1 Problems

Exercise 1. Solve $sin(2x) - sin(x) = 0$ for $0 \leq x < 2\pi$. List all the solutions.

Exercise 2. Solve $cos^2x - 3cosx + 2 = 0$ for $0 \leq x < 2\pi$.

Exercise 3. Solve $cos^2x - 2cosx + 1 = 0$ for $0 \leq x < 2\pi$.

Exercise 4. Solve $4^x - 4{*}2^x + 4 = 0$.

Exercise 5. Solve $e^2x - 5e^x + 6 = 0$.

2 Answer key

Exercise 1. $x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

Exercise 2. $x = 0$

Exercise 3. $x = 0, \pi$

Exercise 4. $x = 1$

Exercise 5. $x = ln2, ln3$

3 Solutions

Exercise 1. Using double angle formula rewrite $sin(2x) = 2sinxcosx$. Then the equation is $2sinxcosx - sinx = 0$ which can be written as $sinx(2cosx - 1) = 0$. The term $sinx$ is 0 when $x = 0, \pi$. The term $cosx = \frac{1}{2}$ when $x = \frac{\pi}{3}, \frac{5\pi}{3}$.

Exercise 2. Let $u = cosx$. Then the equation is $u^2 - 3u + 2 = 0$ which is quadratic. The solutions are $u = 2$ and $u = 1$. Substituting back, there is no x such that $cosx = 2$. There is a unique solution $x = 0$ for $cosx = 1$.

Exercise 3. Let $u = cosx$. Then the equation is $u^2 - 2u + 1 = 0$ which is quadratic and factors into $(u-1)(u+1) = 0$. Then $u = \pm 1$. Substituing back, we want to solve $cosx = \pm 1$. So $x = 0$ and $x = \pi$ are the two solutions.

Exercise 4. $4^x - 4{*}2^x + 4 = 0$ can be rewritten as $2^{2x} - 4{*}2^x + 4 = 0$. Let $u = 2^x$. Then the equation is $u^2 - 4u + 4 = 0$. This can be factored as $(u-2)^2 = 0$ so $u = 2$. Substituting back we get $2^x = 2$ so $x = 1$.

Exercise 5. Let $u = e^x$ then the equation is $u^2 - 5u + 6$ which factors as $(u-2)(u-3) = 0$. So we need to solve $e^x = 2$ and $e^x = 3$. This gives $x = ln2$ and $x = ln3$.