Functions, Domain, and Range

What is a function?

The function maps elements from the domain to the range.

We can think of a function being like a machine that when you put something in you get something else out.

Definition: A function is a relation between a set of inputs and a set of outputs in which each input is related to exactly one output.

- For each element in the domain there is only one element in the range.

NOT A FUNCTION

When we are looking at a graph we use the vertical line test to determine if it is a function.

To pass the vertical line test there is no place on the graph that you can draw a vertical line and hit the graph more than once.

Examples:
Domain: what you are allowed to put into the function
Range: what you get out of a function

What is the domain of:
\[ \sin(x) \rightarrow \text{All real numbers} \]

Notation:
\[ f(x) \quad \text{"A function of x"} \quad \text{whichever is inside} \]
\[ f(\theta) \quad \text{"A function of } \theta \text{"} \quad \text{the parenthesis is the Domain.} \]

ex: \[ f(x) = x^2 \]
Domain = all real numbers
\[ \rightarrow \text{we can square ANY number} \]

Types of Numbers:
1. Counting numbers/Natural Numbers \[ \rightarrow 1, 2, 3, 4, \ldots \]
2. Integers \[ \rightarrow \text{whole } \# \text{s (NOT fractions)} \rightarrow \ldots, -3, -2, -1, 0, 1, 2, \ldots \]
3. Rational Numbers \[ \rightarrow \# \text{s that can be represented by a ratio of an integer and another integer} \]
   \[
   \frac{p}{q} \quad \text{where } p, q \text{ are integers, } q \neq 0
   \]
4. Irrational Numbers \[ \rightarrow \# \text{s that cannot be represented as a ratio of two numbers} \]
5. Real Numbers \[ \rightarrow \text{Rational and Irrational } \# \text{s} \]

ex: \[ f(x) = \sqrt{x} \]
Domain = "positive real \#s and 0"
\[ = x \geq 0 \]
\[ = x \text{ is non-negative reals} \]

Interval Notation:

\[ [0, 10] \quad \text{ex: } \quad \begin{array}{c} 0 \leq x \leq 10 \end{array} \]

\[ (0, 10) \quad \text{ex: } \quad \begin{array}{c} 0 < x < 10 \end{array} \]

\[ [0, 10) \quad \text{ex: } \quad \begin{array}{c} 0 \leq x < 10 \end{array} \]

\[ (4, 6) \quad \text{ex: } \quad \begin{array}{c} 4 < x < 6 \end{array} \]

all \#s between 4 and 6
including 4 and not including 6.
note: we can never include infinity (\(\infty\)) because we can never actually "get to" \(\infty\)

- \([0, \infty)\) \(x \geq 0\)
- \((-\infty, 0]\) \(x \leq 0\)

- "union"
- All reals except \(x = 5\)
- in interval notation: \((-\infty, 5) \cup (5, \infty)\)

*WebAssign makes you use interval notation!*

\[ f(x) = \sqrt{x-3} \]
what is the domain?

\[ x-3 \geq 0 \]
\[ x \geq 3 \quad \text{or} \quad [3, \infty) \]

\[ f(x) = \sqrt{5-4x} \]
- \(5-4x \geq 0\)
- \(-4x \geq -5\)
- \(x \leq \frac{5}{4}\)

\[ \text{Domain is?} \]
\[ \text{Notation: All Real #s = \(\mathbb{R}\)} \]

\[ f(x) = 3\sqrt[3]{x-3} \]
- "cube root"
- All real #s:
- \((-\infty, \infty)\)

\[ f(x) = \frac{5}{x-2} \]
- Domain = all reals except \(x = 2\)
- \((-\infty, 2) \cup (2, \infty)\)

- because we can't have "0" in the denominator!
- \(x-2 \neq 0\)
- \(x \neq 2\)

\[ f(x) = n\sqrt{x} \]
- if \(n\) = positive, even
- Domain = \([0, \infty)\)
- if \(n\) = positive, odd
- Domain = \((-\infty, \infty)\)

\[ n \neq 0 \]
\[
\frac{5}{\sqrt{x-2}}
\]

- Domain: 
  - \(x > 2\)
  - Or
  - \((2, \infty)\)

- We can't have 0 in the denominator AND you can't take the square root of a negative number.

\[
\frac{\sqrt{x-2}}{5}
\]

- Domain: 
  - \(x - 2 \geq 0\)
  - Or
  - \(x \geq 2\)
  - Or
  - \([2, \infty)\)

\[
\frac{x-5}{\sqrt{x-3}}
\]

- Domain: 
  - \(x - 3 > 0\)
  - Or
  - \((3, \infty)\)

- We can be ANYTHING!

\[
\frac{\sqrt{x-3}}{x-5}
\]

- Domain: 
  - \(x - 5 \neq 0\)
  - Or
  - \(x \neq 5\)
  - And
  - \(x - 3 \geq 0\)
  - Or
  - \(x \geq 3\)

- "\(x\) can be any \# greater than 3 except 5"
What is the Domain?

\[ \cos x \neq 0 \]
\[ x \neq \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2} \]

or

\[ x \neq \frac{\pi}{2} + n\pi \text{ where } n = \text{integer} \]

**EX:** \( f(x) = \tan x = \frac{\sin x}{\cos x} \)

what is the Domain?

\[ \cos x \neq 0 \]
\[ x \neq \frac{\pm \pi}{2}, \frac{\pm 3\pi}{2}, \frac{\pm 5\pi}{2} \]

**EX:** \( f(x) = \sqrt{\frac{x}{x^2 - 9}} \)

What is the Domain?

\[ x - \pi > 0 \]
\[ x > \pi \]

**D:** \( \notin \mathbb{R} \) or \( (\pi, \infty) \)

**EX:** \( f(x) = \frac{\sqrt{x}}{x^2 - 9} \)

\( x \geq 0 \) AND \( x^2 - 9 \neq 0 \)
\( x^2 \neq 9 \)
\( x \neq \pm 3 \)

**D:** \( \notin \mathbb{R} \) or \( [0, 3) \cup (3, \infty) \)

we don't need to worry about \( x \neq -3 \) since \( x \geq 0 \) already.

**EX:** \( f(x) = \frac{x}{x^2 - 9} \)

\( x^2 - 9 \neq 0 \)
\( x^2 \neq 9 \)
\( x \neq \pm 3 \)

**D:** \( \notin \mathbb{R} \) or \( (-\infty, -3) \cup (-3, 3) \cup (3, \infty) \)
ex: \( f(x) = \frac{5}{\sqrt{x^2 - 9}} \)

What is the Domain?

\( x^2 - 9 > 0 \)

\((x + 3)(x - 3) > 0 \)

\[ \begin{array}{ccc}
-3 & - & 3 \\
+ & - & + \\
\end{array} \]

test #’s in each interval to see if the function is positive or negative.

So \( D: \frac{5}{x < -3 \text{ or } x > 3} \)

or

\((-\infty, -3) \cup (3, \infty)\)