MAT 132 HW 38-40

1. Problems

1. Find the Taylor polynomials of degree two approximating the given function centered at the given point.
$$f(x) = \sin(3x), \qquad a = 1$$
2. Find the Taylor polynomials of degree two approximating the given function centered at the given point.
$$f(x) = x^{\frac{1}{3}}, \qquad a = 4$$
3. Find the Taylor polynomials of degree two approximating the given function centered at the given point.
$$f(x) = \ln(x), \qquad a = 2$$
4. Find the Maclaurin series of
$$f(x) = e^{2x}.$$
5. Find the Maclaurin series of
$$f(x) = x\cos(x).$$

2. Answer Key

1. $\sin(3) + 3\cos(3)(x-1) + \frac{-9\sin(3)}{2}(x-1)^2$.
2. $4^{\frac{1}{3}} + \frac{1}{3}4^{\frac{-2}{3}}(x-4) + \frac{-1}{9}4^{\frac{-5}{3}}(x-4)^2$.
3. $\ln(2) + \frac{1}{2}(x-2) + \frac{-1}{8}(x-2)^2$.
4. $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$.
5. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$

3. Solutions

1. $f(x) = \sin(3x), a = 1$
$$f(1) = \sin(3), f'(1) = 3\cos(3), f''(1) = -9\sin(3).$$
Thus, the taylor polynomical of degree two is
$$T_2(x) = \sin(3) + 3\cos(3)(x-1) + \frac{-9\sin(3)}{2}(x-1)^2.$$
2. $f(x) = x^{\frac{1}{3}}, a = 4$
$$f(4) = 4^{\frac{1}{3}}, f'(4) = \frac{1}{3}4^{\frac{-2}{3}}, f''(4) = \frac{-2}{9}4^{\frac{-5}{3}}.$$
Thus, the taylor polynomical of degree two is
$$T_2(x) = 4^{\frac{1}{3}} + \frac{1}{3}4^{\frac{-2}{3}}(x-4) + \frac{-1}{9}4^{\frac{-5}{3}}(x-4)^2.$$

3. $f(x) = \ln(x), a = 2$
$$f(2) = \ln(2), f'(2) = \frac{1}{2}, f''(2) = \frac{-1}{4}.$$
Thus, the taylor polynomial of degree two is
$$T_2(x) = \ln(2) + \frac{1}{2}(x-2) + \frac{-1}{8}(x-2)^2.$$
4. We know that the Maclaurin series of e^x is $\sum_{n=0}^{\infty} \frac{x^n}{n!}$. Thus, as $f(x) = e^{2x}$ we just substitute $2x$ in for x and get
$$\sum_{n=0}^{\infty} \frac{(2x)^n}{n!} = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}.$$
5. We know that the Maclaurin series of $\cos(x)$ is $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$. Thus, as $f(x) = x\cos(x)$ we just multiply this series by x to get
$$x \cdot \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}.$$