How to See Things in the Most Efficient Way

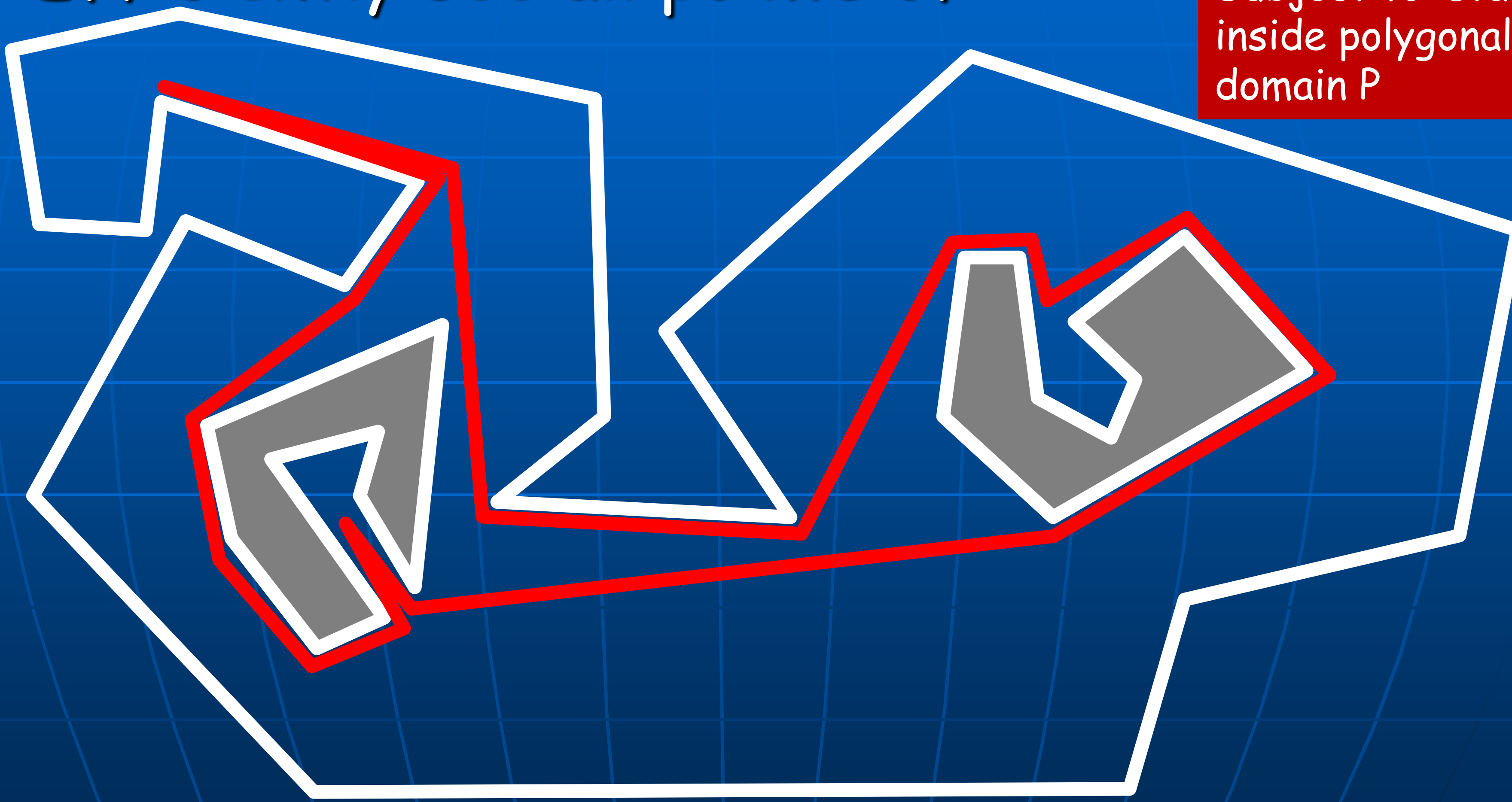
Joe Mitchell
Stony Brook University

Math Colloquium, Feb 20, 2014

Watchman Route Problem

- Efficiently see all points of P

Subject to: stay inside polygonal domain P



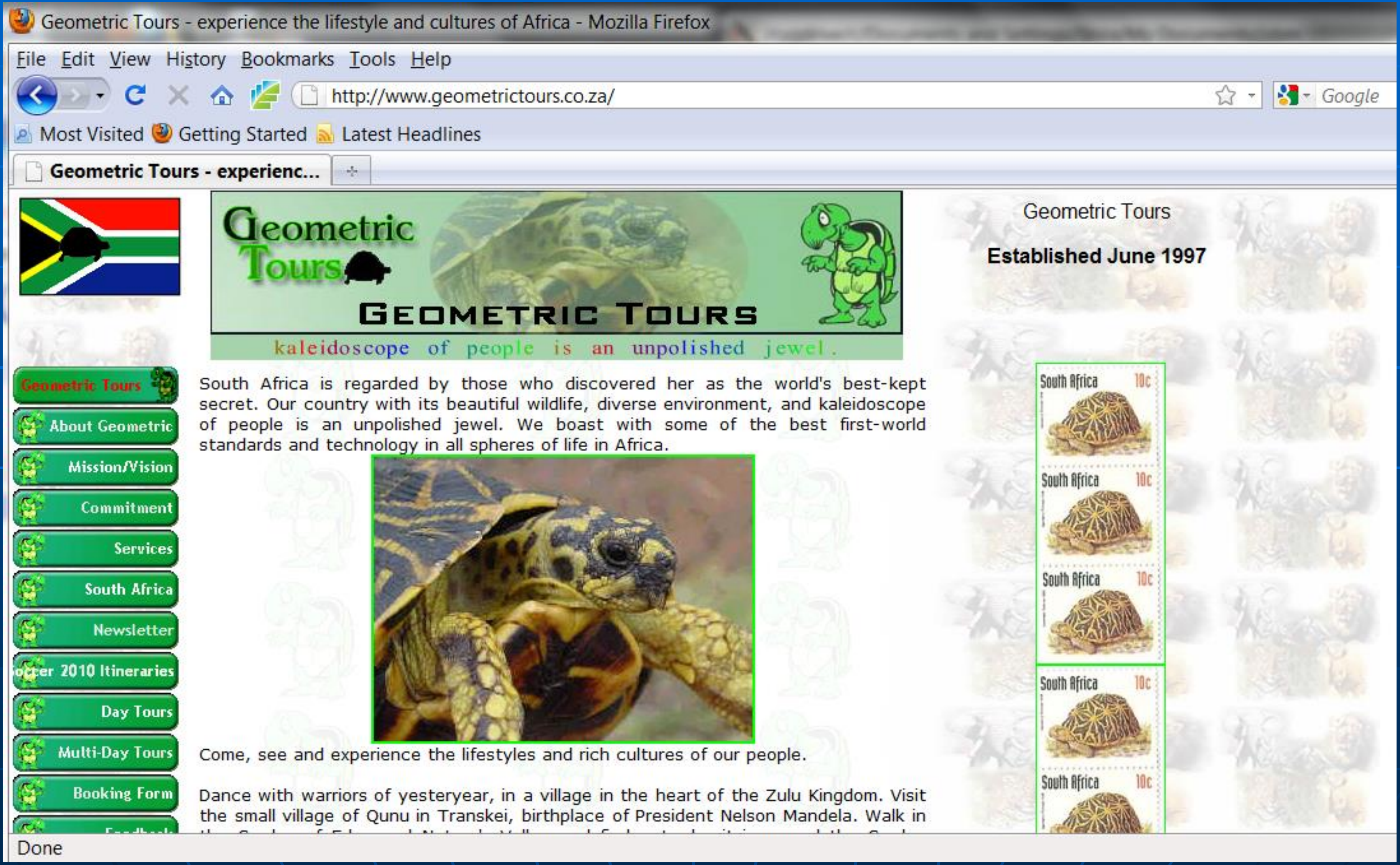
Watchman Route Problem (WRP)

General Problem

Find an "optimal" set X that "sees" all of a set Y

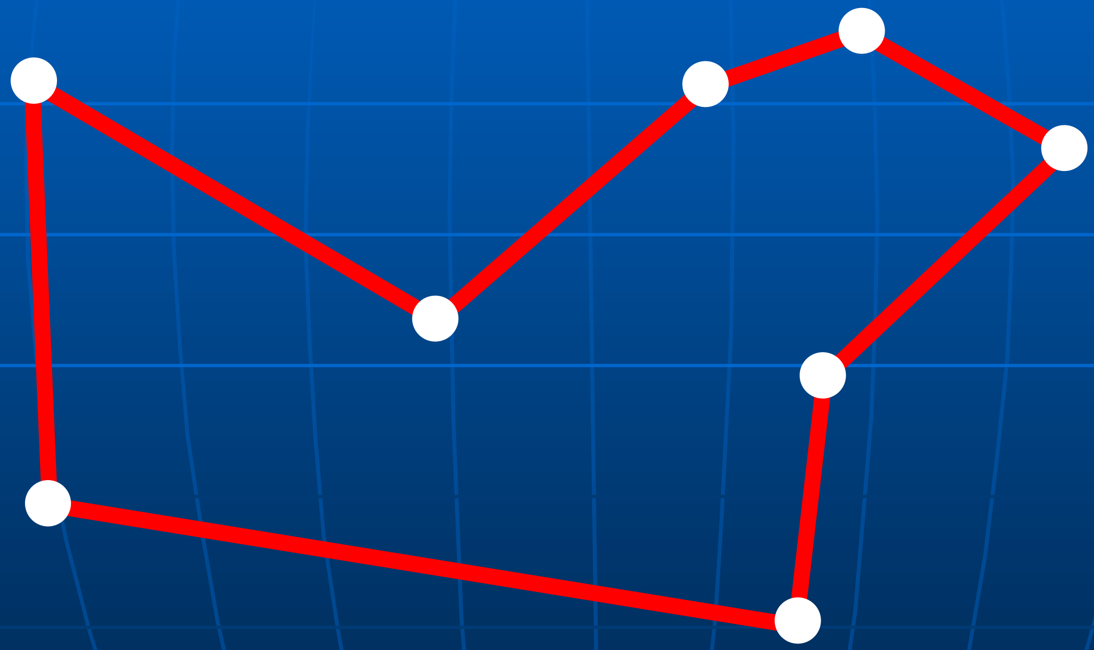
Geometric Covering Tours

What are "geometric covering tours"?



Covering Tours

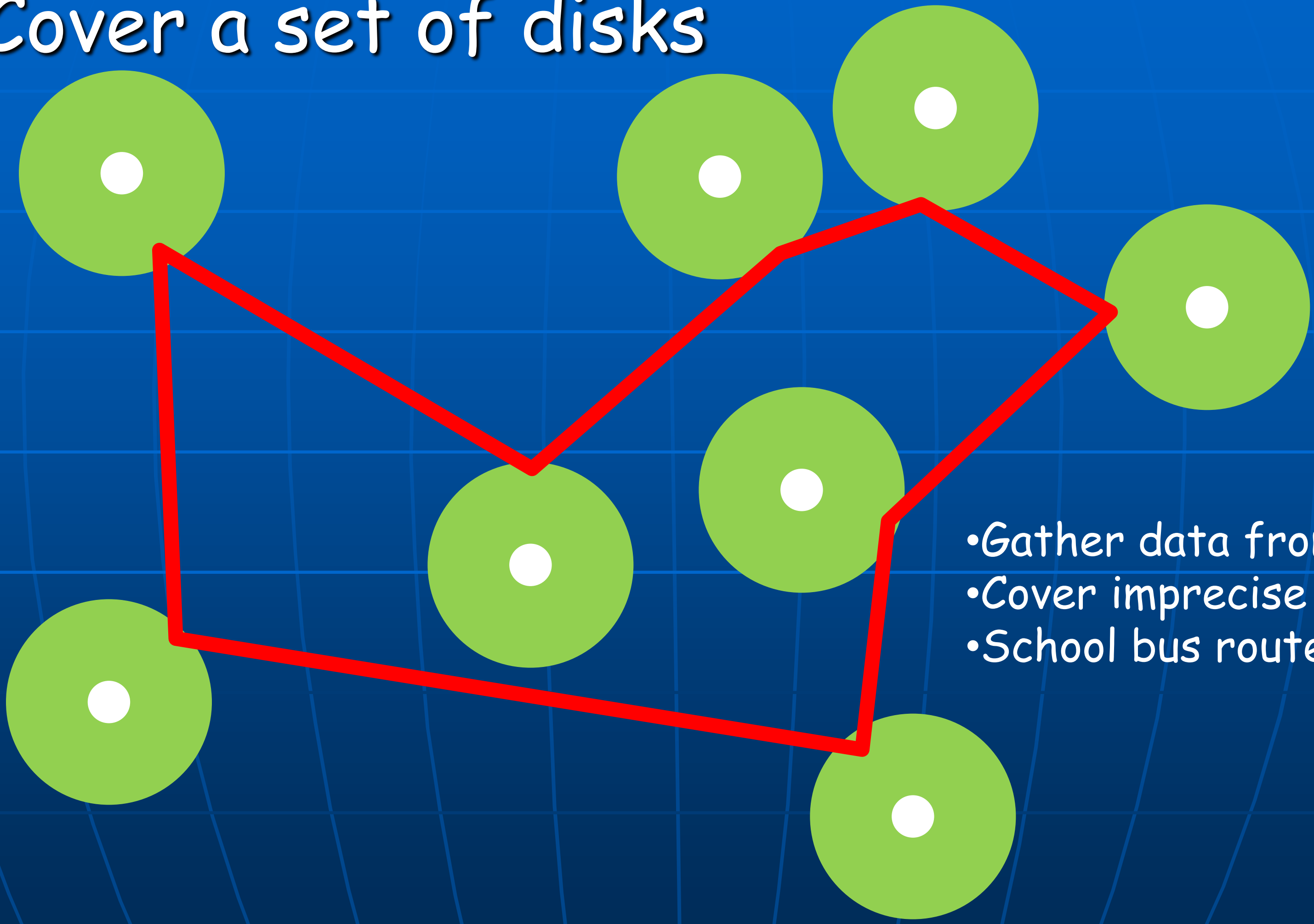
- Cover a point set S



Just geometric TSP

Covering Tours

- Cover a set of disks

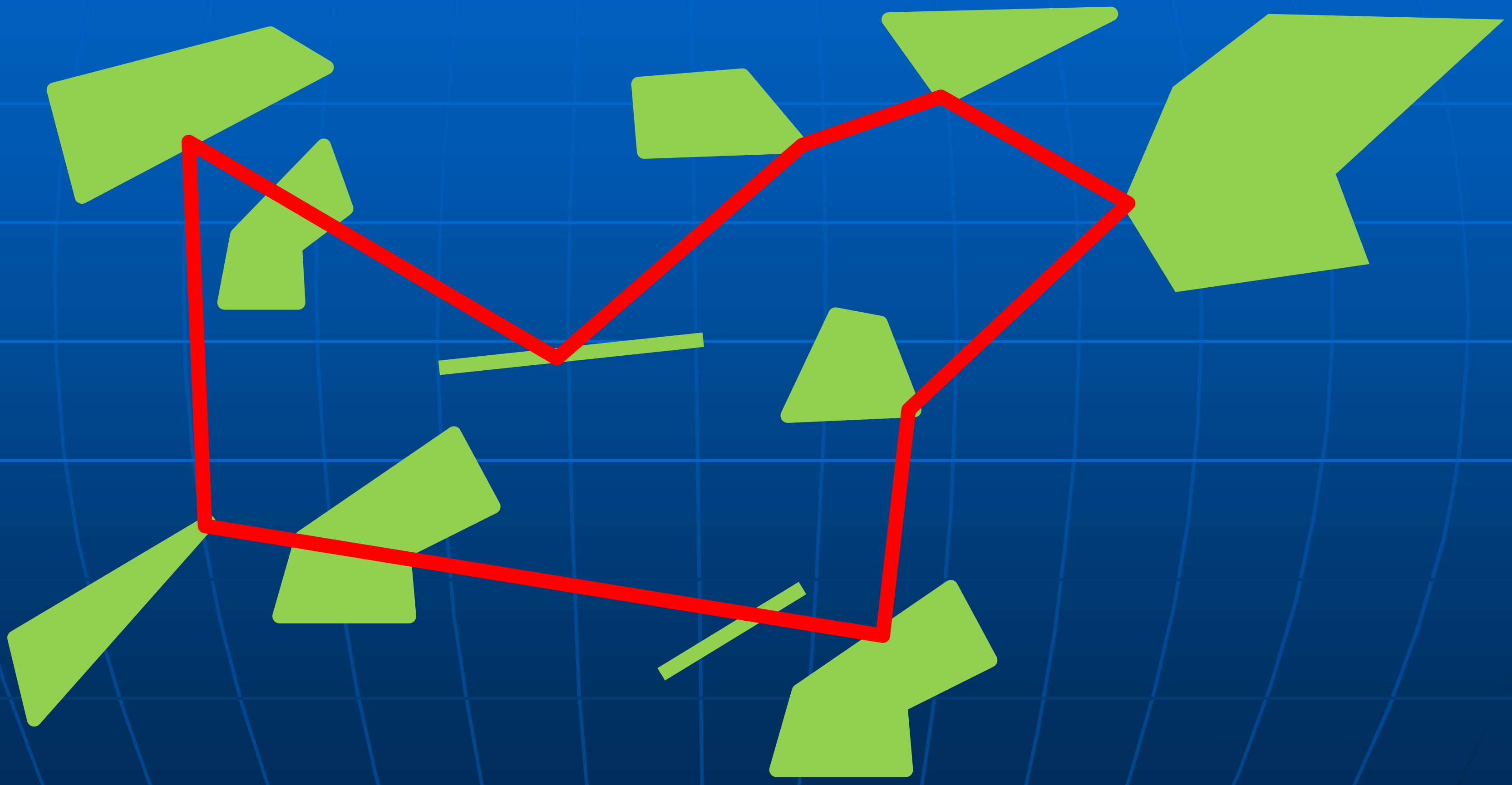


- Gather data from sensors
- Cover imprecise points
- School bus route

TSP with (circular) neighborhoods

Covering Tours

- Cover a set of polygons

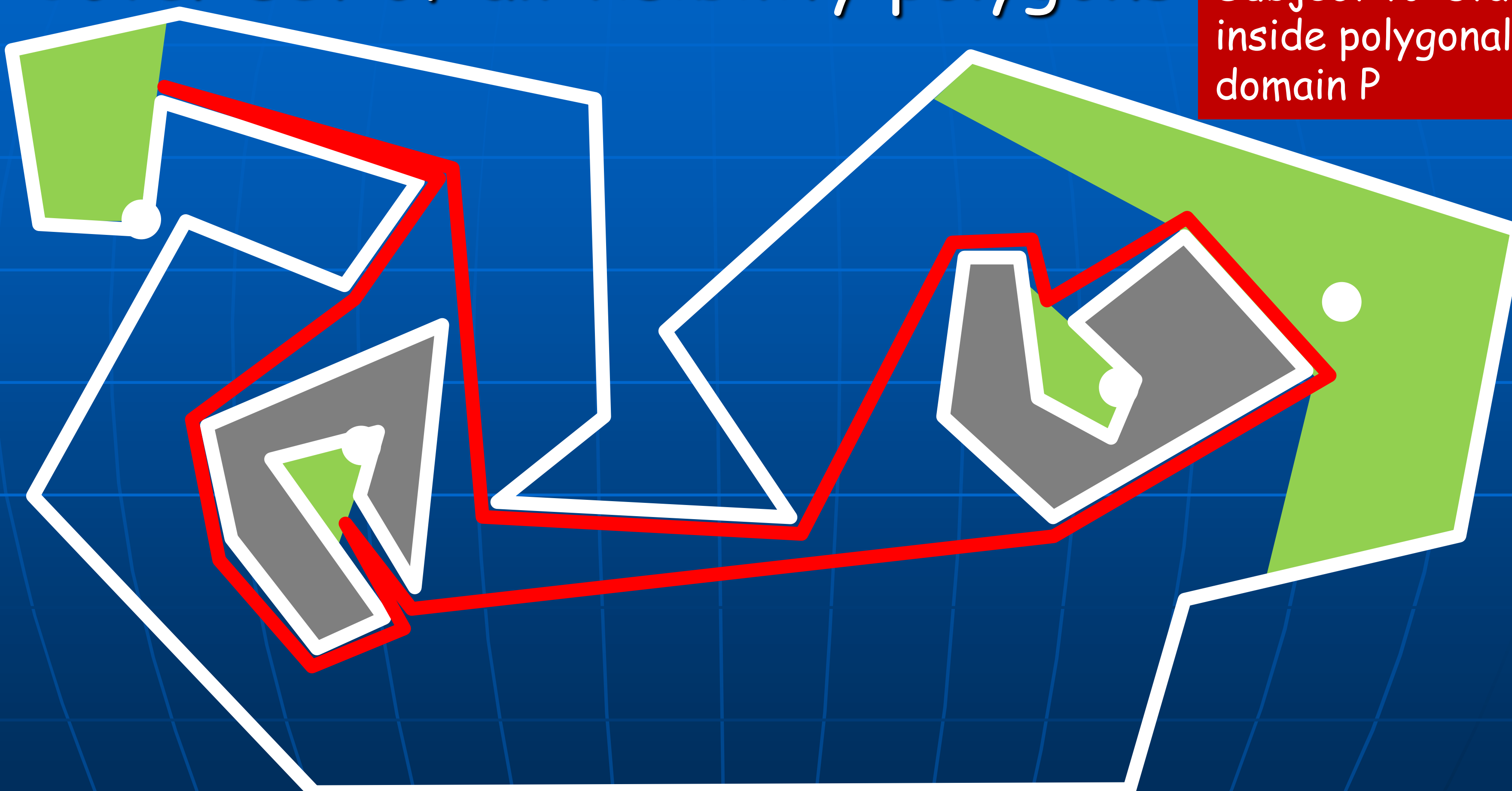


TSP with neighborhoods (TSPN)

Covering Tours

- Cover set of all visibility polygons

Subject to: stay inside polygonal domain P



Watchman Route Problem (WRP)

Watchman Route Problem
WRP

Geometric Covering Tour on the set of all visibility regions, VP(p), for all p in domain

Motivations from Robotics, etc

Exploration Strategies for a Robot with a Continously Rotating 3D Scanner

Elena Digor, Andreas Birk, and Andreas Nüchter

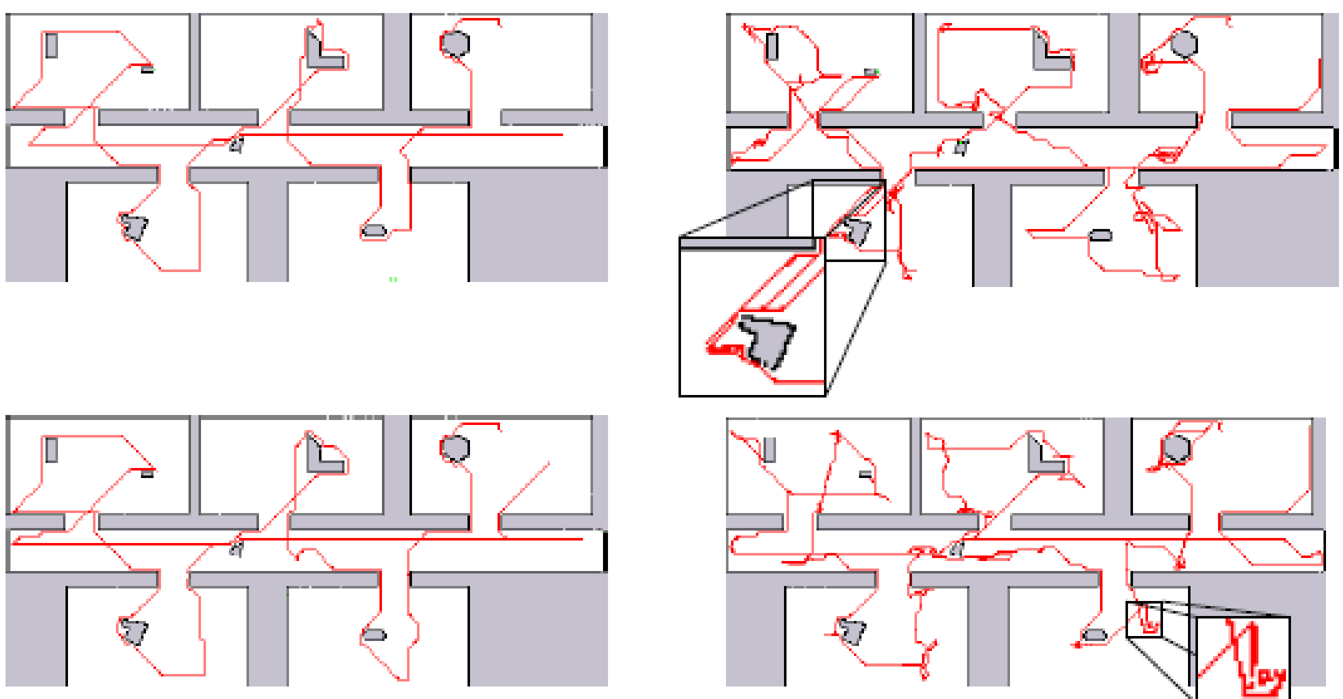


Fig. 7. Results in the office map with clutter. From left to right: Stop-scan-replanning-go, Scan-replanning-go, Continuously-replanning-with-stopping, and Continuously-replanning-go strategy.

Some History

- SoCG 1986: Chin and Ntafos
  - NP-hardness in 2D,3D;
  - O(n) in rectilinear, simple polygons

Revisited:[Dumitrescu, Toth 2012]

- DCG 1988: Chin and Ntafos:
  - $O(n^4)$ for *anchored* WRP in *simple* polygon
- ISA 1991, IJCGA 1993: Tan, Hirata, Inagaki:
  - $O(n^3)$ for *anchored* WRP in *simple* polygon
- ISAAC 1993: Tan, Hirata:
  - $O(n^2)$ for *anchored* WRP in *simple* polygon (D&C)
- ISAAC 1993: Carlsson, Jonsson, Nilsson:
  - $O(n^3)$ for *floating* (unrestricted) WRP in simple polygon

Some History (cont)

- FCT 1997: Hammar, Nilsson: all prior algorithms require *exponential* # of adjustments! First attempt to fix…
- IJCGA 1999: Tan, Hirata, Inagaki: DP to remove exponential behavior: $O(n^4)$ for *anchored* WRP
- DCG 1999: Carlsson, Jonsson, Nilsson: $O(n^6)$ for *floating* WRP
- IPL 2001: Tan: $O(n^5)$ for *floating* WRP
- STOC 2003: Dror, Efrat, Lubiw, M: Touring Polygons Problem: $O(n^3 \log n)$ for *anchored*, $O(n^4 \log n)$ for *floating*

OPEN: Improve these bounds?

Bottom Line

- Watchman Route in simple n-gon: Exact algorithm, time $O(n^3 \log n)$ for *anchored*, $O(n^4 \log n)$ for *floating*

OPEN: Improve these bounds?

- *NP-hard* in polygons with holes and in 3D

WRP Approximation

- Simple polygons:
  - Sqrt(2)-approx, O(n), for anchored [Tan, DAM 2004]
  - 14(π+4)=99.98-approx, O(n log n), for floating [Carlsson, Jonsson, Nilsson, TR 1997]
  - 2-approx, O(n), for floating [Tan, TCS 2007]
  - 4-approx, O(n^2), for min-link [Alsuwaiyel, Lee, IPL 1995]
- Polygons with holes?

This Talk: O($\log^2$ n), Ω(log n)

  - O(log n)-approx, rectilinear, rectangle-visibility
- WRP in 3D: No constant-factor, unless P=NP

[Safra, Schwartz 2003]

Ω(log n), even for terrains

WRP Taxonomy

- Type of domain
  - Without/with holes in 2D; arrangements/networks
  - Terrains (2.5D), 3D, higher
- Anchored vs floating
- Variations on visibility
  - Bounded view distance
  - Robust visibility, α-visibility, rectangle-visibility
- # of watchmen, min-max vs min-sum
- Metric/objective function
  - Euclidean length, link length, scan cost, etc
- Offline vs online

Two Key Aspects of WRP

Coverage

Connectivity

Hence, "Geometric Covering Tours/Trees"

Related Problems in Graphs

- Connected vertex cover

  aka "min-cost tree cover" (edge-dominating)

  [Arkin, Halldórsson, Hassin, 1993] 3.55-approx

  [Fujito, 2012] 2-approx, trimming an MST

- Connected dominating set (vertex-dominating)

  [Guha, Khuller]

- Group Steiner tree/TSP aka "one-of-a-set" TSP/MST
  - $O(\log^2 n \log k)$-approx for k groups

    [Fakcharoenphol et al 2003]

  - (3/2)s-approx if each set of size ≤ s

    [Slavik 1997 "errand scheduling"]

Related Geometric Problems

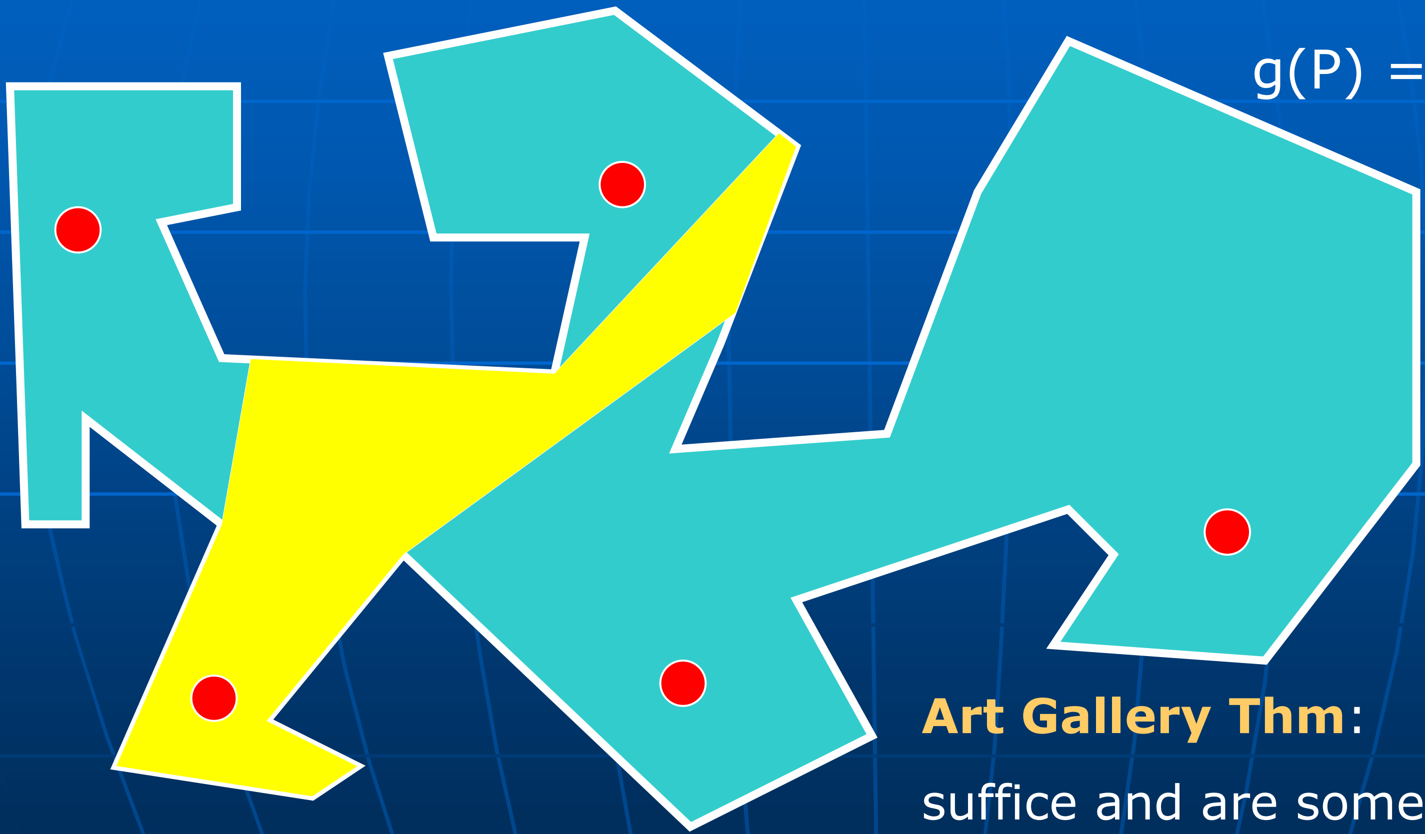
- **Guard cover**: min # guards (stationary)

The Art Gallery Problem

Determine a small set of "guards" to see all of a given n-vertex polygon P

NP-hard, even in simple polygon

g(P) = min # guards

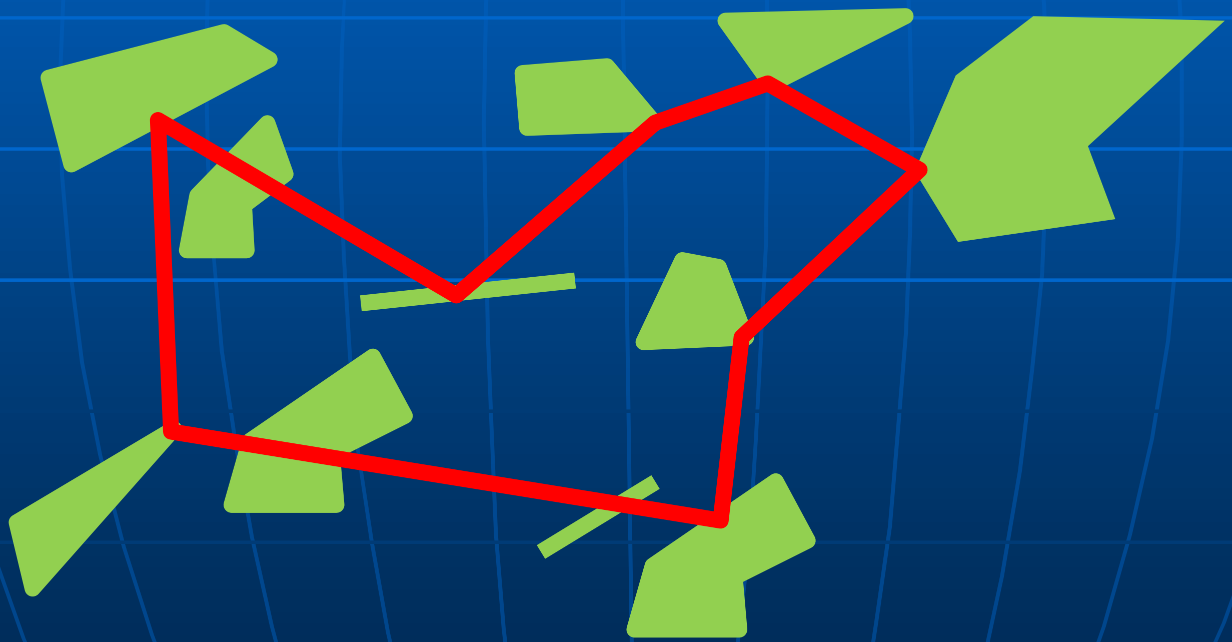


Art Gallery Thm: $\left\lfloor \frac{n}{3} \right\rfloor$ guards suffice and are sometimes necessary

Motivation: Sensor coverage, security

Related Geometric Problems

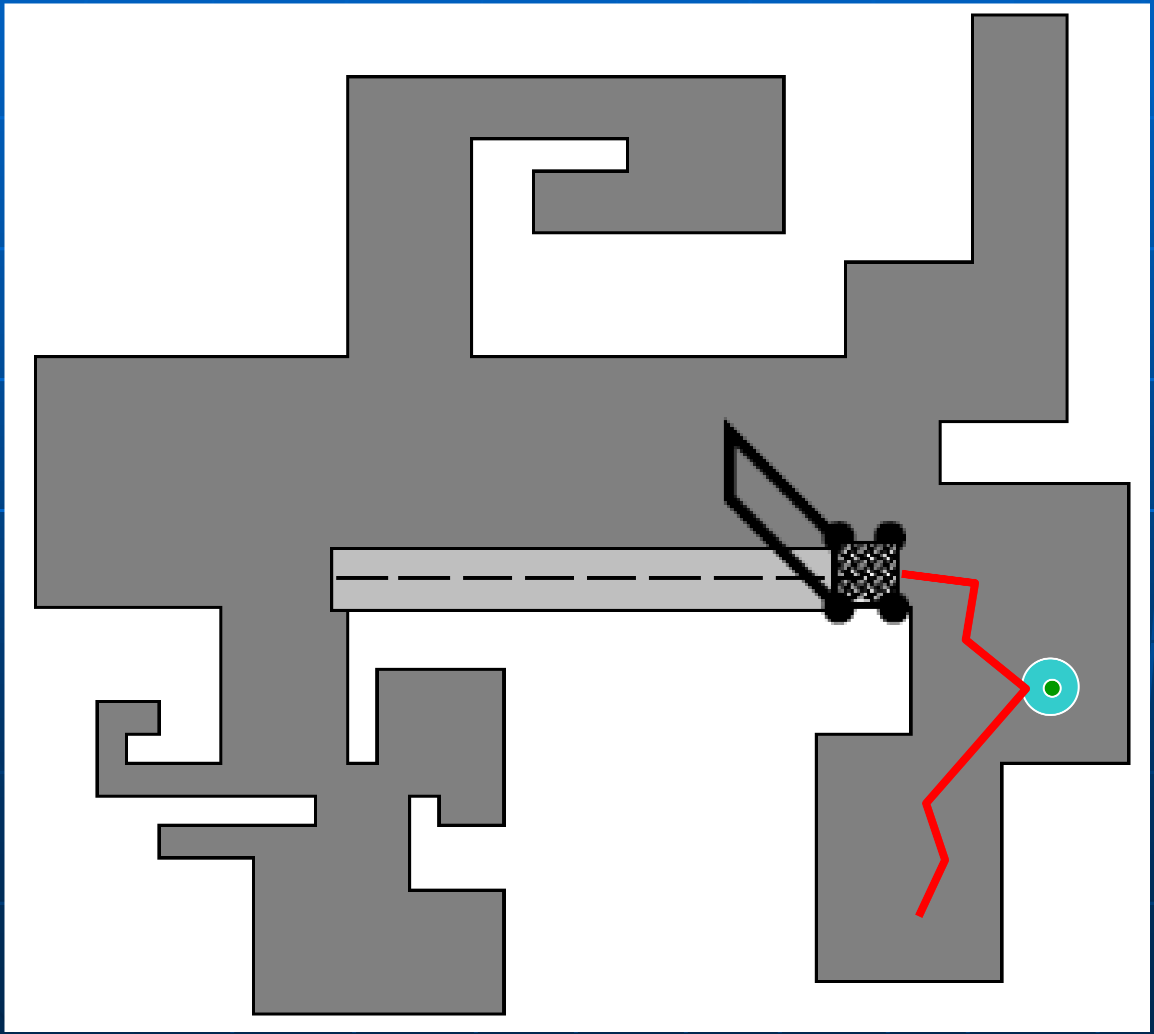
- **TSP with Neighborhoods** (TSPN)



(AND obstacles)

Related Geometric Problems

- Lawnmower/Milling



Best method of **mowing** the lawn?

NC-machining: **milling** a pocket.

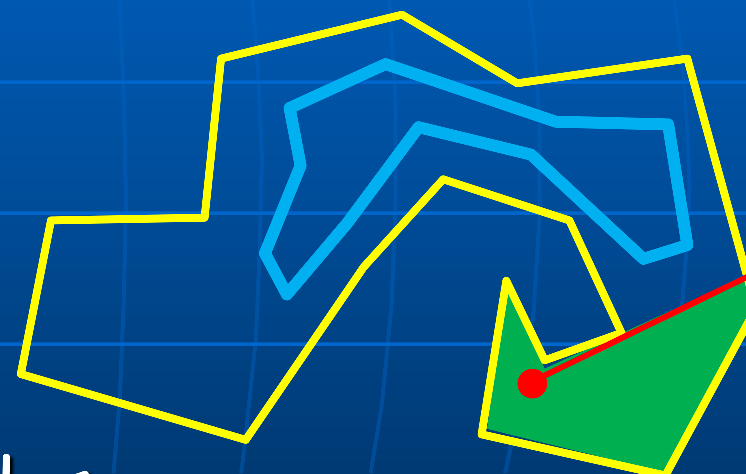
[Ntafos, CGTA 1992]: d-sweeper: must be within distance d to see a point

TSPN: Visit the disk centered at each blade of grass

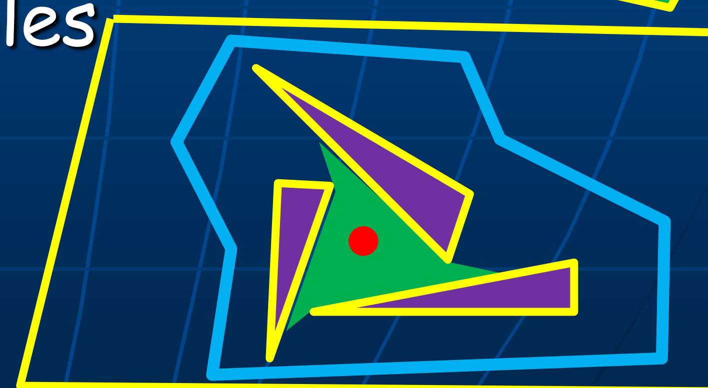
Understanding Structure

How Much Needs to be Covered?

- Must visit VP(p) for ***all*** p in P
- **Q**: Is it enough for the tree/tour to see all vertices of P?
  - YES, in simple polgyon P



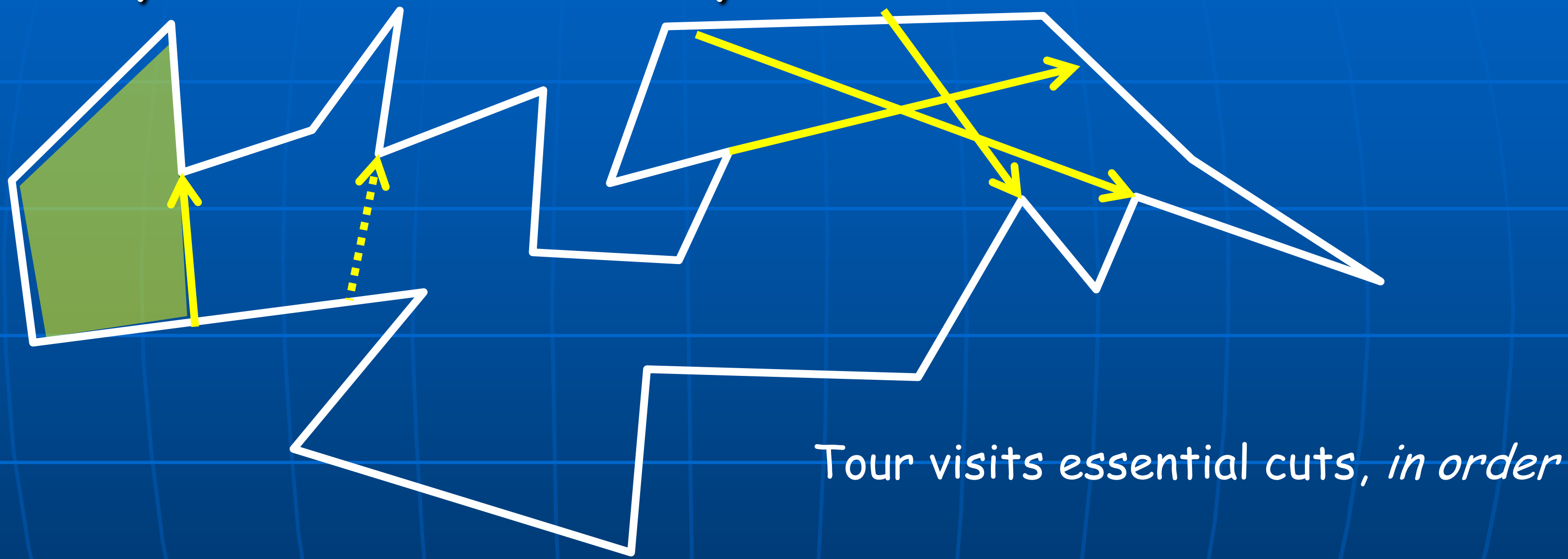
  - NO, in polygons with holes



Not even enough to see all of the boundary of P

WRP Structure in Simple Polygons

- Cuts, essential cuts, corner



- One can compute all essential cuts: O(n)

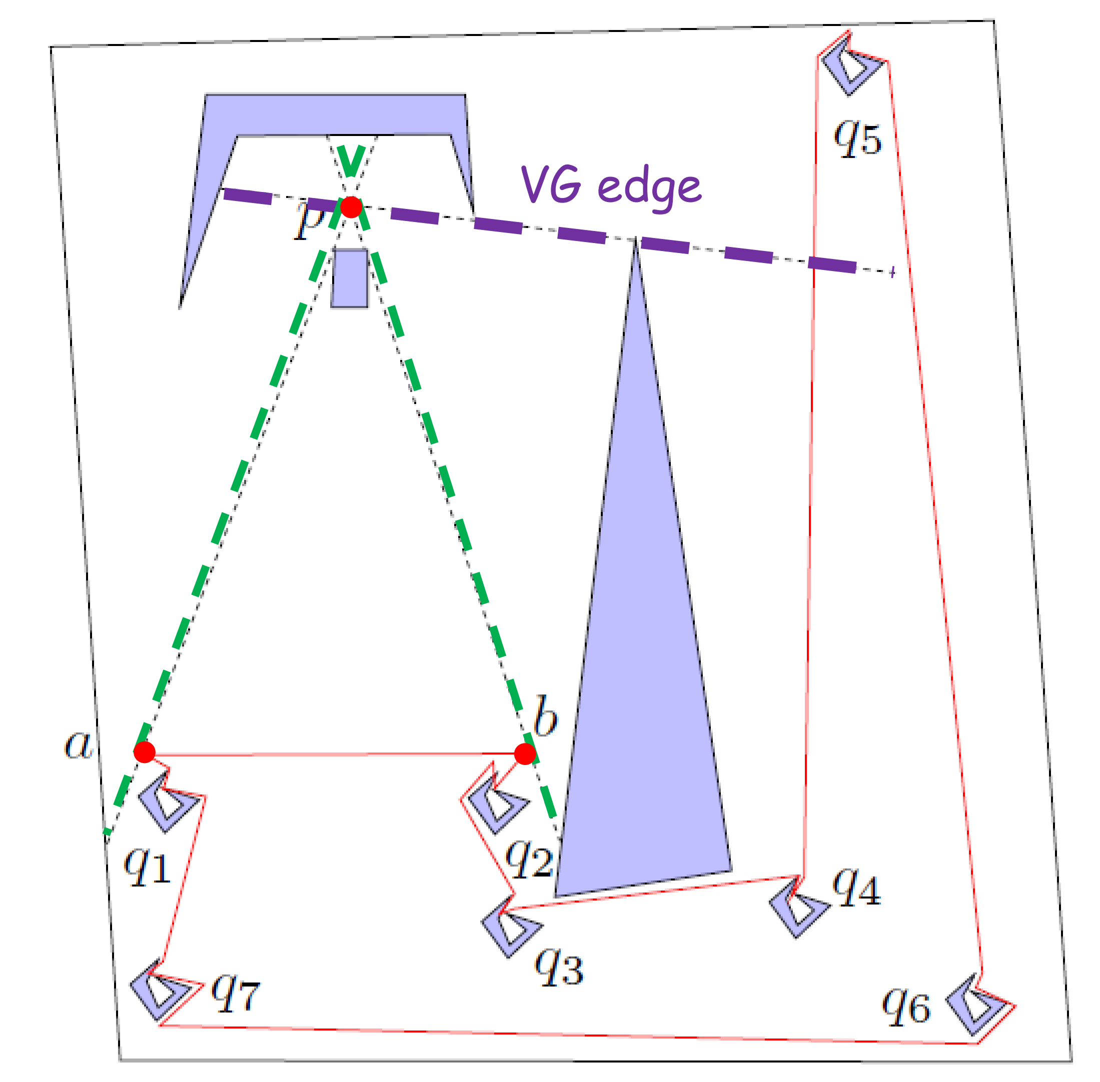
[Tan, 2007]

WRP Example: Effect of Holes

Complicating Issue:

Tour reflects off of segments that are not readily known (e.g., edges of P, VG edges)

Reminiscent of art gallery problem



Bounds on WRP Tour Length

- Upper bound on length of tour, in terms of h (# holes), per(P) and diam(P)

$$O(per(P) + \sqrt{h} \cdot diam(P))$$

tight for polygons P with $per(P) > c \cdot diam(P)$, for any fixed $c > 2$

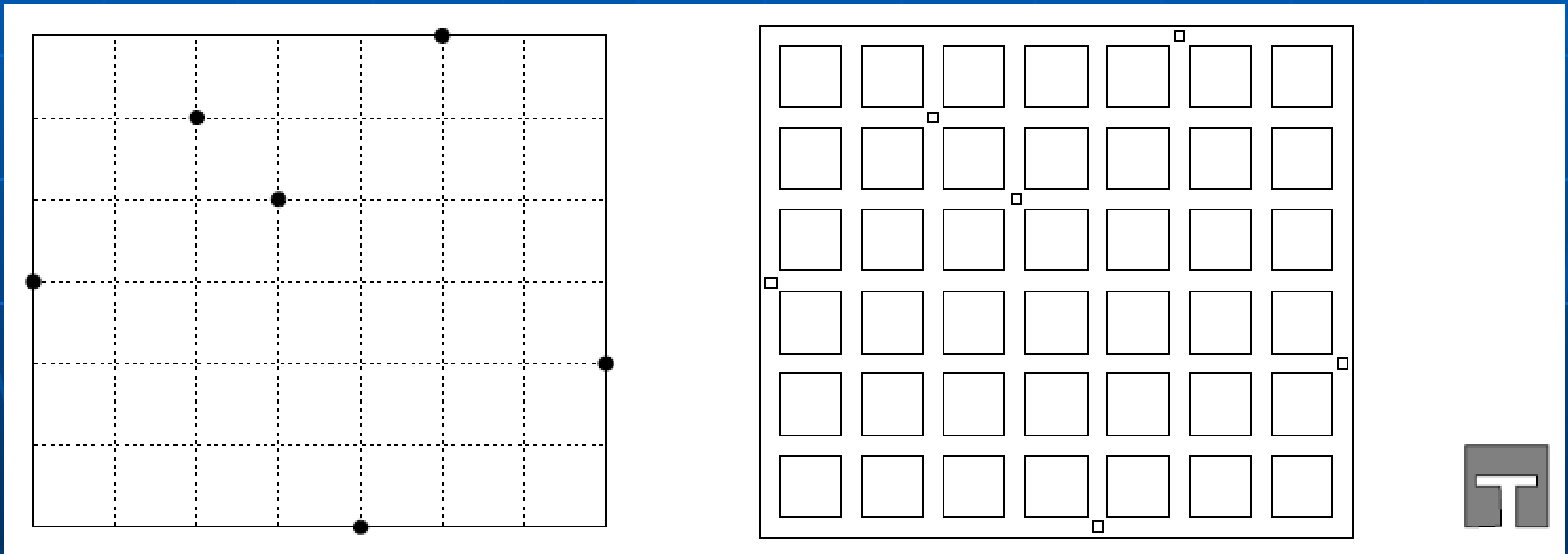
[Dumitrescu, Toth, *CCCG 2010*, *CGTA 2012*] Also bounds in 3D

[Czyzowicz,Ilcinkas,Labourel,Pelc, SWAT 2010]
Exploring an *unknown* domain. Also bounds in terms of area(P) in limited visibility model

- Given P, can compute in O(n log n) time

WRP in Polygons with Holes

- Rectilinear polygon with holes: **NP-hard**
  - From geometric TSP in L_1 metric



[Dumitrescu, Toth]

WRP in Simple Polygons

- Best time bounds based on modelling as "Touring Polygons Problem" (TPP)

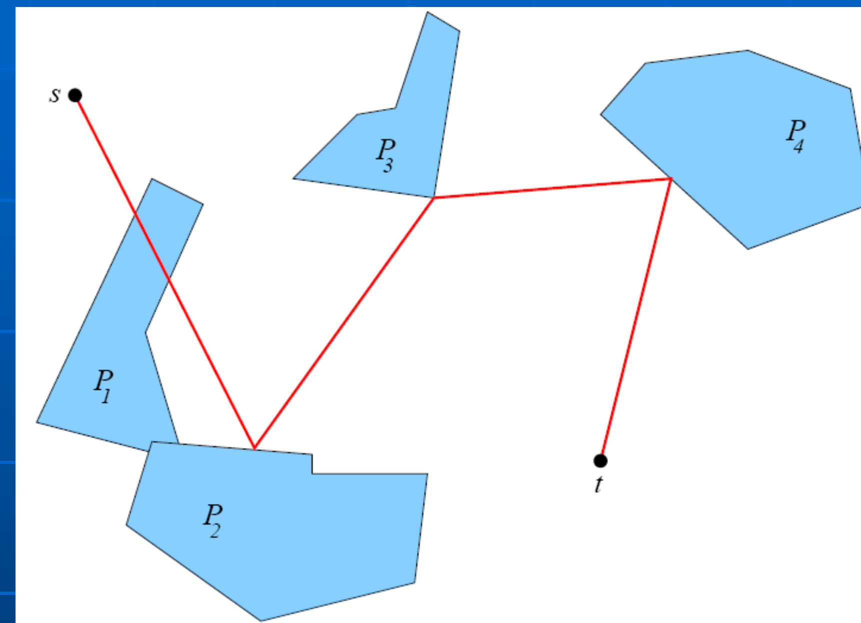
[Dror,Efrat,Lubiw,M, STOC 2003]

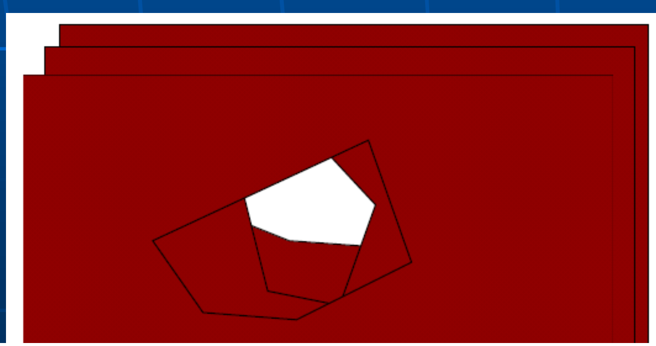
Ordered Covering Tours/Paths

- Order given [DELM, 2003]

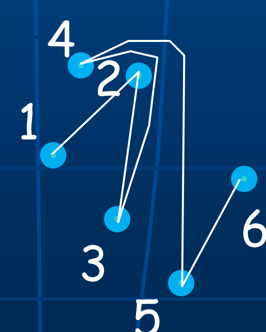
  Convex: poly-time

  Non-convex, overlapping: NP-hard
- Related to 3D shortest paths



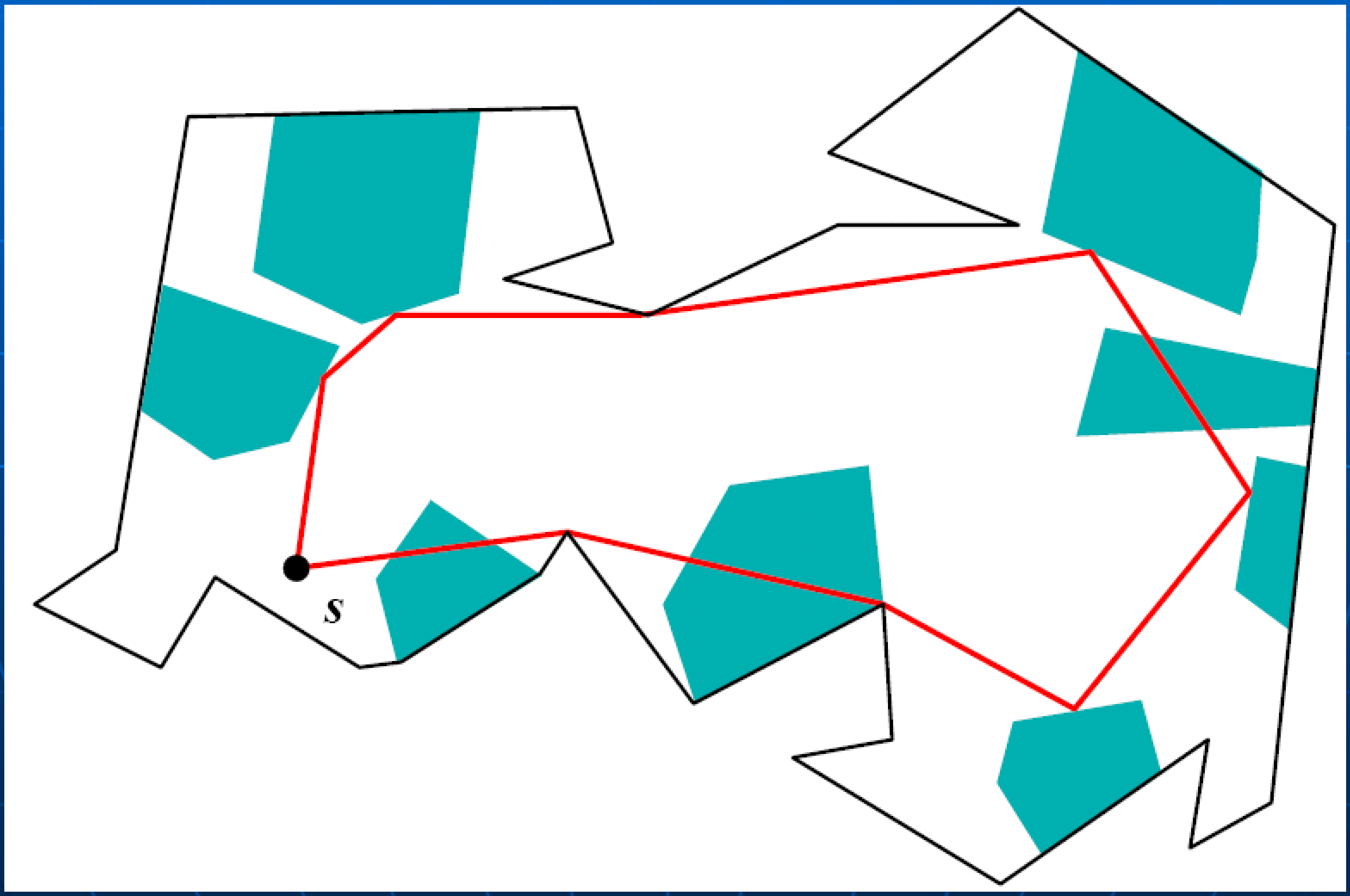


Q: Disjoint non-convex?

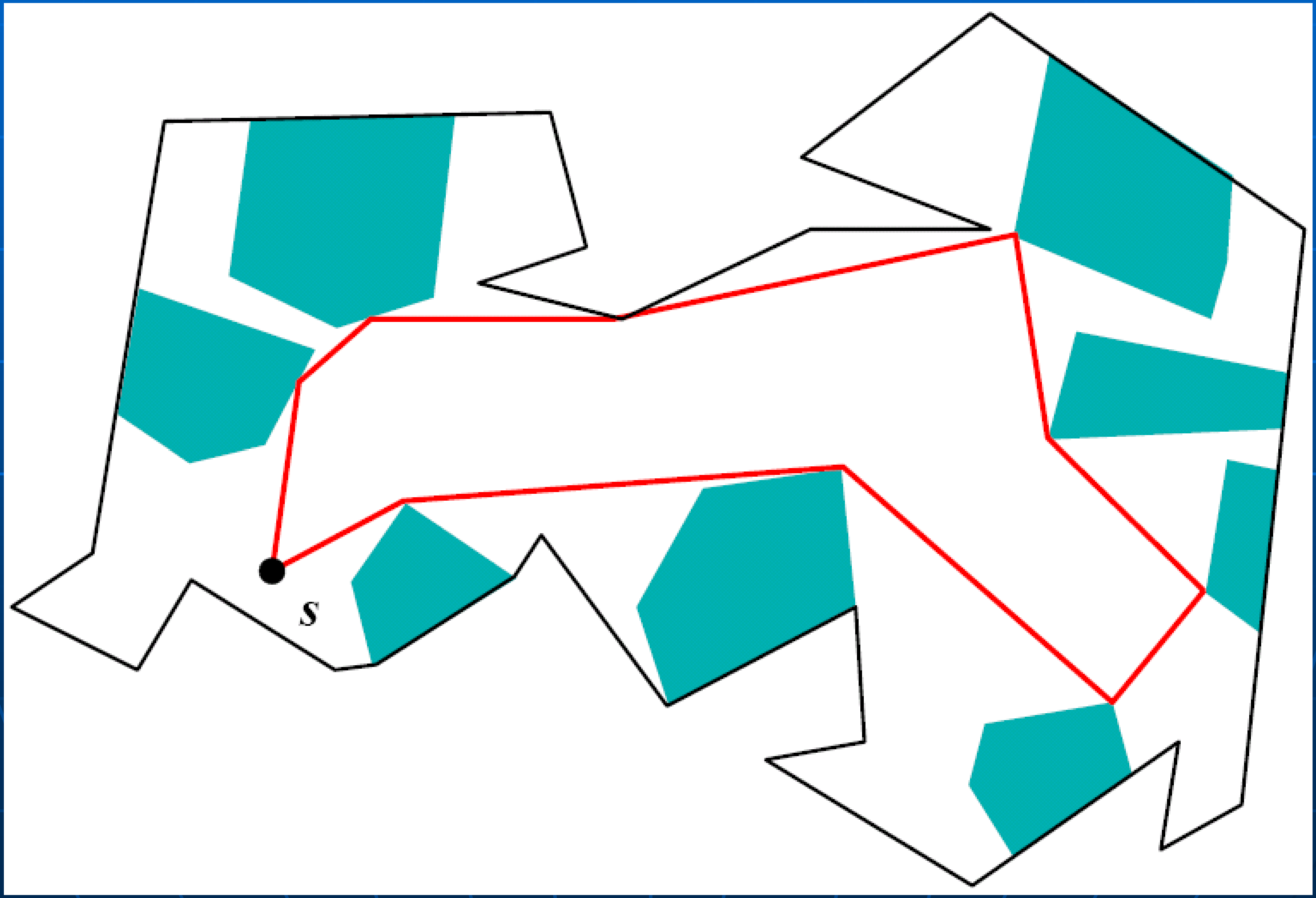


Q: Shortest **simple** tour, even for points?

Safari Problem

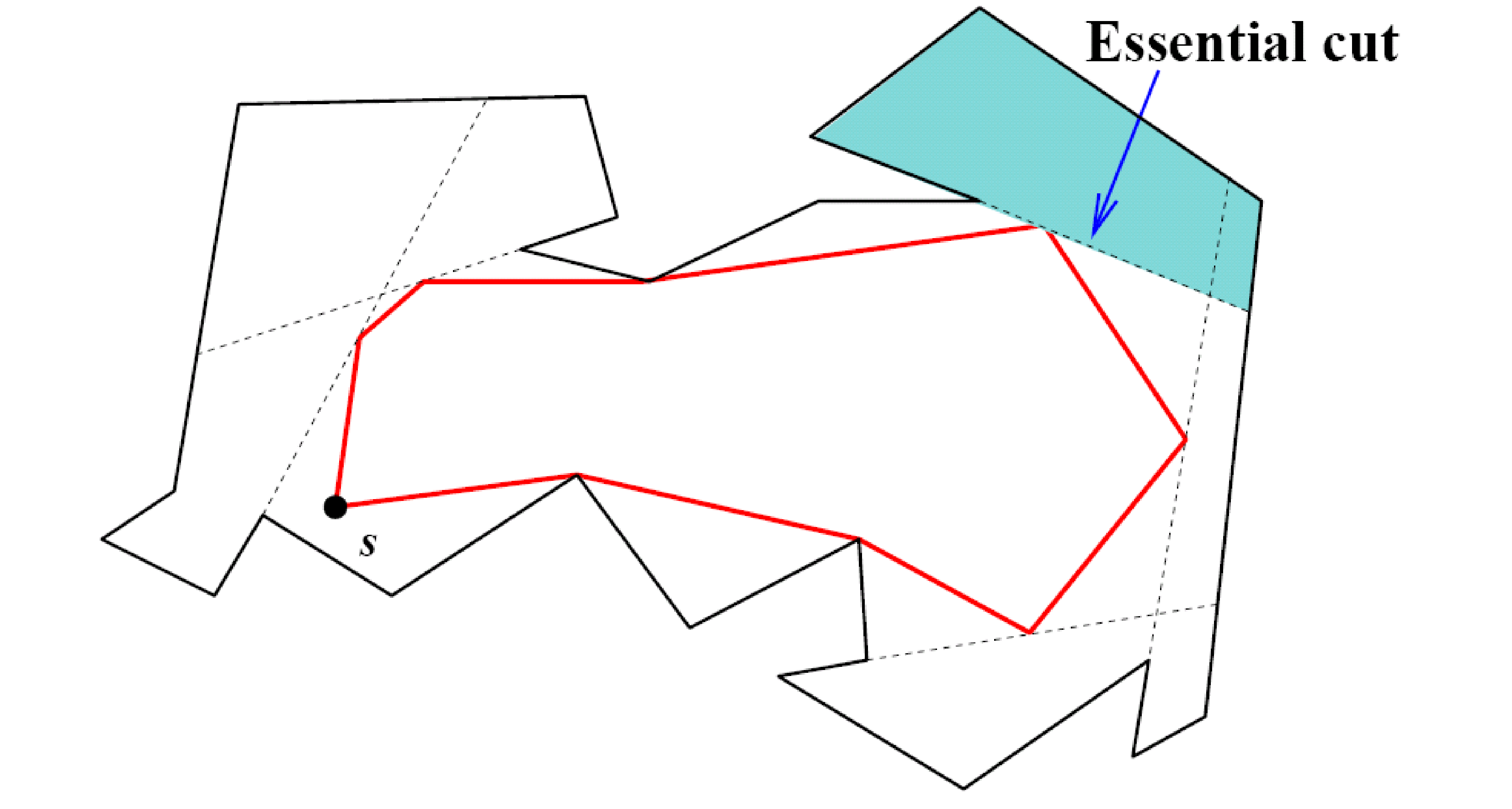


Zookeeper Problem



Watchman Route Problem

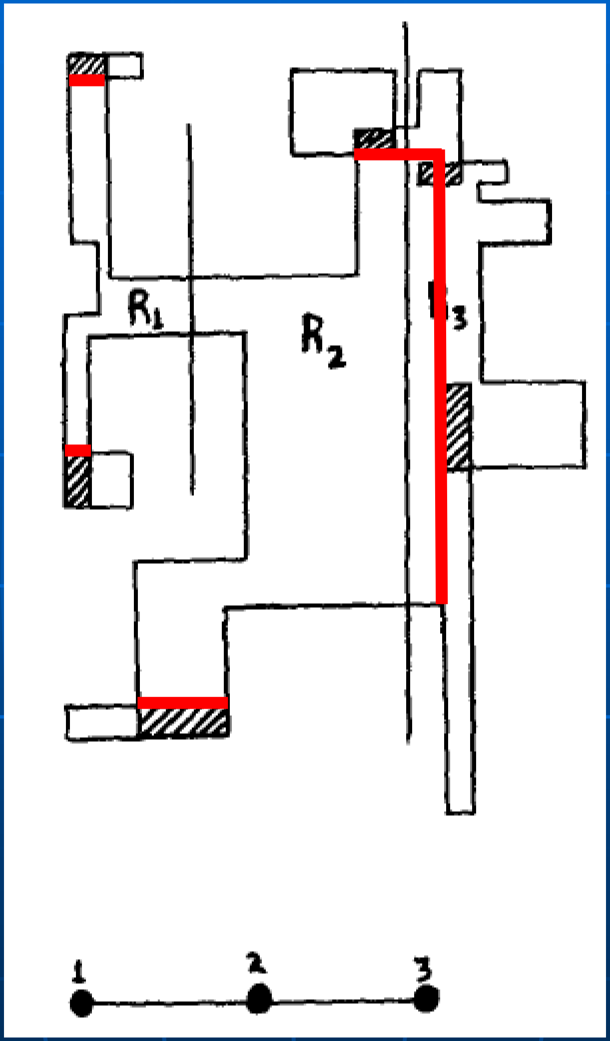
- Find a shortest tour for a guard to be able to see all of the domain



Fact: The optimal path visits the essential cuts in the order they appear along ∂P.

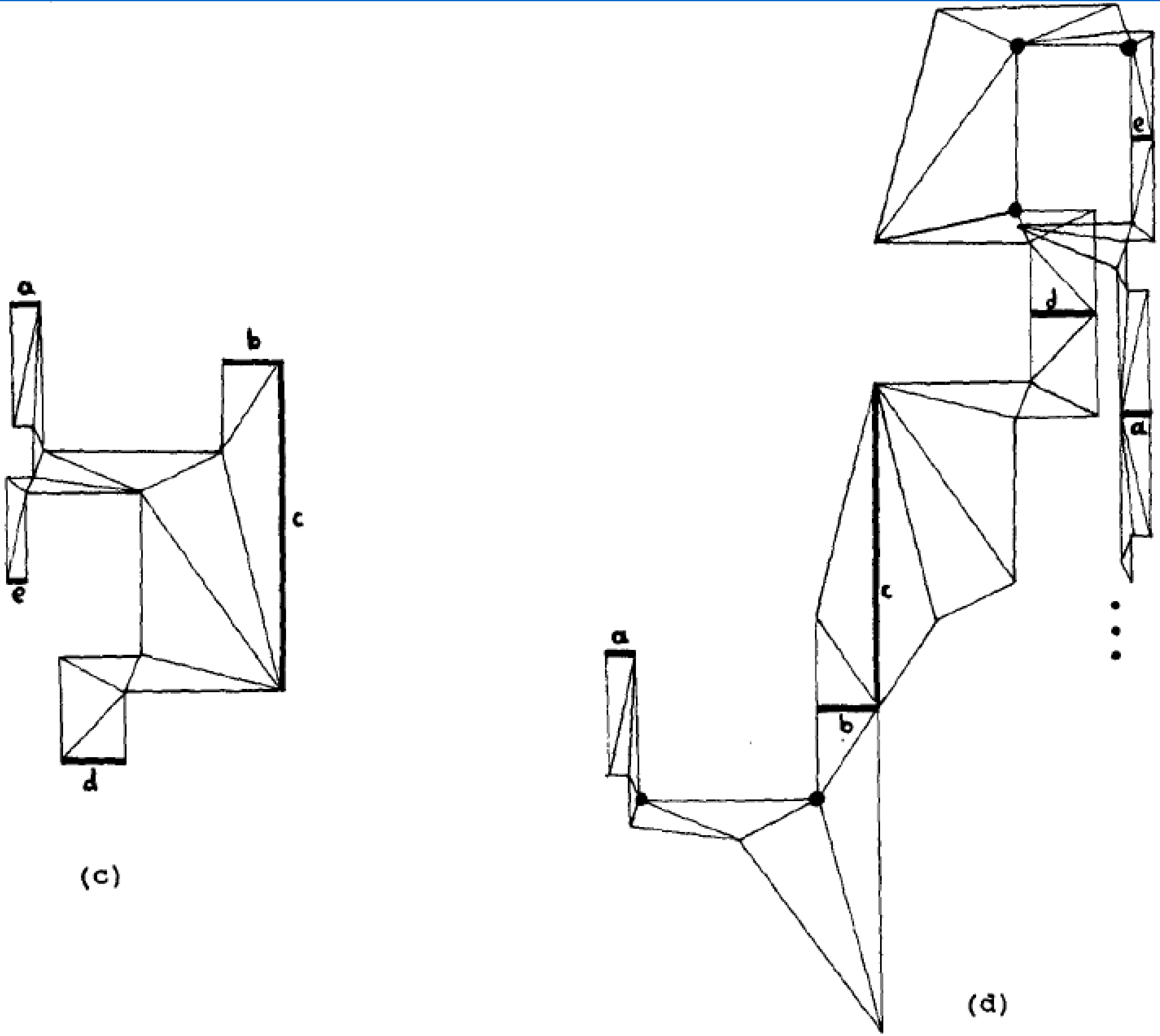
Special Cases of WRP

- (1) Simple, rectilinear polygons:
  O(n) time



Essential Cuts

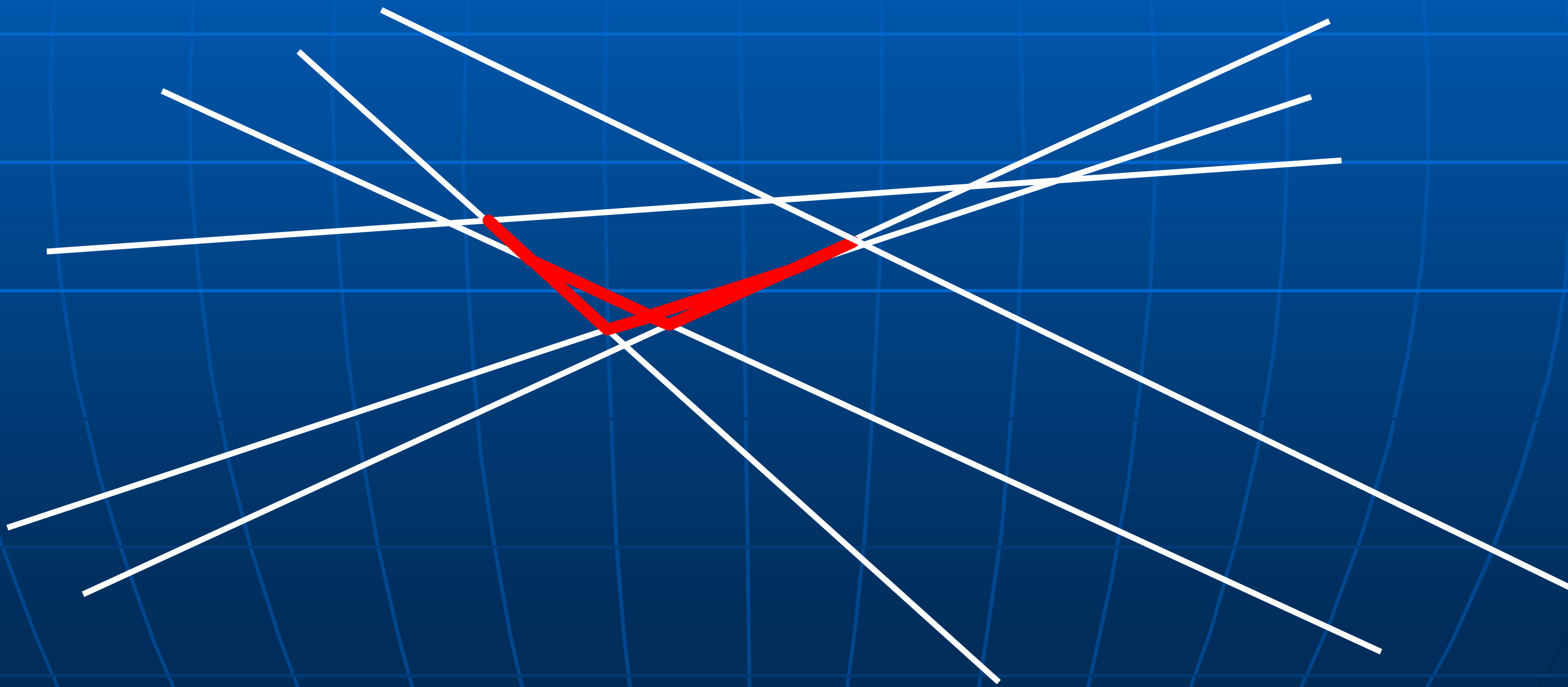
[Chin, Ntafos, SoCG 1986]


a
b
c
e
d
(c)
a
b
c
d
e
a
(d)

Special Cases of WRP

- (2) Watchman on an arrangement of lines
  - Exact polytime algorithm (DP to search for CH)

[Dumitrescu, M, Zylinski, SWAT 2012]

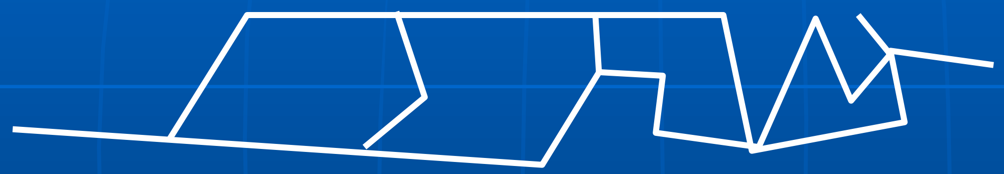


Special Cases of WRP

- (3) Thin polygons (PSLG's), segment arrangements

NP-hard

"Frank's Problem"



  - polylog-approx using one-of-a-set TSP on sets of collinear vertices along straight paths

$O(\log^3 n)$-approx [Dumitrescu,M,Zylinski, SWAT, 2012]
1.5c-approx if straight corridors have < c vertices

This Talk: $O(\log^2 n)$

  - 2-approx if no straight corridors (collinear adjacent edges)

Connected vertex cover, [AHH], [Fujito]

  - O(1)-approx if axis-parallel segments

New

Hardness of Approximation: WRP in Polygons with Holes

- $\Omega(\log n)$: From **Set-Cover**:
  - Sets S_1, S_2, …, S_m, and elements $U=\{x_1, x_2, \ldots, x_n\}$

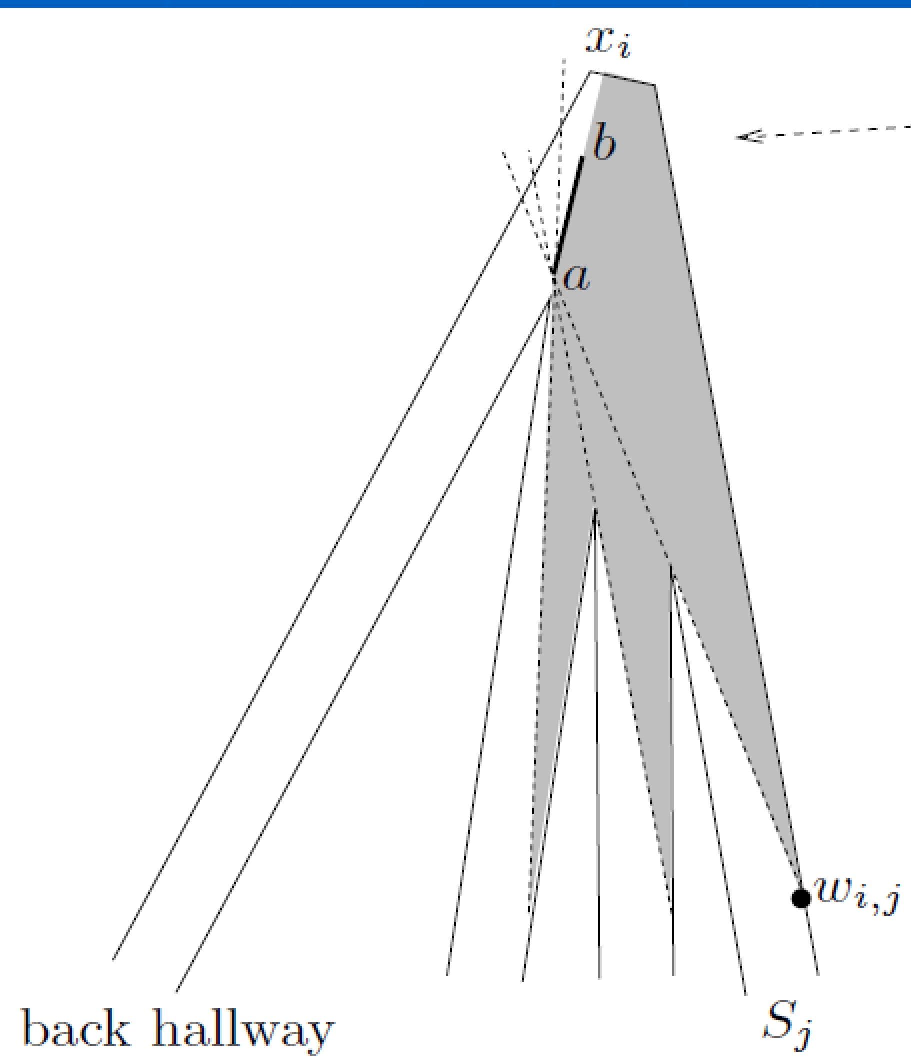
THEOREM 7.1. *The watchman route problem in a planar polygonal domain cannot be approximated in polynomial time within a factor $c \log n$, for some constant $c > 0$, assuming $P \neq NP$.*

Same holds for WRP on terrains (2.5D)

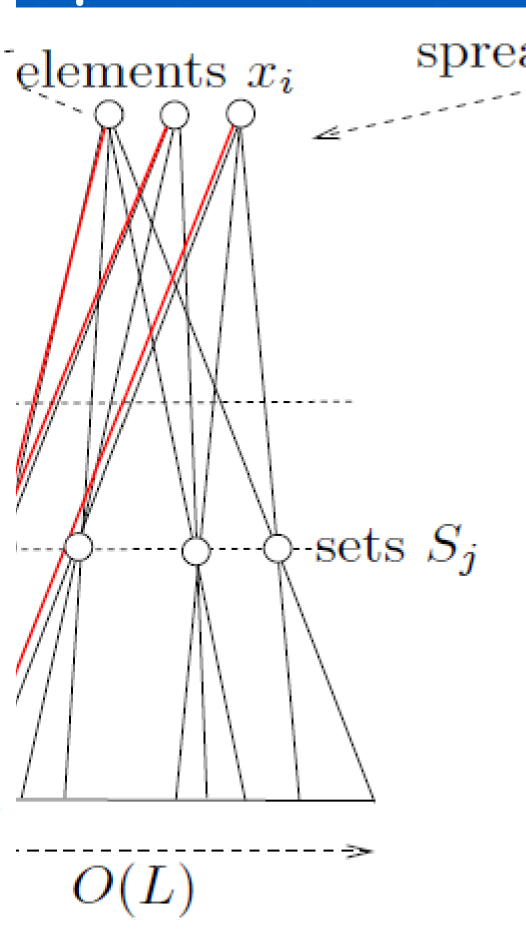
Hardness of Approximation

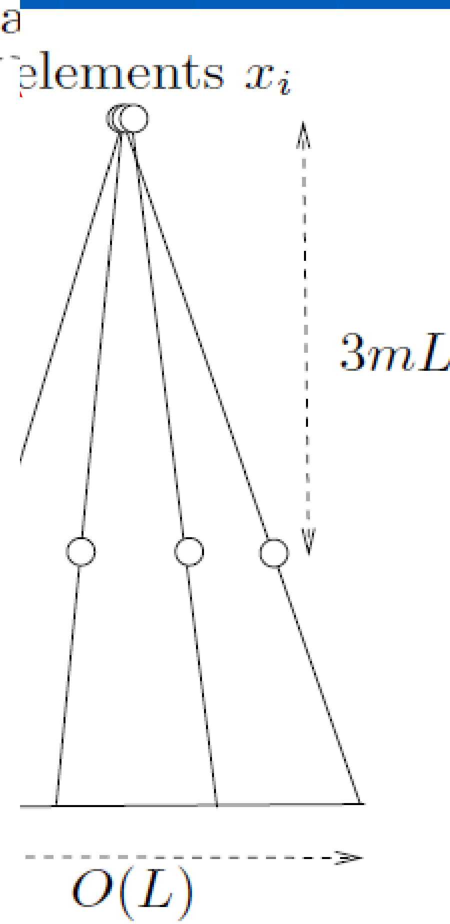
Zoom in to element x_i

From SetCover



Spread out elements x_i





General Case: Polygonal Domain (2D)

- **Theorem**: The WRP has an $O(\log^2 n)$-approximation algorithm.

Main Ideas

- Localization: Consider a polynomial # of "minimal outer-illuminating squares" (MOIS), B, that OPT passes near/through
- Discretization: Show that the continuous problem can be discretized, using an appropriate grid

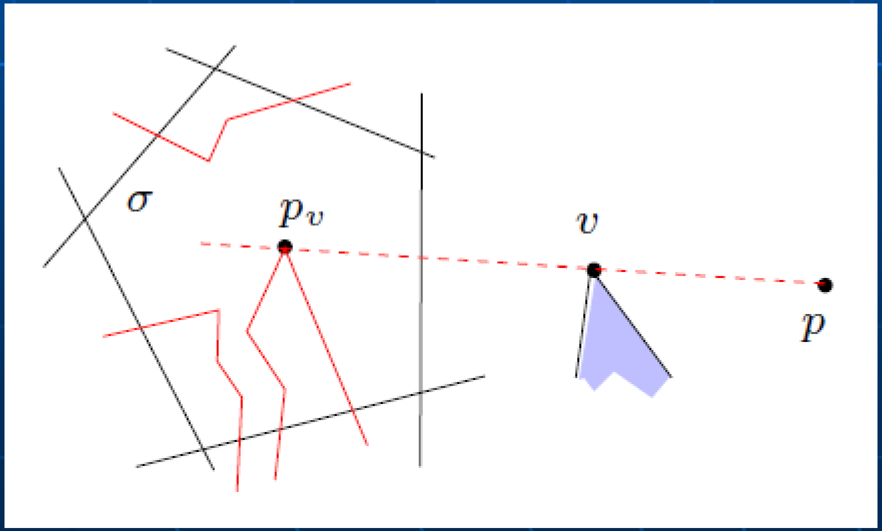
Main Ideas

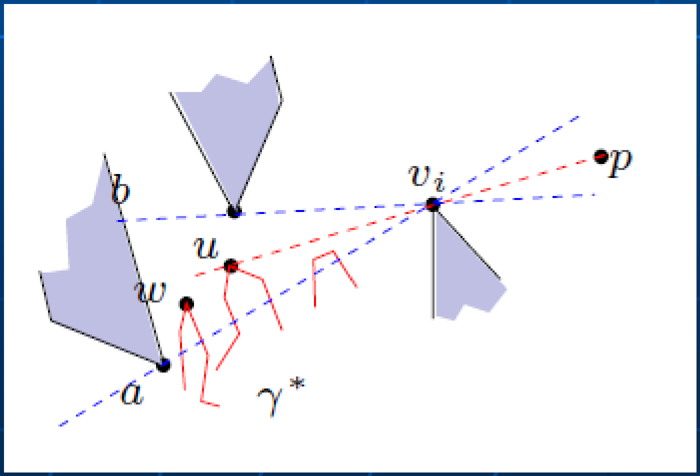
- Solve 2 separate problems:
  - OWRP: Outer WRP: Find a short tour within P that sees all of P that is outside the tour.
    - Discrete-OWRP: exact DP algorithm
    - OWRP: PTAS
  - IWRP: Inner WRP: For a given simple closed curve, γ, within P, augment γ (if needed) into a short network that sees all of P that is inside γ.
    - $O(\log^2 n)$-approx
- Combine

Structure of OPT

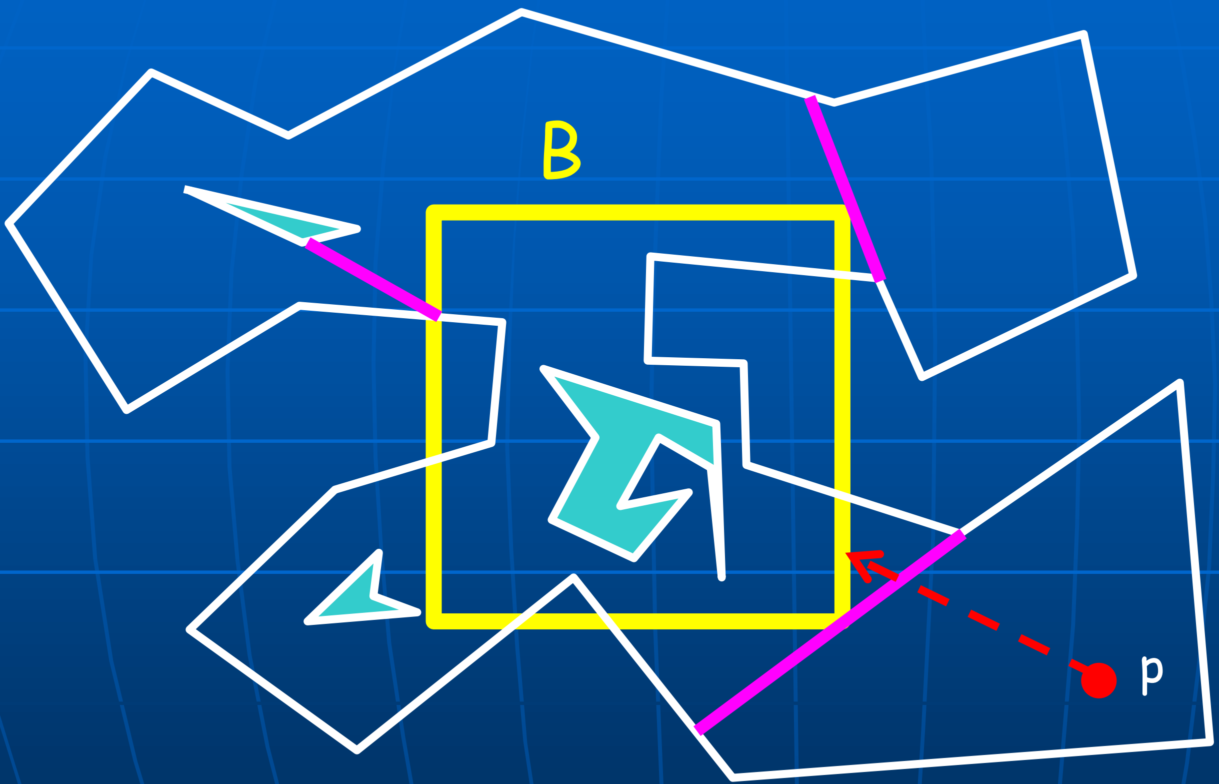
- **Lemma**: OPT for WRP/OWRP/IWRP is polygonal, complexity $O(n^2)$

Conjecture: $O(n)$





Minimal Outer-Illuminating Squares (MOIS)



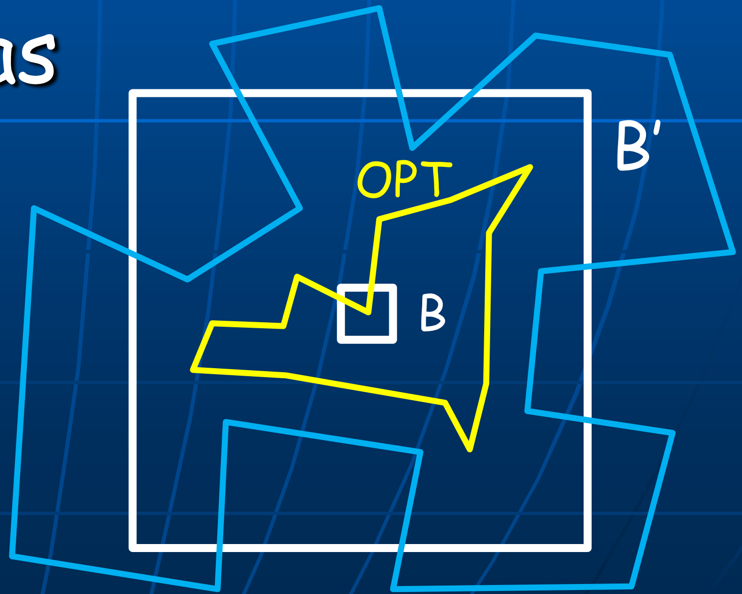
- **Lemma**: There are a polynomial # of MOIS's, B

Pf: Square has 3 degrees of freedom

Localization

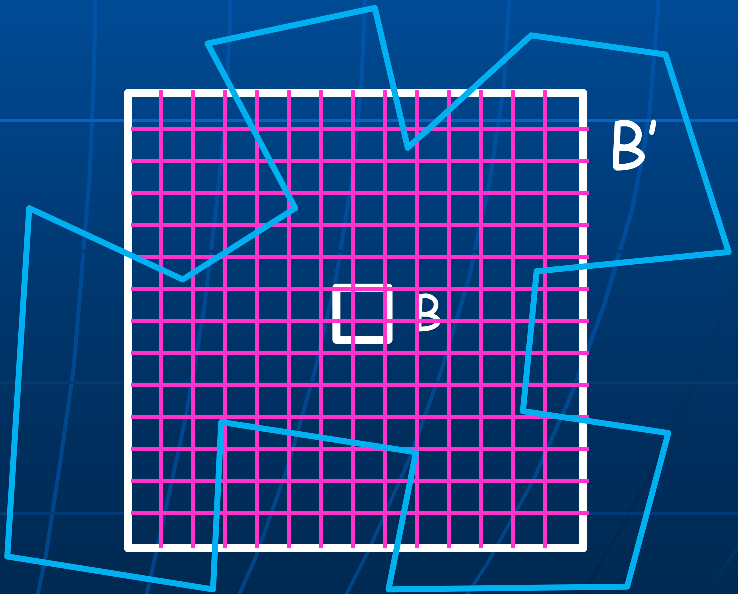
- **Lemma**: If B is a MOIS within BB(OPT), then OPT lies within an enlarged box, B′, centered on B, of size O(n|B|).
- **Pf**: Vertical decomposition of P within B

Each of the O(n) faces has diam = O(|B|); Traversing the edges of all faces sees all of P, inside and out of B



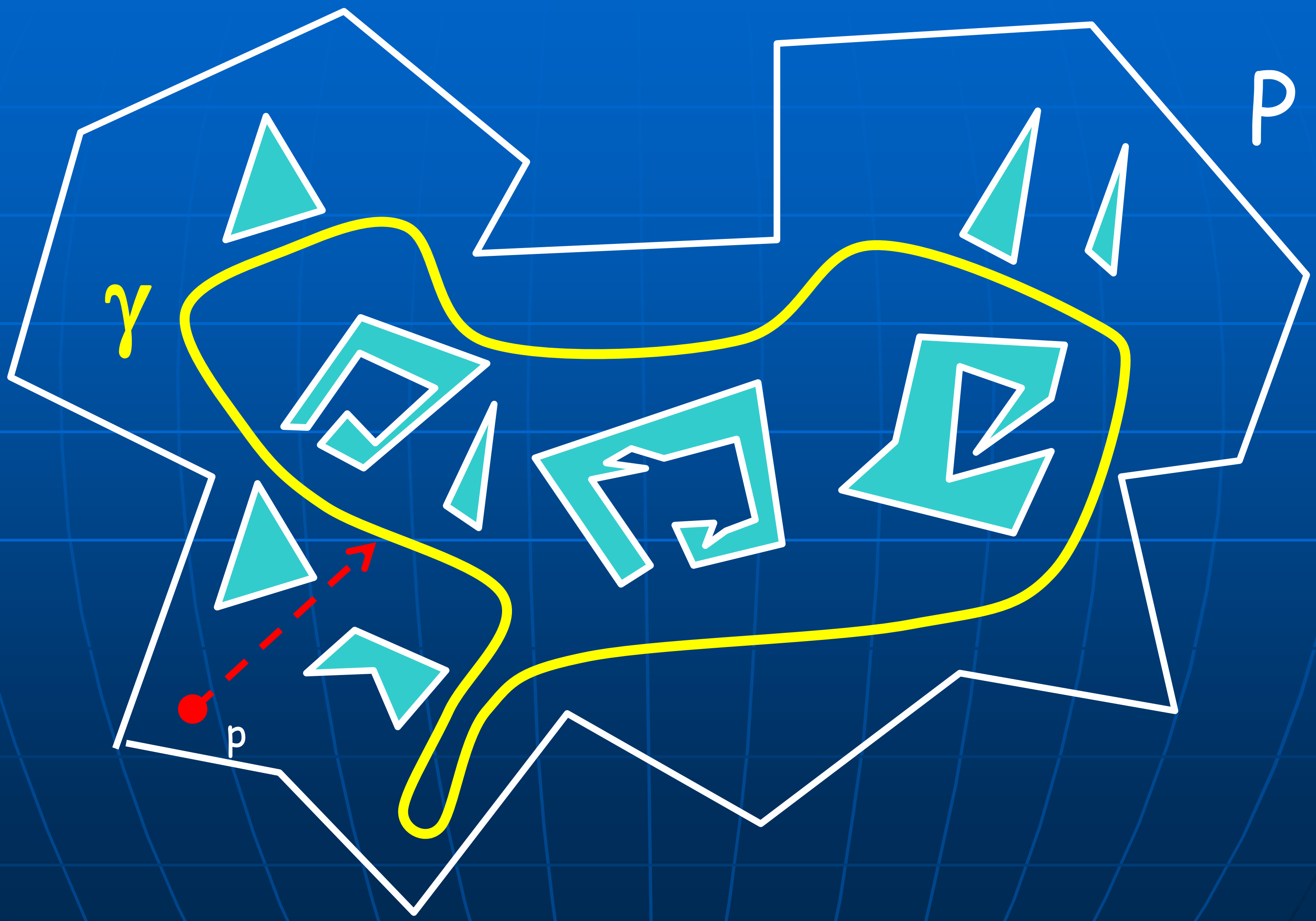
Discretization

- **Lemma**: OPT can be rounded to have vertices on a grid partitioning of the enlarged square, B′, of resolution $\varepsilon|B|/n^2$. The rounding increases its length by factor $(1+\varepsilon)$



Grid refinement of the vertical decomposition of P within B′

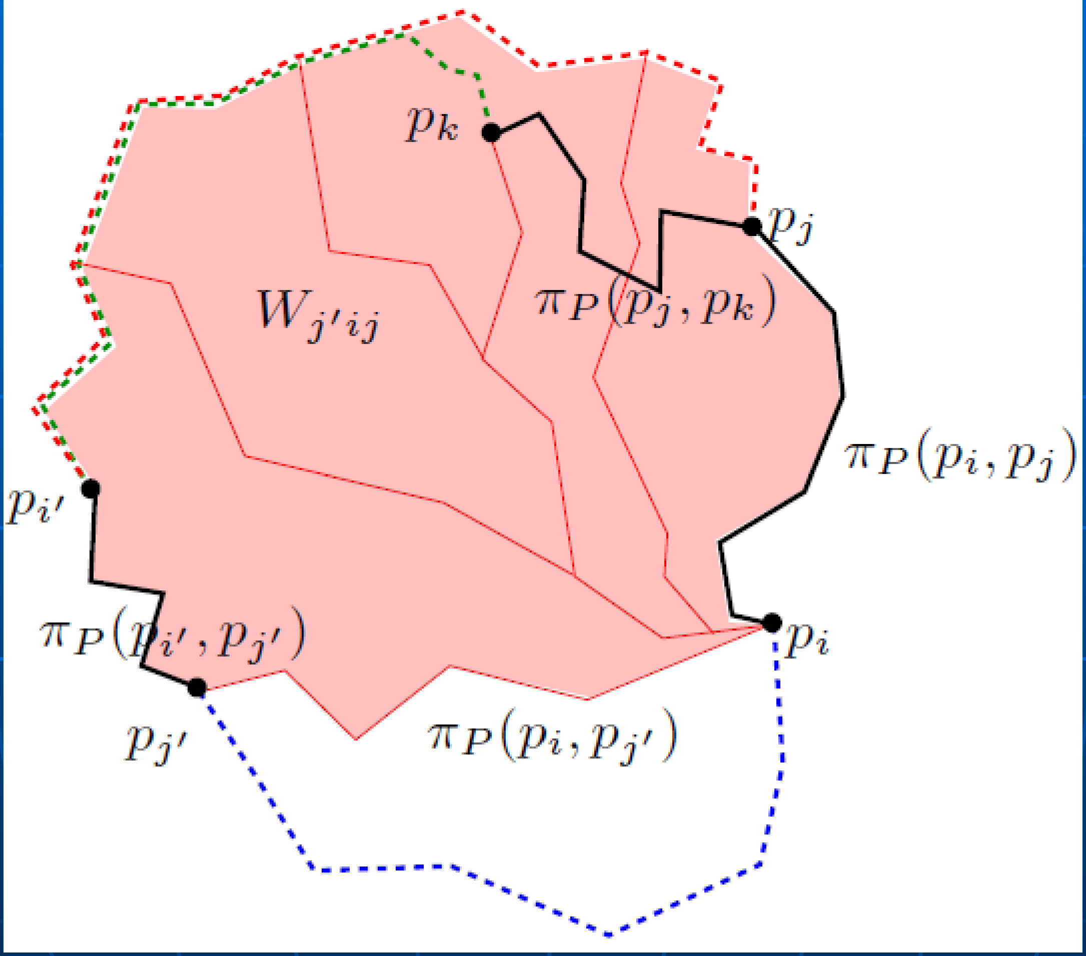
Outer WRP



Outer WRP

- **Lemma**: OPT is geodesically convex (wrt P)
- **Goal**: Search for min-perimeter geodesically convex, outer-illuminating cycle
- Discretize first: Constrain vertices to be among a given set, S, given by the grid discretization (for given choice of B, B')
- "Discrete-OWRP": Exact DP algorithm

SubProblem(i',j',i,j)



$$f(i', j', i, j) = \min_{p_k \in S_{i',j',i,j}} \{|\pi_P(p_j, p_k)| + f(i', j', j, k)\},$$

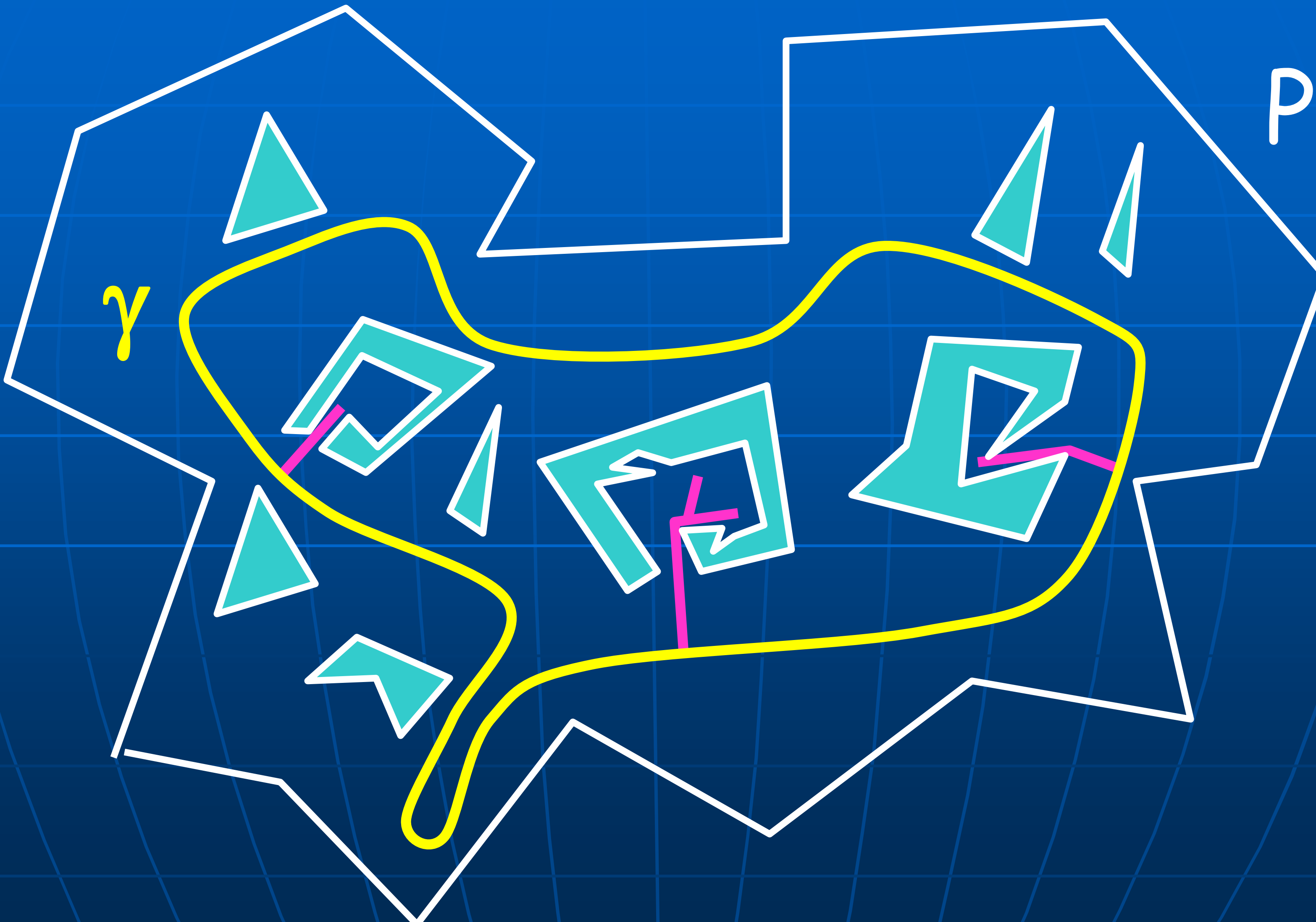
$$f(i', j', i, j) = 0 \text{ if } i' = j$$

Outer WRP

- **Theorem**: The Discrete-OWRP can be solved exactly in poly-time
- **Corollary**: The OWRP has a PTAS
- **Corollary**: The WRP on rays is poly-time

Since it is discrete already

Inner WRP



Inner WRP

- Theorem: The IWRP, for given P and γ, has an $O(\log^2 n)$-approx

Geodesic Triangles

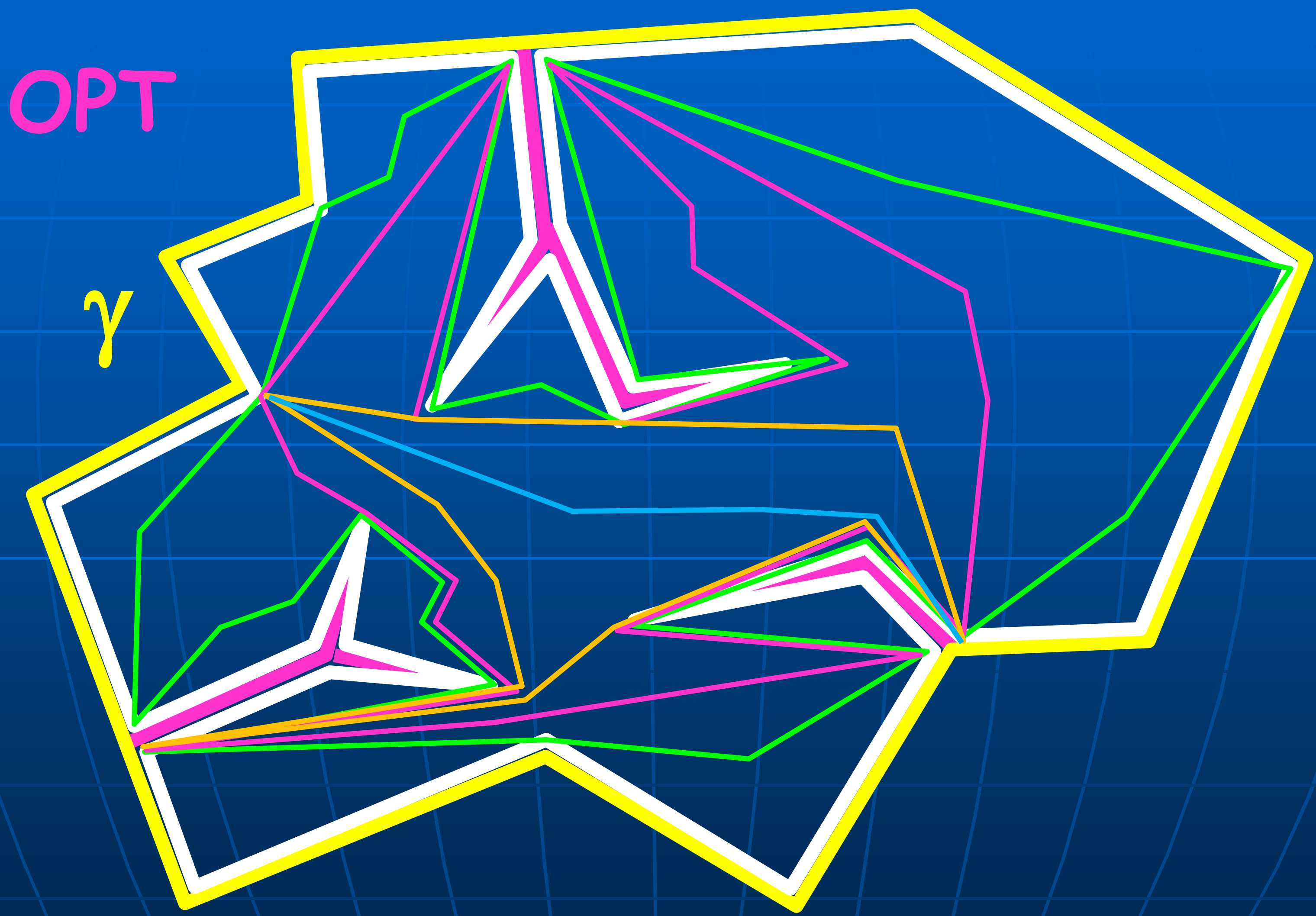
- Geodesic with respect to P, γ

- A triangle Δ is *inner-illuminating* if it sees all of P within Δ

Hierarchical Geodesic Triangulation

- **Lemma**: For a simple closed polygonal curve γ in P, there exists a geodesic triangulation of γ of length $O(|\gamma| \log n)$
- In particular, the *hierarchical geodesic triangulation* of γ works
- Note 1: If γ is inner-illuminating, then so are all geodesic triangles in any geodesic triangulation of it.
- Note 2: It suffices to work with discrete choices, on the B,B'-grid

Inner WRP



Hierarchical geodesic triangulation

Set Cover Formulation: IWRP

- Consider the set of all $O(n^3)$ inner-illuminating geodesic triangles within γ.
- Consider the arr of all-pairs geodesic paths in γ, between grid points/vertices
- Cover all cells within γ with a min-weight set of inner-illuminating geodesic triangles.
- **Lemma**: The boundaries of any such cover is a connected network.

Inner WRP

- Our (greedy) covering: $O(OPT_{cover} \log n)$
- We know one way to cover with length $O(OPT_{IWRP} \log n)$ – just use hierarchical geodesic triangulation of OPT_{IWRP}
- Thus, $OPT_{cover} < O(OPT_{IWRP} \log n)$
- Thus, our solution $< O(OPT_{IWRP} \log^2 n)$
- **Theorem**: IWRP has an $O(\log^2 n)$-approx

Conjecture: $O(\log n)$-approx
Use variant of guillotine method

Overall Algorithm

- Enumerate each MOIS, B
- For each B:
  - Construct grid, cells of size $\varepsilon|B|/n^2$ within the enlarged B (size $O(n|B|)$)
  - DP: Solve Discrete-OWRP, giving cycle γ
  - Solve IWRP within γ
- **Theorem**: The WRP has an $O(\log^2 n)$-approximation algorithm

Conjecture: $O(\log n)$-approx
Use variant of guillotine method