	WELCOME TO MAT 511 **Fundamental Concepts of Mathematics** **Fall 2017**

Time and place: TuTh at 5:30PM in Earth and Space, room 177

Introduction: The course will be concerned with a brief history of mathematics; sets, functions and logic; construction of number systems; mathematical induction. The main focus of the course will be the construction and writing of mathematical proofs.

Text Book: *A Transition to Advanced Mathematics* (8th edition) by D. Smith, M. Eggen and R. St. Andre, Cenage Learning (c) 2015

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Office Hours: Tuesday 2:30-4:30, Th 2:30-3:30 or by appointment

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Student Lecture: Each student will be required to give a lecture on a topic chosen through consultation with the professor.

Homework: Homework will be assigned every week. Doing the homework is a *fundamental* part of the course work. Problems should be handed in in lecture.
1st assignment: page 91, problems 4bdfhj, 6bdef, 9, 11cde,12,13, 14bd; page 101, problems 1bdf, 3beh, 12bc due September 7
2nd assignment: page 123, problems 4bde, 5achq,9, 11; page 133, problems 1ab, 3, 6a, 8ab, 9, 12ab due September 14
3rd assignment: page 147, problems 2bdf, 3, 7, 8, 17bcd, 23, 24abc due September 26
4th assignment: page 171, problems 1abfk, 4, 6ae, 7, 11, 17; page 186, problems 1bgj, 6abcde, 8b, 9c, 10 due October 3
5th assignment: page 209, problems 1bc, 3ae, 10ad, 13ac, 16ab; page 218, problems 1ade, 2ach, 5abcd, 17ad due October 17
6th assignment: page 227, problems 1bf, 5, 6, 10a, 13c, 14ae; page 236, problems 1be,3ac, 7cd due October 24
7th assignment: page 248, problems 2abh, 3aceh, 5eghp, 7, 8ac, 11ab, 12bd; page 257, problems 2abf, 4abc due October 31
8th assignment: page 349, problems 1dfh, 3dfh, 4bc, 6a, 10, 17 due November 21
9th assignment: page 360, problems 1ac, 2ab, 4abc, 5abc, 9, 10abc, 12ac, 13, 15 due

November 30
10th assignment:
11th assignment:

Grading Policy: The overall numerical grade will be computed by the formula **25% Homework + 25 % Student Lecture + 50% Final Exam**

Final Exam: Tuesday, December 19, 11:15AM-2:45PM

FINAL EXAM REVIEW: Exam Review
N. B. Use of calculators is not permitted in any of the examiniations.

Special Needs: If you have a physical, psychological, medical or learning disability that may impact on your ability to carry out assigned course work, I would urge you to contact the staff in the Disabled Student Services office (DDS), Room 113, humanities, 632-6748/TTD. DSS will review your concerns and determine, with you, what accommodations are necessary and appropriate. All information and documentation of disabilities is confidential.

Review sheet for MAT511, Fall 2017

Define the converse, inverse and contrapositive of an implication. Prove that the converse is true iff the inverse is true.

Define quaternions. Define the conjugate and absolute value of a quaternion. Prove that the absolute value of the product of two quaternions is the product of their absolute values.

Prove that the sum of the squares of two positive odd numbers is not divisible by four.

State and prove the Pythagorean theorem.

Define the distance between two points in the plane if the points are given as (a, b) and (c, d). Consider the unit circle centered at $(0, 0)$ in the plane and let each point on it be $(\cos\alpha, \sin\alpha)$ for some angle α. Using the distance between points prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

Define the union and intersection of two sets. Note that a non-empty set S is finite iff for some natural number n, there exists a bijection from S to the set $\{1, 2, 3, \ldots, n\}$. An infinite set a set that is not empty or finite. Prove that if S is infinite, then there exists an injection of the set of natural numbers into S.

Define an equivalence relation, and equivalence classes. Explain how an equivalence relation on a set S gives a decomposition of a S into disjoint subsets – a partition of S.

Define the cardinality of a set S. Call it $|S|$. Define $|S| \leq |T|$ where S and T are sets. Define the power set of a set S. Prove that the natural numbers and the rational numbers have the same cardinality.

State the Schroeder-Bernstein theorem about cardinality of sets.

Explain the principle of mathematical induction and use it to prove formulas for the sum of the first n natural numbers and the sum of the squares of the first n natural numbers. Let M be a map made by lines (not line segments) in the plane. Use induction to prove that M can be colored by two colors. Use induction to show that that any non-empty set of natural numbers has a least element. Show by example that this is not true for real numbers.

State and prove (by induction) the binomial theorem which gives a formula for the nth power of x + y.

Define a sequence of real numbers. Define a Cauchy (or cozy) sequence. State the least upper bound property for real numbers. Show by example that it does not hold for rational numbers.

Show that for any positive number ϵ and any real number r, there exists a natural number n, such that $n\epsilon > r$. Also show that for any positive number ϵ, there exists a natural number n such that $1/n < \epsilon$. Let l be the least upper bound of the set of all rational numbers whose square is less than 2. Prove that $l^2 = 2$.

Define the limit of a sequence and define

$$\lim_{x->a} f(x)$$

for a real-valued function f.

Find

$$\lim_{x->0} \left(\frac{\sin x}{x} \right)$$

and prove your answer.

Find

$$\lim_{x->0} \sin(1/x)$$

or prove that the limit does not exist.

Define a field, an ordered field and a complete ordered field.

Define open and closed sets of real numbers. Define interior points, exterior points and boundary points of a set of real numbers S. Define an open cover and a finite subcover of a set S. State and prove that Heine-Borel theorem.

Define a sequence and subsequence. State and prove the Bolzano-Weierstrass theorem.

Define the Euler characteristic of a polyhedron. Prove that the Euler characteristic of a convex polyhedron is 2. Prove that there are exactly five regular polyhedra.