Exponential Growth & Decay

Recall:

<table>
<thead>
<tr>
<th>Date</th>
<th>Deposits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 1</td>
<td>.01</td>
</tr>
<tr>
<td>Jan 2</td>
<td>.03 = (.01)(3)</td>
</tr>
<tr>
<td>Jan 3</td>
<td>.09 = (.01)(3)(3) = (.01)(3²)</td>
</tr>
<tr>
<td>Jan 4</td>
<td>.27 = (.01)(3³)</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td>Jan 10</td>
<td>.01(3⁹) “10th Day”</td>
</tr>
</tbody>
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"Half life" Radio Active Elements Decay
"Radio Active Decay"

How long it takes to have half of what you had before

Ex: The half life of an element is 5 days. What percent is remaining after 20 days?

Day 0 → 100 grams
Day 5 → 50 grams
Day 10 → 25 grams
Day 15 → 12.5 grams
Day 20 → 6.25 grams

\[
\frac{6.25 \text{ grams}}{100 \text{ grams}} = 6.25\% \\
A = 100 \left(\frac{1}{2}\right)^{\frac{t}{5}}
\]

This formula will tell us how much is left at any time t

General Formula:

\[
A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{K}}
\]

K = half-life
A₀ = starting amount
A(t) = amount at time t
The half-life of an element is 60 hours. If there are 400 gms originally, how much will there be after 10 hours?

\[ A(10) = ? \]
\[ A_0 = 400 \]
\[ t = 10 \]
\[ k = 6 \]

\[ A(10) = 400 \left( \frac{1}{2} \right)^{\frac{10}{60}} = 125.99 \text{ gms} \]

An acceptable answer on an exam ble you will not have a calculator.

If originally you have 180 gms of an element, and after 10 hours you have 40 gms, how many will you have after 20 hours?

A step problem -> (1) figure out the decay rate

(2) solve for what they ask for

\[ y = a \cdot b^x \]

General exponential formula

\[ a = \text{initial amount} \]
\[ b = \text{growth/decay rate} \]

Use info they tell us to get these two points:

(0, 180)
(10, 40)

\[ 180 = a \cdot b^0 \]
\[ a = 180 \]

\[ 40 = \frac{180}{b^{10}} \]

\[ b^{10} = \frac{180}{40} \]
\[ b^{10} = \frac{9}{2} \]

\[ b = \left( \frac{9}{2} \right)^{\frac{1}{10}} \]

\[ t = 10 \]

\[ y = 180 \left( \frac{2}{9} \right)^{\frac{10}{10}} \]

\[ y = 180 \left( \frac{2}{9} \right)^{10} \]

\[ y = 180 \left( \frac{2^{10}}{9} \right) \]
If you have 180 grams of an element and 10 hrs later you have 160 grams. How many will you have after 30 hrs?

\[(0, 180) \rightarrow y = ab^x\]

\[\begin{align*}
180 &= a \cdot b^0 \\
180 &= a(1) \\
a &= 180
\end{align*}\]

\[\frac{160}{180} = b^{10}\]

\[b = \left(\frac{1}{3}\right)^{110}\]

\[y = 180 \left(\frac{1}{3}\right)^{30}\]  
\[y = 180 \left(\frac{1}{3}\right)^{3}\]

---

Ex:

If originally you have 24 gms of an element, and 2 hrs later you have 20 gms, how much will you have after 7 hours?

\[(0, 24) \rightarrow y = ab^x\]

\[\begin{align*}
24 &= a \cdot b^2 \\
a &= 24
\end{align*}\]

\[\frac{20}{24} = b^{1/2}\]

\[b = \left(\frac{5}{6}\right)^{1/2}\]

\[y = 24 \left(\frac{5}{6}\right)^{x/2}\]

**Now we know:**

\[y = (24) \left(\frac{5}{6}\right)^{x/2}\]

\[x = \text{hours or time}\]
If you originally have 600 gms of an element and 3 hrs later you have 30 gms, what is the half-life of the element?

Let's now long does it take to be left with \( \frac{600}{2} = 30 \) gms.

\[
y = a b^x
\]

\[a = 600 \rightarrow \text{initial amount}\]

\[
\frac{50}{600} = \left(\frac{5}{6}\right)^x
\]

\[
\left(\frac{5}{6}\right)^\frac{1}{3} = b
\]

\[
y = 600 \left(\frac{5}{6}\right)^\frac{x}{3}
\]

\[
\frac{30}{600} = \left(\frac{5}{6}\right)^\frac{x}{3}
\]

\[
\frac{1}{2} = \left(\frac{5}{6}\right)^\frac{x}{3}
\]

\[
\log\left(\frac{1}{2}\right) = \frac{x}{3} \cdot \log\left(\frac{5}{6}\right)
\]

\[
3 \cdot \frac{\log\left(\frac{1}{2}\right)}{\log\left(\frac{5}{6}\right)} = \frac{x}{3} \cdot 3
\]

\[
x = 3 \cdot \frac{\log\left(\frac{1}{2}\right)}{\log\left(\frac{5}{6}\right)}
\]
ex: Originally you have 20 gms of an element and 8 hrs later you have 10 gms, what is the half-life?

\[ y = a \cdot b^x \]
\[ a = 20 \]
\[ \frac{10}{20} = \frac{20}{20} \]
\[ \left(\frac{4}{5}\right)^{\frac{x}{8}} \]
\[ 10 = \left(\frac{4}{5}\right)^{\frac{x}{8}} \]
\[ \frac{10}{20} = \frac{20}{20} \]
\[ \frac{1}{2} = \left(\frac{4}{5}\right)^{\frac{x}{8}} \]
\[ \log\left(\frac{1}{2}\right) = \frac{x}{8} \cdot \log\left(\frac{4}{5}\right) \]
\[ \log\left(\frac{4}{5}\right) \]
\[ 8 \cdot \frac{\log\left(\frac{1}{2}\right)}{\log\left(\frac{4}{5}\right)} = \frac{x}{8} \]
\[ x = 8 \cdot \frac{\log\left(\frac{1}{2}\right)}{\log\left(\frac{4}{5}\right)} \]

\[ \text{Log Laws:} \]
\[ \log(AB) = \log A + \log B \]
\[ \log\left(\frac{A}{B}\right) = \log A - \log B \]
\[ \log A^x = x \cdot \log A \]

\[ \text{Problem: Expand the following:} \]
\[ \log(x^2y^3) = \log x^2 + \log y^3 \]
\[ = 2 \cdot \log x + 3 \cdot \log y \]
2. \[ \log \frac{x^3}{y^5} = \log x^3 - \log y^5 \]
   \[= 3 \cdot \log x - 5 \cdot \log y \]

3. \[ \log \left( \frac{x^4 \sqrt{y}}{z^2} \right) = \log x^4 \sqrt{y} - \log z^2 \]
   \[= \log x^4 + \log \sqrt{y} - \log z^2 \]
   \[= 4 \cdot \log x + \frac{1}{2} \log y - 2 \log z \]

**Problem:** Write the following as a single log:

1. \[ 3 \log x + 2 \log y - 4 \log z \]
   \[= \log x^3 + \log y^2 - \log z^4 \]
   \[= \log \left( \frac{x^3 y^2}{z^4} \right) \]

2. \[ 4 \log x - 2 \log y + 3 \log z \]
   \[= 4 \log x + 3 \log z - 2 \log y \]
   \[= \log x^4 + \log z^3 - \log y^2 \]
   \[= \log \left( \frac{x^4 z^3}{y^2} \right) \]

*Be careful! Rewrite with all "+" first then "−." This will help you not mess up what goes in the numerator and what goes in the denominator.*