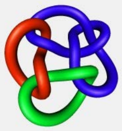


# MAT 539

## Algebraic Topology

**Instructor** Sorin Popescu (office: Math 4-119, tel. 632-8358, e-mail `sorin@math.sunysb.edu`)

**Time and Place** TuTh 09:50am-11:10am, Old Chem 135

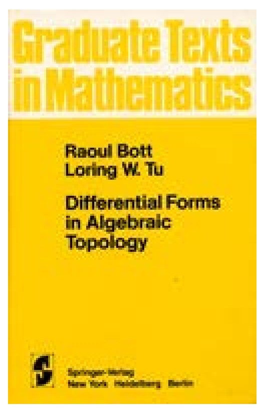
## Prerequisites

A basic introduction to geometry/topology, such as **MAT 530** and **MAT 531**. Thus prior exposure to basic point set topology, homotopy, fundamental group, covering spaces is assumed, as well as a reasonable acquaintance with differentiable manifolds and maps, differential forms, the Poincaré Lemma, integration and volume on manifolds, Stokes' Theorem. We will briefly review some of this material in the first week of classes.

## Textbook

*Differential forms in algebraic topology*, by Raoul Bott and Loring W. Tu, GTM **82**, Springer Verlag 1982.

The guiding principle of the book is to use differential forms and in fact the de Rham theory of differential forms as a prototype of all cohomology thus enabling an easier access to the machineries of algebraic topology in the realm of smooth manifolds. The material is structured around four core sections: de Rham theory, the Cech-de Rham complex, spectral sequences, and characteristic classes, and includes also some applications to homotopy theory.




Other recommended texts:

- *Algebraic Topology: A first Course*, W. Fulton, GTM **153**, Springer Verlag 1995
- *Topology from the Differentiable Viewpoint*, J. Milnor, U. of Virginia Press 1965
- *Algebraic Topology*, A. Hatcher, Cambridge University Press 2002 (also available online at the author's web page)
- *Elements of Homotopy Theory*, G.W. Whitehead, GTM **61**, Springer Verlag 1995

## Course description

The book contains more material than can be resonably covered in a one-semester course. We will hopefully cover the following sections:

- **De Rham theory:** the de Rham complex, orientation and integration, Poincaré lemmas, the Mayer-Vietoris argument, Poincaré duality on an orientable manifold, Thom class and the Thom isomorphism (orientable vector bundle case)
- **The Cech-de Rham complex:** the generalized Mayer-Vietoris argument, sheaves and Cech cohomology, the de Rham theorem, sphere bundles, Euler class, the Hopf index theorem, the Thom isomorphism in general, monodromy
- **Spectral sequences**: basics, spectral sequence of a double complex, products, applications and some explicit computations
- **Homotopy theory:** homotopy groups, long homotopy sequence of a fibration, loop spaces, Eilenberg-MacLane spaces, the Hurewicz isomorphism, a few low dimensional homotopy groups of spheres (Hopf invariant, etc) (all of these, perhaps more only if time permits)

## Homework & Exams

I will assign problems in each lecture, ranging in difficulty from routine to more challenging. Course grades will be based on these problems, class participation, and final exam.

## Software

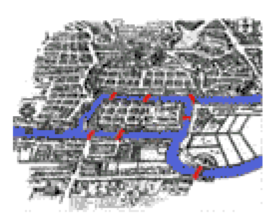
Here are some pointers to software that may be used to visualize topological objects:

- KnotPlot. Download binaries from the following site.
- Java View: a 3d geometry viewer written in Java. Among the demos you may find a Klein Bottle
- LiveGraphics3D: a Java applet to display and rotate three-dimensional graphics.
  Here used to display a version of the *Borromean Rings* (use your mouse to give them a "spin"):
- Geomview: another interactive 3D viewing program.
- JZB: a Java 3D viewer with MATHEMATICA export
- Xj3D: an open source VRML/X3D Toolkit

## Links & 3D-models

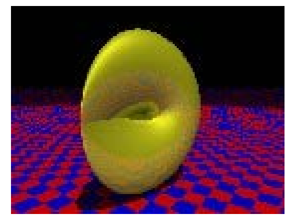
History of topology:

- "Topology enters mathematics": a brief overview of the early developments (MacTutor History of Mathematics archive).
- A Brief History of Topology by E.C. Zeeman
- Stable algebraic topology 1945-1966, by J.P. May



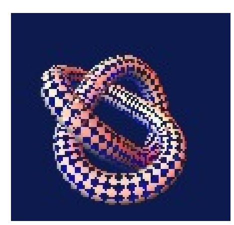
Topological zoo:

- "The Topological Zoo" at the Geometry Center: a visual dictionary of surfaces and other mathematical objects.
- Images of "classical" topological "objects" from the Geometry Center.
- "A Knot Zoo". Here is another one. Or Hyperbolic knots. All these sites are part of an exciting collection of knots and links available at "The KnotPlot Site". Very instructive are also the VRML knot models.
- Raytraced images: Sphere, Torus, projective plane: a Crosscap, a Steiner surface, a Boy surface, and a genus 3 orientable surface.
- VRML models: a Möbius band, a Klein bottle and a Trefoil Knot. Download here a vrml viewer for Linux.
- David Eppstein's "Geometry Junkyard": a collection of pointers, clippings, research blurbs, and other stuffs related to discrete, computational geometry, and topology.
- Paul Bourke's collection of raytraced surfaces. Here is for instance the animation of a transition from a Steiner surface into a Boy surface.
- A picture of the Hopf fibration created by Ken Shoemake. Click here for a better quality TIFF version of the picture. The picture visualizes well the remarkable geometric fact that any two fibres (=circles) of the Hopf fibration are *linked*. Here is another page and an mpeg animation of the Hopf fibration (created with Knotplot).



Art & Topology:

- "Symbolic Sculpture and Mathematics"
- "Mathematics & Knots Exhibition"
- Benno Artmann's Topological Models
- The Scherk-Collins Sculpture Generator: a program to generate Scherk-Collins towers and toroids (by Carlo H. Séquin)
- Helaman Ferguson's sculptures. For instance here is "Klein's modular quartic" which is on the patio of MSRI Berkeley. Or Alexander's horned sphere...
- More art links on Carlo H. Séquin's web site.
- Knots from the Alhambra de Grenada

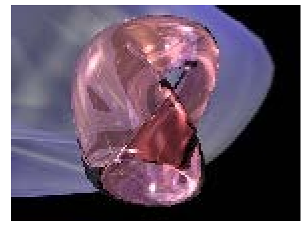


Archives:

- The "Topology Atlas"
- The Hopf Topology Archive

- Rob Kirby's Problems in Low-Dimensional Topology (380 pages)
- An Algebraic Topology Discussion List

Fun:



- Torus and Klein Bottle Games: a collection of Java applets/games played on the surface of a torus or a Klein bottle (chess, tic-tac-toe, crossword puzzles, and more).
- Glass Klein Bottles!

*Sorin Popescu*

*2002-12-29*