More Solving Equations...

1. \(e^{2x} - 6e^x - 7 = 0\)
   - Let \( w = e^x \)
   - \( w^2 - 6w - 7 = 0 \)
   - \((w - 7)(w + 1) = 0\)
   - \(w = 7\) or \(w = -1\)
   - \(e^x = 7\) or \(e^x = -1\)
   - No solution

2. \(e^x - 8e^{-x} = 7\)
   - \((e^x - \frac{8}{e^x} = 7)\) \(\cdot e^x\)
   - \(e^{2x} - 8 = 7e^x\)
   - \(e^{2x} - 7e^x - 8 = 0\)
   - Let \( y = e^x \)
   - \(y^2 - 7y - 8 = 0\)
   - \((y - 8)(y + 1) = 0\)
   - \(y = 8\) or \(y = -1\)
   - \(e^x = 8\) or \(e^x = -1\)
   - No solution

3. \(\sin^2 x - 3\sin x + 2 = 0\)
   - Let \( \sin x = b \)
   - \(b^2 - 3b + 2 = 0\)
   - \((b - 2)(b - 1) = 0\)
   - \(b = 2\) or \(b = 1\)
   - Substitute back:
     - \(\sin x = 2\) or \(\sin x = 1\)
     - No solution

Note: \(e^{2x} = (e^x)^2\)

4. \(e^{2x} - 6e^x - 5 = 0\)
   - Let \( a = e^x \)
   - \(a^2 - 6a - 5 = 0\)
   - Cannot factor -> use quadratic formula
   - \(a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
   - \(a = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-5)}}{2}\)
   - \(a = \frac{6 \pm \sqrt{36 + 20}}{2}\)
   - \(a = \frac{6 \pm \sqrt{56}}{2}\)

6. \(\ln(x + 3) + \ln(x - 2) = 0\)
   - \(\ln((x + 3)(x - 2)) = 0\) 
   - \(x = e^0\)
   - \((x + 3)(x - 2) = e^0\)
   - \((x + 3)(x - 2) = 1\)
   - \(x^2 + x - 6 = 0\)
   - \(x = \pm \sqrt{49 - 36}\)
   - \(x = \pm \frac{7}{2}\)
   - Both positive
6. \( \left( e^x - \frac{6}{e^x} = 5 \right) \cdot e^x \\
e^{2x} - 6 = 5e^x \\
e^{2x} - 5e^x - 6 = 0 \\
\text{Let } y = e^x \\
y^2 - 6y - 6 = 0 \\
(y - 6)(y + 1) = 0 \\
\begin{align*} 
y - 6 &= 0 \\
y + 1 &= 0 \\
y &= 6 \\
y &= -1 \\
\end{align*} \\
\text{Substitute back:} \\
e^x = 6 \\
e^x = -1 \\
\ln(6) = x \\
\text{No solution}

7. \( 4\sin^3x - 3\sin x = 0 \)

\( \sin x \) \( (4\sin^2x - 3) = 0 \)

\( \sin x = 0 \)

\( \sin x = \pm \frac{\sqrt{3}}{2} \)

\( x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \)

8. \( \log_4(2x - 3) - \log_4(3x + 1) = 1 \\
\log_4 \left( \frac{2x - 3}{3x + 1} \right) = 1 \\
4^1 = \frac{2x - 3}{3x + 1} \\
4 = \frac{2x - 3}{3x + 1} \\
x = \frac{2x - 3}{3x + 1} \\
x = 3 \text{ not a solution since } 3(3) + 1 > 0 \\
2x - 3 = 4(3x + 1) \\
2x - 3 = 12x + 4 \\
x = -\frac{7}{10} \\
e^x = e^{-\frac{7}{10}} \\
x = -\frac{7}{10} \\
\text{No solution}

9. \( f(x) = 4e^{2x - 1} \) find \( f^{-1}(x) \)

\( y = 4e^{2x - 1} \)
\( x = 4e^{2y - 1} \)
\( \frac{x}{4} = e^{2y - 1} \)
\( \ln \left( \frac{x}{4} \right) = 2y - 1 \)
\( 2y - 1 = \ln(x) \)
\( 2y = \ln(x) + 1 \)
\( y = \frac{\ln(x) + 1}{2} \)
\( f^{-1}(x) = \frac{\ln(x) + 1}{2} \)

10. \( f(x) = 8^{1 + 3y} \) find \( f^{-1}(x) \)

\( y = 8^{1 + 3y} \)
\( x = 8^{1 + 3y} \)
\( \log_8 x = 1 + 3y \)
\( y = \frac{\log_8 x - 1}{3} \)
\( f^{-1}(x) = \frac{\log_8 x - 1}{3} \)

11. \( f(x) = 5\log_4(2x - 3) \)

\( y = 5\log_4(2y - 3) \)
\( x = 5\log_4(2y - 3) \)
\( \frac{x}{5} = \log_4(2y - 3) \)
\( 4^x = 2y - 3 \)
\( 4^x + 3 = y \)
\( f^{-1}(x) = \frac{4^x + 3}{2} \)
\[ x = 8^{1/3} + 3y \]

\[ \ln x = (1 + 3y) \ln 8 \]

\[ \frac{\ln x}{\ln 8} = 1 + 3y \]

\[ \frac{\ln x - 1}{\ln 8} = 3y \]

\[ y = \frac{\ln x - 1}{3 \ln 8} \]

Another way to solve #10

\[ f(x) = 5x^2 - 3x \]

\[ f(x + h) = 5(x + h)^2 - 3(x + h) \]

\[ f(x + h) - f(x) = 5(x + h)^2 - 3(x + h) - (5x^2 - 3x) \]

\[ = 5(x^2 + 2xh + h^2) - 3x - 3h - 5x^2 + 3x \]

\[ = 5x^2 + 10xh + 5h^2 - 3x - 3h - 5x^2 + 3x \]

\[ = 10xh + 5h^2 - 3h \]

\[ 12 \rightarrow f(x) = 5 \sqrt[3]{4x - 1} \]

\[ y = 5 \sqrt[3]{4x - 1} \]

\[ x = 5 \sqrt[3]{y - 1} \]

\[ \frac{x}{5} = \sqrt[3]{y - 1} \]

\[ \left( \frac{x}{5} \right)^3 = y - 1 \]

\[ \left( \frac{x}{5} \right)^3 + 1 = y \]

\[ f^{-1}(x) = \frac{\left( \frac{x}{5} \right)^3 + 1}{4} \]