MAT 336: History of Mathematics

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Welcome to MAT 336

MAT 336 is a document-based history of mathematics. It moves chronologically from Ancient Egypt and Mesopotamia to the start of the 20th century. In each historical period we will work with documents from that period, in as original a form as we can handle. We will attempt to replicate the original mathematics in each case (doing calculations, for instance, with the tools available in that period); at the same time we will examine that mathematics in the light of present-day science, and with an eye to the lessons it can teach us about how humans do mathematics, and how they learn mathematics.

The prerequisites for the course are MAT 200 or AMS 310. Substantial mathematical experience at the college level, in particular, understanding of proofs, is crucial to moving well through this course.

Assignments include two term papers and an in-class presentation based on the first paper. The term papers will involve critical reading of secondary sources, with explicit reference to the primary. More details and suggestions are available at this page.

For more information, see the course info page.

Important deadlines:

Feb 2	Sign-up for in-class presentation topics
Feb 16	Deadline for e-mail of Paper 1 topic
	Topic must be approved for paper to be accepted.
March 2	Deadline for email of detailed outline of Paper 1
	Outline must include at least one non-course text, non-web reference.
March 19	Paper 1 due
March 30	Deadline for email of 3-sentence description of Paper 2 topic
April 27	Deadline for detailed outline of Paper 2
May 4	Paper 2 due

Any missed deadline: paper gets automatic one-third grade reduction.

MAT 336: History of Mathematics

Course Information

LECTURE

MF 12:50-2:10pm, Mathematics 4-130

INSTRUCTOR

Alexander Retakh
Office: Math Tower 4-108
E-mail: retakh@math.sunysb.edu

Office Hours: M 2:15-3:30, F 11:30-12:45

REQUIRED TEXTS

History of Mathematics: An Introduction by David Burton, 6th edition
Journey through Genius: The Great Theorems of Mathematics by William Dunham

ASSIGNMENTS

Weekly assignments will consist of reading, study of historical documents, and suggested exercises. The detailed list is available on the syllabus page. Suggested exercises are posted on the homework page.

QUIZZES

Each Friday at the beginning of class there will be a 15-minute quiz on the preceding week's material (lectures, reading, suggested exercises, and student presentations). No make-ups will be given for missed quizzes.

TERM PAPERS

Each of you is responsible for two papers, both on the topic of your choice. The first paper (10 pages) will also be the basis of your in-class presentation. The second paper will be longer (15 pages) and should be deeper and more mathematical in content.

You are required to submit the topic proposals and detailed outlines of your papers.

The due dates are:

Feb 16 Deadline for e-mail of Paper 1
Topic must be approved for paper to be accepted.
March 2 Deadline for email of detailed outline of Paper 1
Outline must include at least one non-course text, non-web reference.
March 19 Paper 1 due
March 30 Deadline for email of 3-sentence description of Paper 2 topic.
April 20 Deadline for detailed outline of Paper 2.
May 4 Paper 2 due

Any missed deadline: paper gets automatic one-third-grade deduction (e.g. from B- to C+).

PRESENTATIONS

Starting in week 4 (February 12), each class will include two 15-minute student presentations. The topic of the presentation is identical to the topic of your first paper. You should **sign-up** for an in-class presentation by **February 2**. Your presentation will be graded on the clarity, organization, depth, and your ability to encourage and answer other students' questions.

Grading

Your course grade will be computed as follows:

Weekly quizzes: 30%
Term paper 1: 25%
In-class presentation: 10%
Term paper 2: 30%
Class participation/Attendance: 5%
(Note: No final examination.)

Help Outside Class

The Math Learning Center is located in Math Tower S-240A and offers free help to any student requesting it. It also provides a locale for students wishing to form study groups.

Americans with Disabilities Act

If you have a physical, psychological, medical or learning disability that may impact your course work, please contact Disability Support Services, ECC (Educational Communications Center) Building, room 128, (631) 632-6748. They will determine with you what accommodations are necessary and appropriate. (Note that we cannot make special arrangements for students with disabilities except for those determined by DSS.) All information on and documentation of a disability condition should be supplied to me in writing at the earliest possible time.

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Course Syllabus

Readings are assigned from:

Burton: David Burton, *History of Mathematics: An Introduction*

Dunham: William Dunham, *Journey through Genius: The Great Theorems of Mathematics*

Struik: Dirk Struik, *A Source Book in Mathematics, 1200-1800*, on reserve in Math Library

WEEK	TOPIC & READING
Week 1 (1/22-1/26)	Primitive counting. Positional and non-positional number systems. Babylonian number recording. Reading: Burton 1.1, 1.3 Document: YBC 7289 (Yale Babylonian Collection; Casselman's website at UBC)
Week 2 (1/29-2/2)	Babylonian MultiplicationTables; the decoding of Plimpton 322. Reading: Burton 2.5, 2.6 Document: VAT 7858 (Berlin Museum; Christian Siebeneicher's website at Bielefeld) Document: NBC 7344 (Yale Babylonian Collection; Duncan Melville's website at St.Lawrence University) Document: Plimpton 322 (Columbia Rare Book Collection; David Joyce's website at Clark University, see also Eleanor Robson's analysis) February 2: Sign-up for in-class presentation
Week 3 (2/5-2/9)	Egyptian number recording and Arithmetic. Reading: Burton 1.2, 2.1, 2.2, 2.3 Document: The Rhind Papyrus (St Andrew's website)
Week 4 (2/12-2/16)	Development of Greek mathematics. Reading: Burton 3.1, 3.3, 4.1, 4.2 Document: Euclid's calculation of the volume of a cone (Euclid's Elements website, XII, 10; see diagram of proof.) February 16: Deadline for e-mail with Paper 1 topic.
Week 5 (2/19-2/23)	Mathematics in the Hellenistic world. Reading: Burton 4.3, 4.5 Document: Archimedes "On moments" palimpsest (read all parts 1-4)

Week 6 (2/26-3/2)	Arab, Indian and Chinese mathematics of the first millennium. Reading: Burton 5.3, 5.5 Document: Table of sines from the Aryabhatiya (Narasimha in *Nature* Dec 20/27 2001) Document: Al-Khwarizmi on quadratic equations (Struik) March 2: Deadline for email of detailed outline of Paper 1.
Week 7 (3/5-3/9)	Italian mathematics of the late middle ages and the Renaissance. Reading: Burton 6.2, 7.2, 7.3 Document: Word problems from the 1478 "Treviso Arithmetic" Document: Cardano's solution of the cubic (Dunham)
Week 8 (3/12-3/16)	Mathematics of the humanistic era. Reading: Burton 8.1, 8.2 Document: Descartes on the theory of equations (Struik) Document: Galileo on uniformly accelerated motion (Struik) (see full text at Michael Fowler's website at University of Virginia: part 1, part 2)
Week 9 (3/19-3/23)	The birth of calculus. Reading: Burton 8.3, 8.4 Document: Leibnitz on the Fundamental Theorem of Calculus (Struik) March 19: Paper 1 due.
Week 10 (3/26-3/30)	The beginnig of probability. Reading: Burton 9.1, 9.2, 9.3 Document: Pascal on "Pascal's triangle" (Struik) March 30:Deadline for email of 3-sentence description of Paper 2 topic.
Week 11 (4/9-4/13)	Number theory in the 17-18 centuries. Reading: Burton 10.2, 10.3 Document: Euler's calculation of the sum of the $1/n^2$ series (Dunham) Document: Gauss' proof of the Fundamental Theorem of Algebra (Struik)
Week 12 (4/16-4/20)	Analysis in the 19th century. Reading: Burton 11.3 Document: Cauchy on the derivatives and differentials of functions
Week 13 (4/23-4/27)	The beginning of modern Algebra. Reading: Burton 11.4 Document: Cayley on the definition of Group Document: Cayley on the Cayley-Hamilton Theorem April 27: Deadline for detailed outline of Paper 2.
Week 14	The beginning of set theory. Reading: Burton 12.2, 12.3

(4/30-5/4)	Document: Cantor, "On an elementary question in set theory" (1890-91) Document: Cantor on the cardinality of a power set: |P(S)|>|S| (Dunham) May 4: Paper 2 due.

Developed by Anthony Philips

STONY BROOK UNIVERSITY

MAT 336: History of Mathematics

Suggested Exercises

All exercises are taken from Burton. Assignments will be posted after every class.

WEEK OF	PROBLEMS
1/22-1/26	1.3: 1-5
1/29-2/2	2.5: 1, 2, 7, 9, 11; 2.6: 1, 2, 5
2/5-2/9	2.3: 1-3, 5, 11, 12, 19, 20
2/12-2/16	3.3: 15-18; 4.2: 11, 12
2/19-2/23	4.3: 1, 6, 11, 12; 4.5: 1, 4, 5
2/26-3/2	5.3: 13, 14, 16; 5.5: 1, 2, 10, 11
3/5-3/9	6.2: 1-3, 6; 7.3: 3ab, 5, 8, 9
3/12-3/16	8.1: 12, 13; 8.2: 1, 2, 6, 7
3/19-3/23	8.3: 3, 4, 5; 8.4: 1, 4, 5
3/26-3/30	9.2: 1ac, 2, 9, 10bc; 9.3: 1, 4ab, 7, 11ab
4/9-4/13	10.2: 2, 4, 11; 10.3: 4, 6, 8, 12, 13
4/23-4/27	11.4: 1, 4, 5ab, 6, 8ab; 11.4: 11, 13
4/30-5/4	12.2: 1, 3, 4, 10, 13

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Term Papers

Each of you is responsible for two papers, both on the topics of your choice. You should inform me about the subject of each paper, e-mail a detailed outline, and submit the paper by the appropriate deadlines. Failure to do so will result in an automatic one-third grade deduction (e.g., from B- to C+).

Each paper should contain a bibliography section listing the references (sources) you used while researching and writing your paper. Your bibiliography should contain at least three references. You can use non-assigned sections in Burton as the starting point of your research but should not overrely on them. At least one of your references should be a book not assigned for this course.

You can use both printed and web references. Keep in mind, however, that web sources should not be trusted completely — even well-known "encyclopedic" sites contain many errors.

You can find a number of books on the history of mathematics in the Math/Physics library. Some have been put on reserve. Another good source for printed material is Google Books, where you can search within books unavailable at the library (complete downloads are impossible, though).

Plagiarism: if you don't know what it is, find out and don't do it. Cutting-and-pasting without acknowledging your source is an example of plagiarism. Any student who plagiarizes material on a term paper will receive zero for this paper and may be reported to Academic Judiciary.

The first paper should be at least ten pages in length and deal with a particular subject in the history of mathematics. You should work out at least some of the appropriate mathematics. A clear setting of historical context is also expected. Lengthy biographical sketches are too easy to write and should be avoided. Rather, you should show how a particular mathematical subject was developed in a given historical period.

The second paper should be at least fifteen pages long and on a subject different from the first paper's. You are expected to go deeper into the historical and mathematical setting and show a more critical attitude toward your sources.

Either paper may be used to satisfy upper-division writing requirement. Often a rewrite is necessary.

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Presentations

Starting in week 4 (February 12), each class will include up to two 15-minute student presentations. The topic of the presentation is identical to the topic of your first paper.
You should **sign-up** for an in-class presentation by **February 2**
and **submit** your presentation topic by **February 16**.

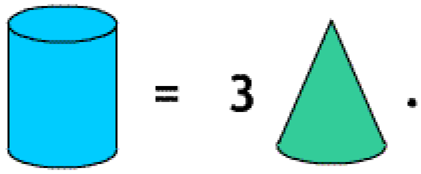
Presentation Schedule

The first two presentations of every week are on Monday; the third and fourth, on Friday.

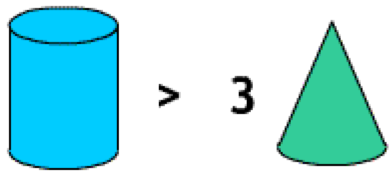
WEEK	PRESENTER & TITLE
Week 4 (2/12-2/16)	1. David Briggs, *Pythagoras* 2. Ramon Fernandez, *Justifications* 3. Anita Mathes, *Construction problems* 4. Lisa Beckel, *Hippocrates' quadrature of the circle*
Week 5 (2/19-2/23)	1. Christina Domanico, *Euclides' work in number theory* 2. Christina Spagnolo, *Use of Euclidean postulates in construction* 3. Dustin Loew, *Diophantus' Arithmetica* 4. Eric Dornicik, *Archimedes' computation of π*
Week 6 (2/26-3/2)	1. Fallon Nugent, *Geometry in ancient India* 2. Hou Li, *Indian method of finding square roots* 3. Steven Lim, *"Nine chapters" of Chinese mathematics* 4. Marie Milach, *Omar Khayyam*
Week 7 (3/5-3/9)	1. Chris Arettines, *Fibbonacci's "Liber Abaci"* 2. Jim Hoffman, *Development of the Fibbonacci series* 3. 4.
Week 8 (3/12-3/16)	1. Molly Law, *Galileo's study of the motion of projectiles* 2. 3. Melisa Bilgel, *Descartes' work on roots of polynomials*

	4. Chris Wolf, *Descartes' theory as basis of calculus*
Week 9 (3/19-3/23)	1. Sean Ferguson, *Newton's law of universal gravitation* 2. Katherine Cruceta, *Newton's use of binomial theorem* 3. Anthony Rubbo, *Leibniz's path to calculus* 4.
Week 10 (3/26-3/30)	1. Olga Boykova, *Prehistory of probability* 2. Camelia Medina, *Bernoulli's work in probability* 3. Suzan Bilgel, *Laplace's inductive reasoning* 4. Renee Goldfarb, *"Student"'s statistics*
Week 11 (4/9-4/13)	1. Casey Necheles, *Fermat* 2. Tom Ferretti, *Fermat's and Euler's theorems* 3. Loren Sanso, *Gauss's work in modular arithmetic* 4. Suren Grigoryan, *Gauss and non-Euclidean geometry*
Week 12 (4/16-4/20)	1. Stephen Mancuso, *Cauchy on limits* 2. Edward Cummings, *Complex analysis* 3. Konstantine Anastasakis, *Fourier series* 4. Solomon Bamiro, *Karl Weierstrass*
Week 13 (4/23-4/27)	1. Dan Stelmach, *Matrix operations* 2. 3. James Lynch, *Unsolvability of quintic equations* 4.
Week 14 (4/30-5/4)	1. Steve Mackey, *Cardinal numbers* 2. Keisuke Yoshii, *Set theory and the axiom of choice* 3. 4.

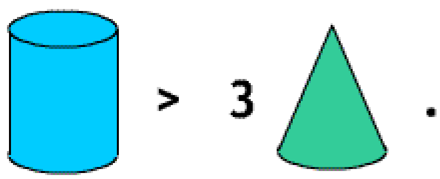
Theorem (Euclid XII, 10):

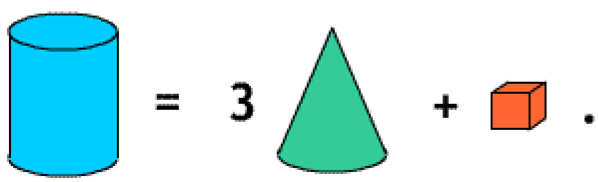


Proof: Suppose not. Then

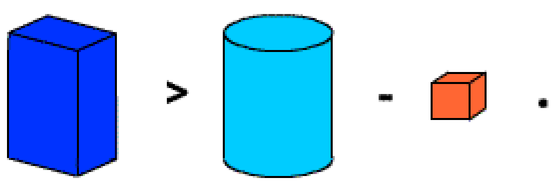
either A: 

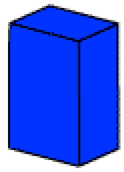
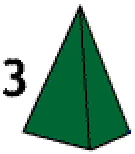
or B: 

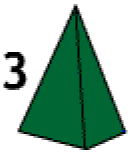
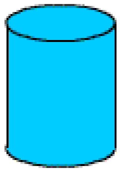
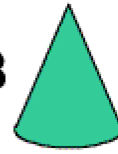
Case A. 

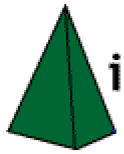
So 

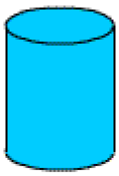
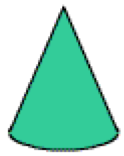
We can subdivide polygon

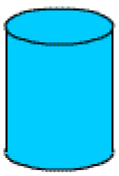
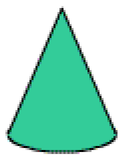
to make 

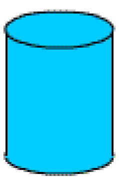
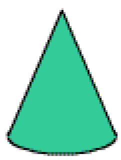
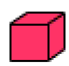
We know that  = 3  .

So 3  >  -  = 3  ,

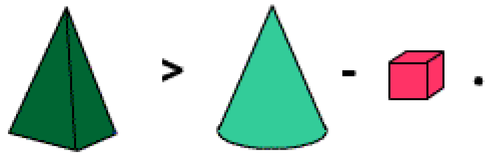
impossible since  inside  .

Case B.  < 3  ,

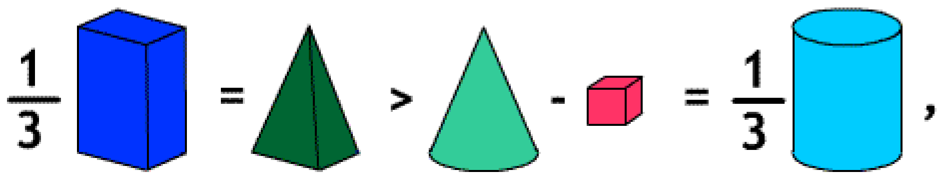
or $\frac{1}{3}$  <  .

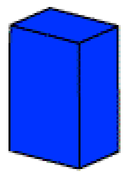
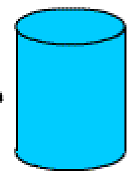
So $\frac{1}{3}$  =  -  .

Subdivide polygon to make



Then



impossible since  inside .

MAT 336 Stony Brook University

Word Problems from the Treviso Arithmetic - 1478

[comments in square brackets]

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1. (f. 35r) 1 and 1/2 pounds of saffron are worth 2 and 1/3 ducats [maybe $230; saffron is currently on sale on eBay for $205/lb.] how much would 1 and 1/4 ounces be worth. [One pound is 12 ounces, one ducat is 24 grossi, one grosso is 32 piccioli. Express your answer in grossi (g.) and piccioli (p.).]
   Answer: If 1 and 1/2 pounds of saffron are worth 2 and 1/3 ducats then 1 and 1/4 ounces are worth g.3 p.28 and 4/9.

2. (f. 44r) I have 9 and 2/3 yards (braccia) of fabric 2 and 3/4 yds wide which I want to make into a robe. I want to line it with fabric 1 and 1/8 yds wide. I ask how much I will need.
   Answer: I will need 23 and 17/27 yds. [The text has the wrong answer, due to a trivial arithmetic error, making 232x11=1552 instead of 2552.]

3. (f. 46v) Three merchants have invested their money as a company. Let's give them names to make this easier to understand. The first is Piero. The other is Polo. The third is Zuanne. Piero put in as his capital 112 ducats [maybe $11200]. Polo put in as his capital 200 ducats. Zuanne put in as his capital 142 ducats. And at the end of a certain period of time they have realized a profit of 563 ducats. I ask how much each man gets so that no one will be cheated.
   Answer: You answer that Piero gets as profit ducats 138 g.21 p.11 and 190/454. Polo gets as profit ducats 248 g.0 p.13 242/454. Zuanne gets as profit ducats 176 g.2 p.7 and 22/454. No one will be cheated. The sum of the profit is duc.563 g.0 p.0/454, which shows that this argument was done correctly.

4. (f. 46v) Two merchants, that is Sebastiano and Jacomo have invested their money to make profit as a company. Sebastiano contributed duc.350 on January 1, 1472. Jacomo contributed duc.500 g.14 on July 1, 1472. On January 1, 1474 they have realized a profit of ducats 622. I ask how much each one gets.
   Answer: You answer that Sebastiano gets as profit duc.300 g.2 p.8 and 327456/417852. Jacomo gets as profit duc.321 g.21 p.23 and 90396/417852. The sum of the profit is duc.622 g.0 p.0/417852 so this sharing was done justly and well.

5. (f. 53v) A merchant has 40 marks of silver alloy [one mark is 8 ounces] with 6 and 1/2 ounces per of pure silver per mark. And he has 56 marks of another alloy, with 5 ounces of pure silver per mark. And he wants to make all of this into coinage which will contain 4 and 1/2 ounces of pure silver per mark. I ask how much coinage he will produce and how much copper he will have to add.
   [Answer: The amount of coinage he can produce is 120 marks. The amount of copper he must add is 24 marks.]

Anthony Phillips
October 15 2006

On an elementary question in set theory

Georg Cantor

[Jahresbericht der Deutsch. Math. Vereing. **1** 75-78 (1890-91)]

In the article entitled: On a property of the set of all real algebraic numbers (Journ. Math. **77** 258) there was presented for the first time a proof that there are infinite sets which cannot be put into one-one correspondence with the set of all finite whole numbers $1, 2, 3, \ldots, \nu, \ldots$ or, as I put it, do not have the cardinality of the number sequence $1, 2, 3, \ldots, \nu, \ldots$. From what is proved in §2 there follows in fact something further, that for example the set of all real numbers in an arbitrary interval $(\alpha \ldots \beta)$ *may not* be represented as a sequence

$$\omega_1, \omega_2, \ldots, \omega_\nu, \ldots.$$

Each of these propositions can be given a much more simple proof, which is independent of considerations about the irrational numbers.

Specifically, let m and w be two different symbols, and let us consider the set M of elements

$$E = (x_1, x_2, \ldots, x_\nu, \ldots),$$

which depend on infinitely many coordinates $x_1, x_2, \ldots, x_\nu, \ldots$, where each of these coordinates is either m or w. Let M be the set of all such elements E.

Among the elemets of M we find for example the following three:

$$E^I = (m, m, m, m, \ldots),$$

$$E^{II} = (w, w, w, w, \ldots),$$

$$E^{III} = (m, w, m, w, \ldots).$$

I now state that such a set M does not have the cardinality of the sequence $1, 2, 3, \ldots, \nu, \ldots$.

This is a consequence of the following proposition:

``Let $E_1, E_2, \ldots, E_\nu, \ldots$ be any infinite sequence of elements of the set M; then there is an element E_0 of M which does not coincide with any E_ν."

For the proof let

$$E_1 = (a_{11}, a_{12}, \ldots, a_{1\nu}, \ldots),$$

$$E_2 = (a_{21}, a_{22}, \ldots, a_{2\nu}, \ldots),$$

$$\ldots$$

$$E_\mu = (a_{\mu 1}, a_{\mu 2}, \ldots, a_{\mu\nu}, \ldots),$$

$$\ldots$$

Here each $a_{\mu\nu}$ is set to be m or w. We will now define a sequence $b_1, b_2, \ldots, b_\nu, \ldots$ in such a way that b_ν is also only m or w and is *different* from $a_{\nu\nu}$.

So if $a_{\nu\nu} = m$ then $b_\nu = w$, and if $a_{\nu\nu} = w$ then $b_\nu = m$.

Let us now consider the element

$$E_0 = (b_1, b_2, b_3, \ldots)$$

of M; we immediately see that the equation

$$E_0 = E_\mu$$

cannot be satisfied for any integer value of μ, because for that particular μ and for all integer values of ν we would have

$$b_\nu = a_{\mu,\nu},$$

and in particular

$$b_\mu = a_{\mu,\mu},$$

which is excluded by the definition of b_μ. From this proposition it follws immediately that the set of all the elements of M cannot be placed in a sequence: $E_1, E_2, \ldots, E_\nu, \ldots$, since we would be faced with the contradiction that the object E_0 would be both an element of M and not an element of M.

This proof is remarkable not only because of its great simplicity, but also because the principle it contains leads immediately to the general proposition that the cardinalities of well defined sets have no maximum; or, equivalently, that for any given set L we can find another set M of larger cardinality than L.

For example, let L be an interval, say the set of all real numbers which are ≥ 0 and ≤ 1.

Then let M be the set of all functions $f(x)$ which only take on the two values 0 and 1, while x runs through all the real values ≥ 0 and ≤ 1.

That M *does not have a smaller* cardinality than L follows from the fact that M has subsets which are of the same cardinality as L, for example the subset consisting of the functions of x which give 1 just for a single value x_0, and 0 for all other values of x.

But M *cannot have the same* cardinality as L, because if it did then the elements of M could be put in one-one correspondence with the variable z, and M could be thought of as a function of the two variables x and z

$$\varphi(x, z)$$

so that each choice of z would give the element $f(x) = \varphi(x, z)$ of M and vice-versa each element $f(x)$ of M would correspond to $\varphi(x, z)$ for some choice of z. But this leads to a contradiction. Because then let us consider the function $g(x)$ which only takes on the values 0 and 1, and which for each x is different from $\varphi(x, x)$; then on the one hand $g(x)$ is an element of M, and on the other hand $g(x)$ cannot be $\varphi(x, z)$ for any choice $z = z_0$, because $\varphi(z_0, z_0)$ is different from $g(z_0)$.

Since the cardinality of M is neither smaller than or the same as that of L, it must be larger than the cardinality of L. (see Crelles Journal **84** 242).

I have already, in ``Foundations of a general theory of sets'' (Leipzig 1883; Math. Ann. Vol. 21) shown, by completely different techniques, that the cardinalities have no maximum; there it is also proved that the set of all cardinalities, when we think of them as ordered by their size, forms a ``well-ordered set'' so that in nature for every cardinality there is a next larger, but also every infinite set of cardinalities is followed by a next larger.

The ``cardinalities'' represent the single and remarkable generalization of the finite ``cardinal numbers;'' they are nothing else but the actual-infinitely-large cardinal numbers, and they inherit the same reality and definiteness as those do; only the relations between them form a different ``number theory'' from the finite one.

The further completion of this field is a job for the future.

Translator's note:

- This is a rough translation. I have tried to keep the flavor of the original, but I used ``set'' for Cantor's *Mannigfaltigkeit* (``manifold''), *Gesamtheit* (``totality''), *Inbegriff* (``clutch'') and *Menge*, now standard for ``set,'' which appears twice, in the last two paragraphs. It is interesting that the notations (a, b) and $[a, b]$ for open and closed intervals were not available to Cantor, nor was the possibility of writing f without (x).
- Zermelo, the editor of the collected works where this article was located, remarks: That the set of all cardinalities is a well-ordered system (even if not a ``set'') is in no way ``proven'' in the referenced work; Cantor does not yet have the proof that every set can be well-ordered and therefore that every cardinality is an Aleph.

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