WELCOME TO MAT 341
Applied Real Analysis
Spring 2016

Class Mondays, Wednesdays and Fridays at 9:00am in the Physics Building, Room 113

Introduction: Partial differential equations of mathematical physics: the heat, wave, and Laplace
equations. Solutions by techniques such as separation of variables using orthogonal functions (e.g., Fourier
series, Bessel functions, Legendre polynomials). D'Alembert solution of the wave equation.

Prerequisites: C or higher in the following: MAT 203 or 205 or 307 or AMS 261; MAT 303 or 305 or AMS 361
Advisory Prerequisite: MAT 200

Text Book: David L. Powers, Boundary Value Problems and Partial Differential Equations,

Instructor: Prof. David Ebin
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Office hours: Monday, Wednesday and Friday 10:00-11:00 or by appointment

Grader: Ms. Zhongshan An
Physics D-101
E-mail: zan@math.stonybrook.edu

Office Hours: Wednesday 4-5PM, Monday 5-7PM in the math learning center

Grading Policy: The overall numerical grade will be computed by the formula
20% Homework + 30 % Midterm Exam+ 50% Final Exam

Homework: Homework will be assigned every week. Doing the homework is a fundamental
part of the course work.

1st assignment: page 11, problems 2, 6, 8, 10, 14; page 23, problems 2, 4, 10 due February 10
2nd assignment: page 23, problems 14, 16, 18; page 34, problems 6, 12, 16 due February 17
3rd assignment: page 42, problems 2, 6; page 44, problems 4, 8, 10, 30; page 56, problem 8.
Due February 24
4th assignment: page 63, problems 12; page 70, problems 2, 8; page 77, problems 2, 4; page
82, problems 4, 6, 8 due March 2
5th assignment: page 87, problems 2, 4, 6; page 118, problems 2, 4, 20. Due March 9
Midterm Exam: Friday, March 11

6th assignment: page 139, problems 2, 4, 8; page 148, problems 6, 8; page 153, problems 2, 4; page 162, problems 2, 6, 10 due March 30

7th assignment: page 205, problems 2, 4, 10; page 224, problems 4, 8, 12, 14; page 233, problems 10, 12 due April 6

8th assignment: page 257, problems 31, 32, 33; page 263, problems 2, 8; page 269, problems 6, 8; page 276, problem 10 due April 13

9th assignment: page 293, problems 2, 8, 10; page 311, problem 2; page 317, problem 2; page 321, problems 8, 10, 12 due April 20

10th assignment: page 324, problems 6, 8; page 330, problems 4, 10; page 335, problem 8; page 343, problems 2, 10abc due April 27

11th (last) assignment: page 349, problems 2, 4, 6, 16; page 360, problems 4, 6, 8, 12; page 367, problems 2, 4, 6 due May 4

MIDTERM EXAM REVIEW: Exam Review

Final Exam: Wednesday, May 11, 8:30pm-11:00pm

FINAL EXAM REVIEW: Exam Review

N. B. Use of calculators is not permitted in any of the examinations.

Disability Support Services (DSS)

If you have a physical, psychological, medical or learning disability that may impact your course work, please contact Disability Support Services, ECC (Educational Communications Center) Building, room 128, (631) 632-6748. They will determine with you what accommodations, if any, are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to the following website: http://www.stonybrook.edu/ehs/fire/disabilities

Academic Integrity

Representing another person's work as your own is always wrong. Faculty are required to report any suspected instances of academic dishonesty to the Academic Judiciary. Faculty in the Health Sciences Center (School of Health Technology & Management, Nursing, Social Welfare, Dental Medicine) and School of Medicine are required to follow their school-specific procedures. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at http://www.stonybrook.edu/commcms/academic_integrity/index.html

Critical Incident Management

Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn. Faculty in the HSC Schools and the School of Medicine are required to follow their school-specific procedures.
Review for MAT 341 Midterm
March, 2016

Solve first order linear ode’s both homogeneous and non-homogeneous
Solve \( \ddot{u} + k\dot{u} + pu = 0 \) for \( k \) and \( p \) constants.
Show that there is only one solution \( u \) of the equation above with a given \( u(0) \) and \( \dot{u}(0) \).
Understand superposition for linear equations. (the sum of two solutions is also a solution and a constant times a solution is a solution)
Boundary value problem: Solve \( \ddot{u} + k\dot{u} + pu = 0 \) given \( u(a) \) and \( u(b) \). Is the solution necessarily unique?
Find heat flow in a cylinder and the velocity of water in a pipe using the assumptions made in class.
Define a periodic function.
Define the Fourier series of a function on the interval \((-a,a)\). This includes the formula for the Fourier coefficients.
Define orthogonal and orthonormal sets in a vector space with inner product.
Define the span of a set of vectors.
Define even and odd functions.
Prove that \( \{\sin nx\} \) is an orthogonal set with respect to the inner product:
\[
\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)dx
\]
Define pointwise and uniform convergence for a sequence of functions. Show that the uniform limit of a sequence of continuous functions is continuous.
Let \( \{a_n, b_n\} \) be the Fourier coefficients of a function \( f \). Show that if
\[
\sum_{n=0}^{\infty} |a_n| + |b_n| < \infty
\]
then the Fourier series of \( f \) converges uniformly.
Define convergence in mean.
Given \( f(x) = \sum_{n=1}^{\infty} a_n \cos nx \) find a solution of
\[
\ddot{u} + \alpha\dot{u} + \beta u = f
\]
assuming that \( \alpha \) is not zero.
Review for MAT 341 Final
May, 2016

Solve first order linear ode’s both homogeneous and non-homogeneous
Solve $\ddot{u} + ku + pu = 0$ for $k$ and $p$ constants.
Show that there is only one solution $u$ of the equation above with a given $u(0)$ and $\dot{u}(0)$.
Understand superposition for linear equations. (the sum of two solutions is also a solution and a constant times a solution is a solution)
Boundary value problem: Solve $\ddot{u} + ku + pu = 0$ given $u(a)$ and $u(b)$. Is the solution necessarily unique?
Find heat flow in a cylinder and the velocity of water in a pipe using the assumptions made in class.
Define a periodic function.
Define the Fourier series of a function on the interval $(-a,a)$. This includes the formula for the Fourier coefficients.
Define orthogonal and orthonormal sets in a vector space with inner product.
Define the span of a set of vectors.
Define even and odd functions.
Prove that $\{\sin nx\}$ is an orthogonal set with respect to the inner product:
$\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)dx$
Define pointwise and uniform convergence for a sequence of functions. Show that the uniform limit of a sequence of continuous functions is continuous.
Let $\{a_n, b_n\}$ be the Fourier coefficients of a function $f$. Show that if
$$\sum_{0}^{\infty} |a_n| + |b_n| < \infty$$
then the Fourier series of $f$ converges uniformly.
Define convergence in mean.
Given $f(x) = \sum_{1}^{\infty} a_n \cos nx$ find a solution of
$$\ddot{u} + \alpha \dot{u} + \beta u = f$$
assuming that $\alpha$ is not zero.
Understand the heat equation for the temperature in a uniform rod as a function of one space variable and time. Understand both transient and steady-state solutions
Solve the heat equation using Fourier series in the case of insulated ends and in the case of fixed temperature at the ends
Show that
$$\partial_t u = \partial_x^2 u$$
$u(x,t) = f(x)$ has a unique solution assuming that $0 < x < 1$ and assuming that $u(0,t) = a$ and $u(1,t) = b$.
Understand that the solution of the wave equation
$$\partial_x^2 u = \frac{1}{c^2} \partial_t^2 u$$
with zero boundary data approximates the motion of a string with fixed ends. Show that for any differentiable functions $\psi$ and $\phi$, $u(x, t) = \psi(x + ct) + \phi(x - ct)$ solves the wave equation above.

Understand Laplace’s equation (the potential equation) in rectangular, polar, cylindrical and spherical coordinates.

Same for the wave and heat equations.

Derive the potential equation in polar coordinates.

Use the separation of variables technique together with sequences of orthogonal functions to construct solutions to all of these equations.

The sequences should include Fourier series, Bessel functions and or Legendre polynomials as appropriate to each problem.

Find a function on a square with given values at the corners which satisfies the potential equation.

Solve $\Delta u = H$ where $H$ is a constant. Is the solution unique?

Let $\Omega$ be a bounded domain in the plane. Assume that

$$\Delta u = \lambda^2 u$$

with $u$ equal to zero on the boundary of $\Omega$. Prove that $u$ is zero.

Define spherical coordinates and know the range of each variable; write the transformation from spherical to rectangular coordinates.

Define the Laplace transform and compute the transforms of $e^{kt}$ and of polynomials. Also compute the transforms of sine, cosine, sinh and cosh.