Introduction to Trigonometry

1. Right Triangle Trig

\[
\frac{y}{x} = \frac{B}{A} = \frac{C}{D}
\]

These ratios hold because all three of these triangles are similar.

These 3 triangles are similar because they all have the same angle measures \((20^\circ, 70^\circ, 90^\circ)\).

We use trigonometry to give meaning to these ratios!

The Basics

\[
\sin x = \frac{A}{C}
\]
\[
\cos x = \frac{B}{C}
\]
\[
\tan x = \frac{A}{B}
\]

"SOH CAH TOA"

\[
\sin x = \frac{\text{opposite}}{\text{hypotenuse}}
\]
\[
\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}
\]
\[
\tan x = \frac{\text{opposite}}{\text{adjacent}}
\]

Note: adjacent = side next to the angle
hypotenuse = the longest side; the side across from the right angle

Pythagorean Theorem: A formula that relates the 3 sides of a right triangle

\[a^2 + b^2 = c^2 \quad (c \text{ is the hypotenuse})\]
example: Given \( \triangle ABC \)

1. \[ a^2 + b^2 = c^2 \]
2. \[ b^2 + c^2 = a^2 \]
3. \[ 30 + 8^2 = a^2 \]
4. \[ 100 = c^2 \]
5. \[ \pm 10 = c \]

\[ \boxed{c = 10} \]

(1) find the length of side \( c \)

(2) find \( \sin x \), \( \cos x \), \( \tan x \)

\[ \sin x = \frac{6}{10} \]
\[ \cos x = \frac{8}{10} \]
\[ \tan x = \frac{6}{8} \]

use SOH CAH TOA to find these ratios.

example: Given \( \triangle DEF \)

1. find the missing side
2. find \( \sin x \), \( \cos x \), \( \tan x \)
3. find \( \sin y \), \( \cos y \), \( \tan y \)

\[ \begin{align*}
\sin x &= \frac{12}{13} \\
\cos x &= \frac{5}{13} \\
\tan x &= \frac{12}{5} \\
\sin y &= \frac{5}{13} \\
\cos y &= \frac{12}{13} \\
\tan y &= \frac{5}{12}
\end{align*} \]

notice \( \sin x = \cos y \) and \( \cos x = \sin y \)

sine and cosine are "co-functions" of angle \( x \) and \( y \) are complementary angles (they add to \( 90^\circ \))

connecting right triangle trig to the unit circle

\[ \sin \theta = \frac{y}{r} \]
\[ \cos \theta = \frac{x}{r} \]

\[ (x, y) \to (\cos \theta, \sin \theta) \]

\[ \theta \]

\[ \sin \theta \]

\[ \cos \theta \]

\[ \theta \]

this allows us to connect trig to all real numbers.
# Special Right Triangles

### Equilateral Δ:

- $\sin 30^\circ = \frac{x}{2x} = \frac{1}{2}$
- $\cos 30^\circ = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$
- $\tan 30^\circ = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$

### Square:

- $\sin 45^\circ = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $\cos 45^\circ = \frac{x}{x\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $\tan 45^\circ = \frac{x}{x} = 1$

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"All Students Take Calculus"
**Helpful Table**

<table>
<thead>
<tr>
<th></th>
<th>$30^\circ$</th>
<th>$45^\circ$</th>
<th>$60^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sin</strong></td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
</tr>
<tr>
<td><strong>Cos</strong></td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td><strong>Tan</strong></td>
<td>$\frac{1}{\sqrt{3}}$</td>
<td>1</td>
<td>$\sqrt{3}$</td>
</tr>
</tbody>
</table>

*Note: $\tan x = \frac{\sin x}{\cos x}$*