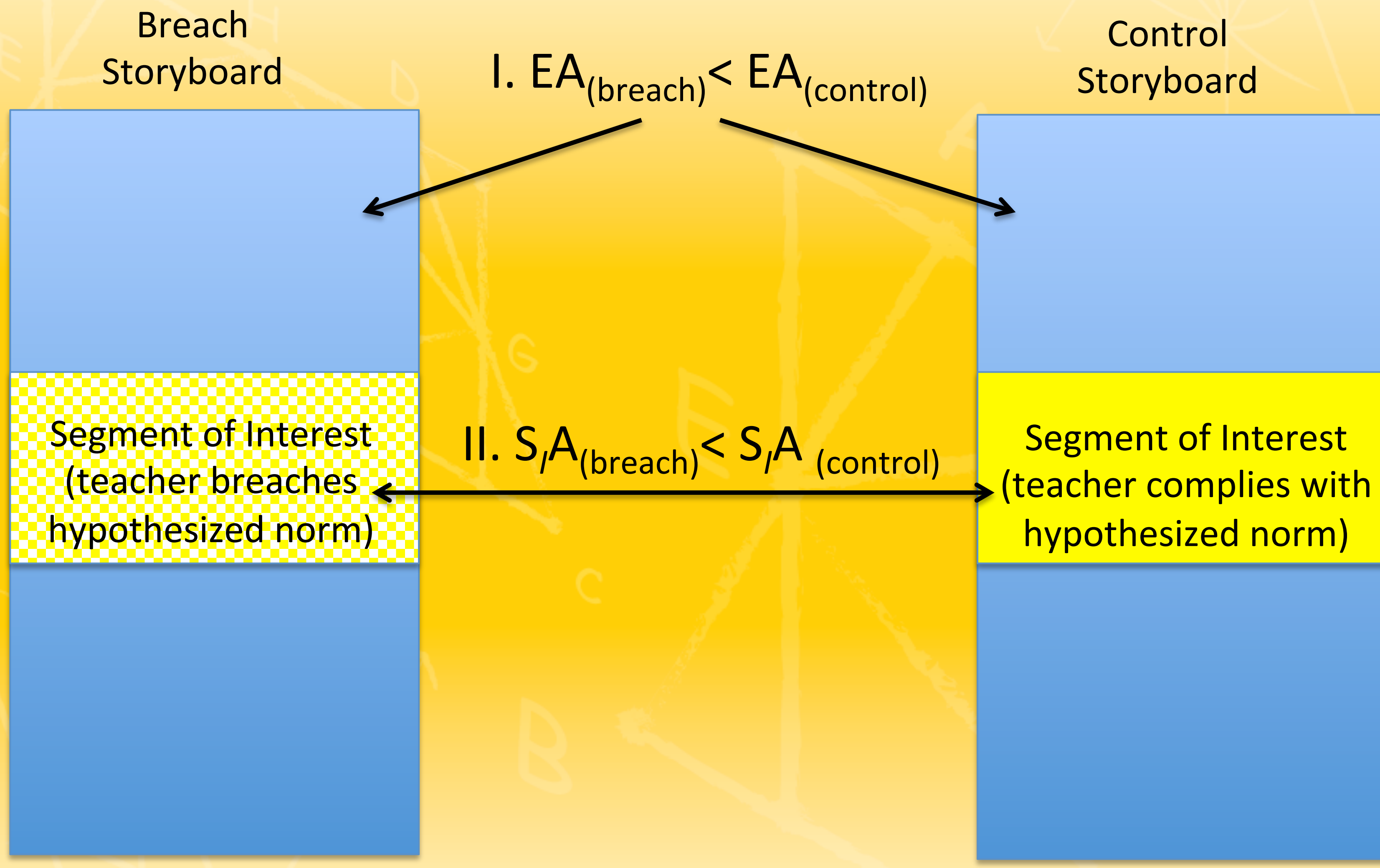
Hypotheses (II)

Breach Storyboard

I. $EA_{(breach)} < EA_{(control)}$

Control Storyboard

Hypotheses (II)



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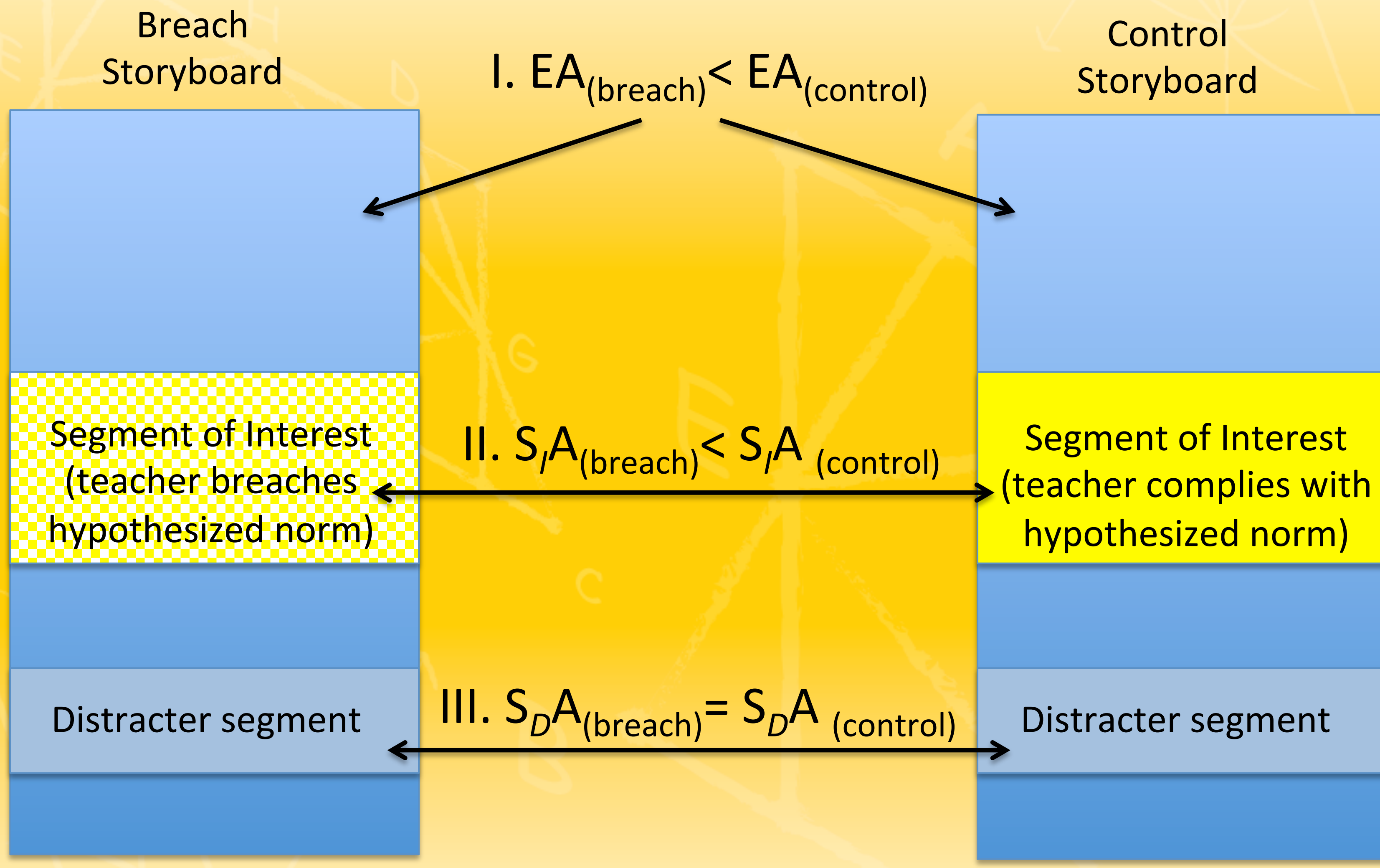

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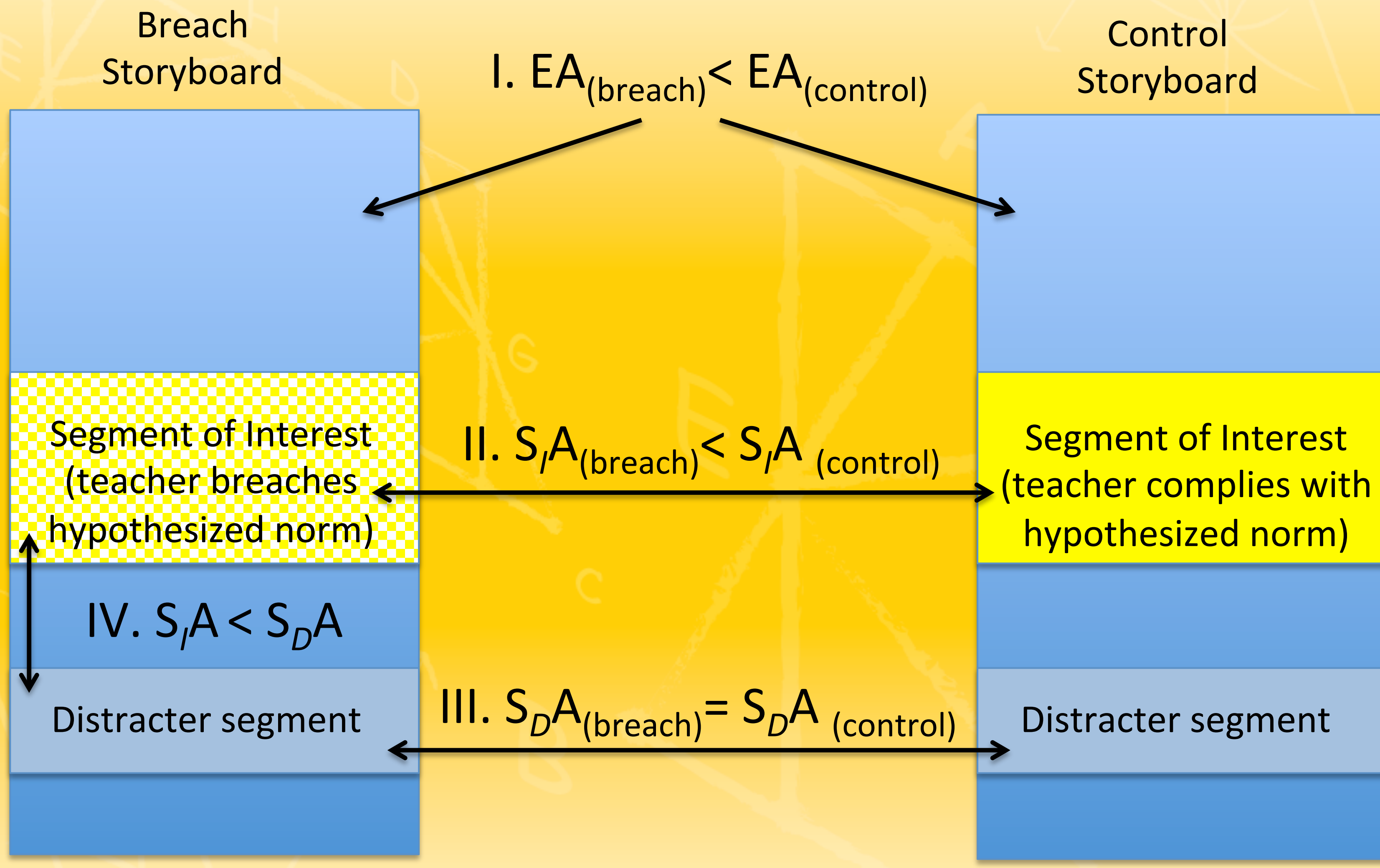

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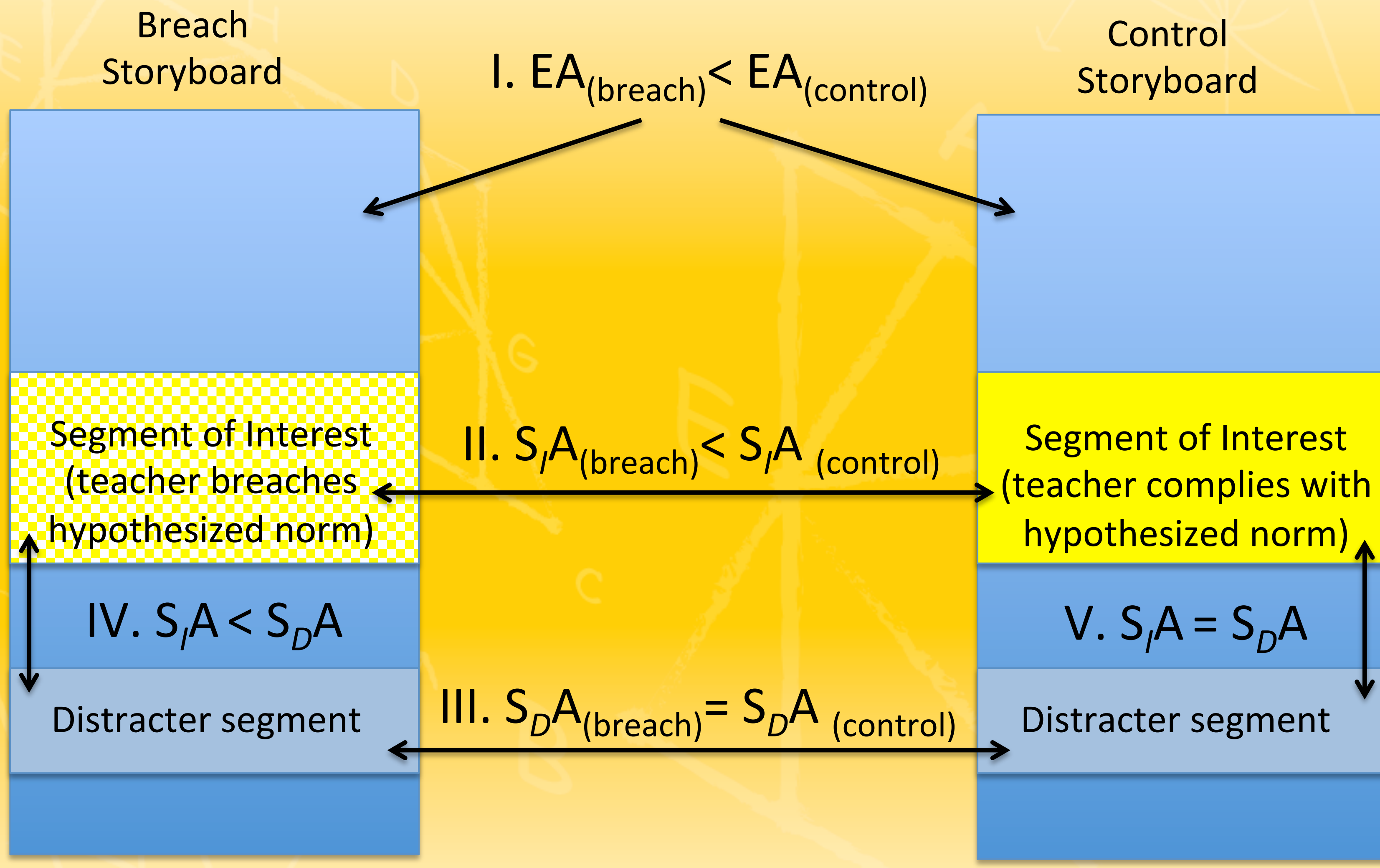

Results

Hypotheses I, II, III

Less Details Stories (BG1 vs. CG2)

Rating	Breach	Control	μ_1-μ_2	p
EA	3.4	4.6	-1.2	**.001**
$S_I A$	2.3	4.5	-2.2	**<.001**
$S_D A$	4.2	4.5	-0.3	.374

More Details Stories (BG2 vs. CG3)

Rating	Breach	Control	μ_1-μ_2	p
EA	3.4	4.4	-1.0	**.006**
$S_I A$	3.3	4.2	-0.9	**.009**
$S_D A$	3.8	4.3	-0.5	.115

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Hypotheses IV, V

Group	Story	S_IA	S_DA	μ_1-μ_2	*p*
1 ***n*=16**	Breach	2.3	4.2	-1.9	**<.001**
	Control	4.8	4.9	-0.1	.704
2 ***n*=13**	Breach	3.3	3.9	-0.6	**.024**
	Control	4.4	4.5	-0.1	.819
3 ***n*=15**	Breach	2.8	4.4	-1.6	**<.001**
	Control	4.2	4.4	-0.2	.499

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Example Open Responses

Less Details Breach:

- "A proof shouldn't leave **any piece** up to the imagination or interpretation"

- "When you do proofs, **you can't assume anything**"

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Less Details Breach:

- “A proof shouldn't leave **any piece** up to the imagination or interpretation”
- “When you do proofs, **you can’t assume anything**”

More Details Breach

- “A 99 step proof? This teacher is **being a bit ridiculous on the thoroughness** of the proof”
- “The teacher got caught up in some minutia of the problem and is **getting overly detailed about steps that really are not relevant in the solving of the problem.**”
- “**We assume the steps** he wants to add [when] given the picture”

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Discussion

- The differences in ratings suggest that:
  - Teachers recognize normative ways of checking the details of a proof
  - The details of a proof get checked in mathematically specific ways
- The details norm presents a curiosity:
  - Participants objected to the *less details* breaches, on the grounds that every step in a proof needs to be explicitly stated
  - Participants objected to the *more details* breaches, on the grounds that the teacher was seeking too much detail in the proof.

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Significance

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- generate basic knowledge about mathematics teaching
- help reform efforts contend with the paradox of "change without difference" (Woodbury & Gess-Newsome, 2002), by
- providing a foundation for designing instructional interventions that are aligned with the realities and constraints of existing practice (Cobb, Zhao, & Dean, 2009; Hora & Ferrare, 2013)

Implications for Research

This study of geometry instruction:

- shows that instruction can be described without analyzing a representative sample of classroom video

- shows the viability of using an experimental design to investigate the aspects of instruction that are salient for teachers

Implications for Teaching

The results of the study suggest that

- secondary geometry teachers have a shared sense of how proofs get presented and checked in geometry classrooms

- there are opportunities in geometry classrooms for students to develop discipline-specific communication skills.

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