Lecture 18

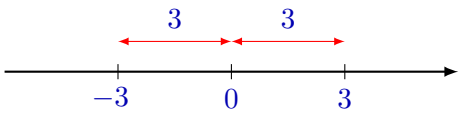
Absolute Value

Absolute value of a number

The **absolute value** of a number is the **distance** between this number and 0 on the number line.

The absolute value of a number a is denoted by $|a|$.

For example, $|3| = 3$, $|-3| = 3$, $|0| = 0$.



In general,

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Observe that if a is negative, then $-a$ is **positive**.

For example, if $a = -5$, then the formula above gives $|-5| = -(-5) = 5$.

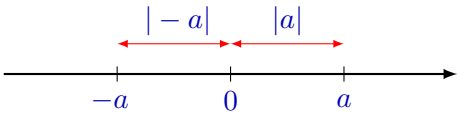
Properties of absolute value

- The absolute value of a number is **non-negative** (positive or zero):

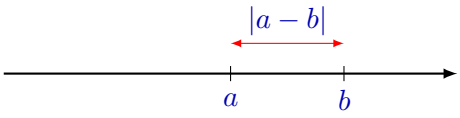
$$|a| \geq 0$$

- A number and its opposite have the same absolute values:

$$|a| = |-a|$$



- The distance between numbers a and b on the number line is given by $|a-b|$



Examples

Example 1. Calculate $|-6+|-2-3||$.

Solution. $|-6+|-2-3||=|-6+|-5||=|-6+5|=|-1|=1$.

Example 2. Which number is greater, $|-2|$ or -3?

Solution. Since $|-2|=2$, and $2>-3$, we get $|-2|>-3$.

Example 3. Find the distance between the numbers -7 and -3 on the number line.

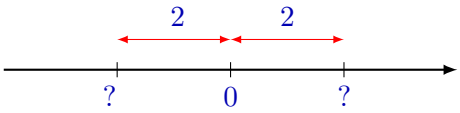
Solution. The distance between two numbers is given by the **absolute value** of the difference between them:

$$|-7-(-3)|=|-7+3|=|-4|=4.$$

Linear equations involving absolute value

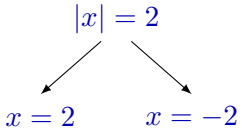
Example 1. Solve the equation $|x|=2$.

Solution. We have to find **all** values of the unknown x for which $|x|=2$, that is, all numbers which are located at the distance of 2 from 0.



These numbers are 2 and -2.

It is convenient to write down our solution as follows:



Answer. $x=2$ or $x=-2$.

Linear equations involving absolute value

Example 2. Solve the equation $|3x-1|=2$. Check your answer by substitution.

Solution.

$$|3x-1|=2$$

$$3x-1=2 \quad \textbf{or} \quad 3x-1=-2$$

$$3x=3 \qquad\qquad 3x=-1$$

$$x=1 \quad \textbf{or} \quad x=-1/3$$

Check now that both $x=1$ and $x=-1/3$ satisfy the original equation.

Plug in $x=1$:

$|3\cdot 1-1| \stackrel{?}{=} 2$

$|2| \stackrel{?}{=} 2$

$2 \stackrel{\checkmark}{=} 2$

Plug in $x=-1/3$:

$\left|3\cdot\left(-\frac{1}{3}\right)-1\right| \stackrel{?}{=} 2$

$|-1-1| \stackrel{?}{=} 2$

$|-2| \stackrel{?}{=} 2$

$2 \stackrel{\checkmark}{=} 2$

Answer. $x=1$ or $x=-1/3$.

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Linear inequalities involving absolute value

Example 1. Solve the inequality $|3x-1|<2$.
Give your answer in interval notation. Show the solution on the number line.

Solution. The inequality means that
the number $3x-1$ is on the distance **less** than 2 units from 0.

Therefore, this number should be in between -2 and 2:

$$-2<3x-1<2.$$

This double inequality is nothing but a system of inequalities:

$$-2<3x-1<2 \iff \begin{cases} -2<3x-1 \\ 3x-1<2 \end{cases} \iff \begin{cases} -2+1<3x \\ 3x<2+1 \end{cases} \iff$$

$$\begin{cases} -1<3x \\ 3x<3 \end{cases} \iff \begin{cases} -1/3<x \\ x<1 \end{cases} \iff -1/3<x<1$$

Answer. $(-1/3, 1)$

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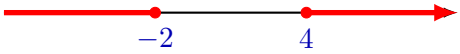
Linear inequalities involving absolute value

Example 2. Solve the inequality $|1-x| \geq 3$.
Give your answer in interval notation. Show the solution on the number line.

Solution. The inequality means that
the number $1-x$ is on the distance **more** than or equal to 3 units from 0.

Therefore, the number $1-x$ should be ≥ 3 **or** ≤ -3:

$$\begin{array}{rcl} 1-x \geq 3 & \text{or} & 1-x \leq -3 \\ -x \geq 2 & \text{or} & -x \leq -4 \\ x \leq -2 & \text{or} & x \geq 4 \end{array}$$



The solution is the **union** of two intervals: $(-\infty, -2) \cup (4, \infty)$.

Answer. $(-\infty, -2) \cup (4, \infty)$

Summary

In this lecture, we have learned

- ☑ what **absolute value** of a number is
- ☑ what the **properties** of absolute value are
- ☑ how to solve **linear equations** involving absolute value
- ☑ how to solve **linear inequalities** involving absolute value