Chemistry (chemical reactions)
Medicine (drug administration, dialysis)
Ecology. (filtration of pollution)
Problem. A tank contains 20 kg salt dissolved in 5000 I of water. Brine containing 0.03 kg of salt per liter or water enters the tank at a rate of $25 \mathrm{l} / \mathrm{min}$. The solution is kept mixed and drains from the tank at the same rate. How much salt remains in the tank after half an hour?

Sol.


$$
20 \mathrm{~kg} \text { of salt }
$$

$$
5,0001 \text { of water }
$$

Plan:
Introduce variables and f-us Compose DE IVP
Solve it
Answer the question
Analyse the result
let $y(t)$ be the amount of salt in the tank after $t$ min Given: $\quad y(0)=20 \mathrm{~kg}$ (initial cord.) $t=0$ initial time
Find: $\quad y(30)=$ ?
Compose DE: $\underbrace{\frac{d y}{d t}}=($ rate in) $-($ rate out $)$
the rate of chang
of amount of salt
rate in $=$ concentration $\times$ rate of admission of tho brice e $=$

$$
\text { traction } x \text { rate of } 0.03 \frac{\mathrm{~kg}}{\mathrm{l}} \cdot 25 \frac{\mathrm{e}}{\mathrm{~min}}=0.75 \frac{\mathrm{~kg}}{\mathrm{~min}}
$$

rate out $=$ concentration $\times$ rate of discharge of the mixture $=$

$$
\begin{aligned}
& \text { rate out }=\begin{array}{r}
\text { concentration } \\
\frac{y(t)}{5000} \frac{\mathrm{~kg}}{l} \cdot 25 \frac{l}{\mathrm{mih}}
\end{array}=\frac{y(t)}{200} \frac{\mathrm{ly}}{\mathrm{~min}} \\
& \operatorname{IVP} \int \frac{d y}{d t}=0.75-\frac{y}{200} \quad \text { DE } \quad y(30)=\text { ? }
\end{aligned}
$$

Solve $D E$ :

$$
\begin{aligned}
& \frac{d y}{d t}=\frac{150-y}{200} \\
& \frac{d y}{150-y}=\frac{d t}{200} \\
& \int \frac{d y}{150-y}=\int \frac{d t}{200} \\
& -\ln |150-y|=\frac{t}{200}+C_{1} \\
& \ln |150-y|=-\frac{t}{200}-C_{1} \\
& |150-y|=e^{-\frac{t}{200}-C_{1}} \\
& 150-y=C e^{-\frac{t}{200}} \\
& y=150-C e^{-\frac{t}{200}} \left\lvert\, \begin{array}{l}
\text { gen. sol. } \\
\text { of } D E
\end{array}\right.
\end{aligned}
$$

Initial coral.:

$$
\begin{aligned}
& 20=\underset{t}{y(0)} \underset{t}{c}=150-\underbrace{e_{1}^{0}}_{1} \Rightarrow C=130 \\
& y(t)=150-130 e^{-\frac{t}{200}} \\
& \text { sol. of }
\end{aligned}
$$

$e^{\frac{-}{200}}$
$y=130 e^{200}$


What is 150?

$$
\begin{aligned}
& 150=\underbrace{0.03}_{\text {concentruin }} \cdot \underbrace{5000}_{\text {val of }}
\end{aligned}
$$

$$
\underset{\uparrow}{y(30)}=150-130 \cdot e^{-\frac{30}{200}} \approx \xlongequal{38.1}(\mathrm{~kg})
$$

