WELCOME TO MAT 315
Advanced LinearAlgebra

Spring 2018

Time and place:

Lecture: TuTh 2:30-3:50PM in Frey Hall 316

Recitation: Monday 9AM Physics P117 with Mr. Ou

Introduction: This is an advanced mathematically rigorous course with complete proofs. Topics covered will include vector spaces, linear transformations, eigenvalues, dual spaces and inner products, bilinear and quadratic functions and quotient vector spaces Also the trace, determinant, charateristic and minimal polynomial of a linear operator and the Jordan form if time permits.

Text Book: *Linear Algebra Done Right* (3nd edition) by Sheldon Axler, Springer, (c) 2015

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Office Hours: Tuesday and Thursday 1:00-2:30, or by appointment

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Assistant: Mr. D. Ou, (dsou@math.stonybrook.edu) Office Hours: Tues 5:00-6:00PM in Math Tower 3-105. Also Monday 10AM-11AM and Thursday 1:00-2:00PM in the Math Learning Center.

Homework: Homework will be assigned every week. Doing the homework is a *fundamental* part of the course work. Problems should be handed in in your recitation section.

1st assignment: Page 11, Problems 1, 10, 11; Page 17, Problems 1, 2 Due the week of January 29

2nd assignment: Page 24, Problems 1ac, 5, 6, 7, 8, 12, 19, 20, 23, 24 Due the week of February 5

3rd assignment: Page 37, Problems 1, 5, 6; Page 43 Problems 3, 8; Page 49 Problem 13; Page 58 Problems 8, 9. Due the week of February 19

4th assignment: Page 67, Problems 1, 2, 4, 5, 6, 7, 9, 13, 17, 22 Due the week of February 26

5th assignment: Page 78, Problems 2, 4, 5, 6, 12; Page 88, Problems 7. 9, 16, 17, 19 Due the week of March 5

6th assignment: page 98, problems 1, 3, 6, 8, 15; page 113, problems 3, 6, 8, 13, 20 Due the week of March 19.

7th assignment: page 113, problems 23, 31; page 129, problem 2, 3, 5, 6, 7, 8, 9, 10 Due the week of March 26

8th assignment: page 129, problem 11; page 138, problems 1, 2, 3, 8, 9, 12, 15, 16, 22 Due the week of April 2

9th assignment: page 138, problems 23, 24ab, 29, 32, 34; page 153, problems 1a, 3, 4, 8, 19 Due the week of April 9

10th assignment: page 160, problems 1, 2, 5, 6, 7, 8, 9, 10, 14, 16, Due the week of April 16

11th assignment: page 175, problems 4abc, 5, 10, 11, 16, 31; page 189, problems 2, 13, 16, 17acd due the week of April 23

12th assignment: page 201, problems 2, 4, 5, 7, 8, 9, 10, 11, 12, 14 due the week of April 30

Grading Policy: The overall numerical grade will be computed by the formula **15% Homework + 15% First Midterm Exam+ 20% 2nd Midterm Exam + 50% Final Exam**

Midterm Exams: February 15 in class, March 27 in class

MIDTERM EXAM REVIEW: Exam Review

Final Exam: Monday, May 14, 11:15AM-1:45PM

FINAL EXAM REVIEW: Exam Review

N. B. Use of calculators is not permitted in any of the examinations.

Disabilities: If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact Disability Support > Services or call (631) 632-6748. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to the Evacuation Guide for People with Physical Disabilities.

Academic Integrity: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instances of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website.

Critical Incident management:
Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn.

Review for Midterm

Description of real and complex numbers as fields. Complex conjugate. Absolute value of a complex number.

Definition of a vector space over a field; e. g., the real or complex numbers.

Subspaces, sums and direct sums.

Span, linear independence, bases and dimension.

Prove that if a space has n linearly independent vectors, then a spanning set must have at least n elements. It follows that any two bases have the same number of elements.

Linear maps, their null spaces and ranges. The rank plus nullity theorem and its proof.

The matrix of a linear map. Matrix multiplication and composition of linear maps.

Invertible maps and their matrices. Injective and surjective maps.

Given a linear map $T : V \to W$, find bases of V and W that make the matrix of T as simple as possible. It should have r ones and the rest zeros, where r is the rank of the map.

Equivalence relations, equivalence classes and partitions of a set. Show that an equivalence relation on a set gives a partition of the set and vise versa.

Direct sums, products and quotients of vector spaces.

The quotient map $\pi : V \to V/U$.

Linear functionals, dual spaces and the dual of a linear map. Show that the $(TS)' = S'T'$ where $'$ means "dual".

Subspaces and their annihilators. Dimension of an annihilator. Rank and nullity of the dual of a linear map.

Bases and dual bases. Given bases of V and W, and dual bases of V' and W', and a linear map $T : V \to W$ what is the relation between the matrix of T and the matrix of T'?

Polynomials with real or complex coefficients, or with coeffieients in a finite field. The degree of a polynomial. The division algorithm for polynomials. Prove that r is a root of a polynomial iff $(x - r)$ divides the polynomial.

The distinction between polynomials and polynomial functions, and its importance if the coefficients are in a finite field.

The fundamental theorem of algebra for polynomials with complex coefficients. (You needn't know the proof.)

Review for Final

Everything that is on the Midterm exam review sheet.

Definition of an inner product for real and complex vector spaces.

Schwartz inequality and its proof

Triangle inequality

Parallelogram equality

Norms and how an inner product gives rise to a norm

Orthogonal and orthonormal sets, orthonormal bases

Proof that a basis is self-dual iff it is orthonormal

Gram-Schmidt procedure to get an orthonormal set

If an operator from V to itself has a basis for which it's matrix is upper trianglular, then it has an orthonormal basis for which its matrix is upper triangular.

Subspaces and their perpendicular subspaces.

Orthogonal projections and the closest point in a subspace

Linear functionals

The representation theorem which states: Let V be a finite dimensional vector space. Then for any linear functional $\phi : V \to \mathbf{F}$, there exists a vector $w \in V$ such that $\phi(v) =< v, w >$ for all $v \in V$. Know how to prove this and show by example that it may not be true if V is not finite dimensional.

Definition of the adjoint T^* of a linear transformation T.

The matrix of T^* with respect to orthonormal bases is the conjugate of the transpose of the matrix of T

Self-adjoint and normal operators

Self-adjoint operators have only real eigenvalues and they have an orthonormal basis of eigenvectors.

T is normal iff Tv and T^*v have the same norm for all vectors v.

If T is normal and λ is an eigenvalue of T, then $\bar{\lambda}$ is an eigenvalue of T^* with the same eigenvector.

Spectral theorem for normal operators on a complex vector space and for self-adjoint operators on a real vector space

Positive operators and their square roots

Show that a positive operator has a unique positive square root.

Isometries: Note that an isometry is normal and all its eigenvalues have absolute value one.