WELCOME TO MAT 313
Abstract Algebra

Fall 2018

Time and place:

Lecture: TuTh 11:30AM-12:50PM in Earth and Space 181

Introduction: This is an advanced mathematically rigorous course with complete proofs. Topics covered may include:

1. Groups: Symmetry Groups, Finite groups, Cyclic groups and Permutation groups; Subgroups, especially normal subgroups, cosets and factor groups; Group homomorphisms; The fundamental theorem of Abelian groups

2. Rings: examples of rings, Integral domains, Ideals and Factor rings, Ring homomorphisms, Polynomials and factorization, Unique factorization domains

3. Fields: Vector spaces and expansion fields, Algebraic and transcendental extensions, Finite fields, Grometric constructions

Text Book: *Comtemporary Abstract Algebra* (9th edition) by Joseph A. Gallian, Houghton Mifflin, (c) 2017

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Office Hours: Tuesday and Thursday 10:00-11:00AM, or by appointment

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Assistant: Mads Villadsen

Office Hours: Monday 11:30AM to 12:30PM Math Tower S240A.

Friday 12PM to 14PM in the Math Learning Center

Or by appointment

Homework: Homework will be assigned every week. Doing the homework is a *fundamental* part of the course work. Problems should be handed in in your lecture.

1st assignment: Page 23, Problems 4, 6, 10, 14, 30, 38 due Sept. 6

2nd assignment: Page 37, Problems 2, 16; page 54, problems 4, 8, 10, 16, 20, 34 due Sept. 13

3rd assignment: Page 68, Problems 6, 10, 36; page 85, problems 22, 30, 36; page 112, problems 2, 28 due Sept. 20

4th assignment: Page 132, Problems 2, 4, 12, 14, 20, 44, 60, 64 also find the non-abelian group with the smallest number of elements. Write its multiplication table. due Sept. 27

5th assignment: Page 150, problems 4,10,18, 20, 26, 42, 46 due October 4th

6th assignment: page 167, problems 12, 18, 26, 34, 36, 38, 42, 72 due October 11

No homework the week of October 15. Exam on October 16 instead.

7th assignment: page 187, problems 4, 6, 14, 16, 22, 26, 32, 38, 48, 62 due October 25

8th assignment: page 205, problems 4, 6, 10, 12, 16, 18, 24, 28, 32, 42, due November 1

9th assignment: page 220, problems 2, 6, 10, 12, 18, 22, 26, 28, 32 due November 8

10th assignment: page 232, problems 2, 6, 20, 30, 32, 50; page 243, problem 6, 8, 16, 42 due November 15

11th assignment: page 256, problems 4, 10, 14, 16, 18, 28, 46, 58, 64, 66 due November 27

12th assignment: page 270, prob;ems 6, 14, 18, 38, 44, 66; page 283, problems 14, 32, 48, 56 due December 4

Grading Policy: The overall numerical grade will be computed by the formula **20% Homework + 30% Midterm Exam+ 50% Final Exam**

Midterm Exam: October 16 in class

Final Exam: Wednesday, December 19, 11:15am-1:45PM

FINAL EXAM REVIEW: Exam Review

N. B. Use of calculators is not permitted in any of the examinations.

Disabilities: If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact Disability Support > Services or call (631) 632-6748. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to the Evacuation Guide for People with Physical Disabilities.

Academic Integrity: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instances of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website.

Critical Incident management:
Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn.

Review for Final

Go over everything on the midterm review sheet.

Well ordering principal and mathematical induction, Division algorithm, Fundamental theorem of arithmetic

Equivalence relations and equivalence classes and partitions

Groups and abelian groups, symmetries of a polygon, dihedral groups, order of a group and order of an element of a group.

$Z(G)$, The center of a group. Show that for a given group G, if $G/Z(G)$ is cyclic, then G is abelian.

Subgroups and normalizers and centralizers thereof

The Lagrange coset theorem and its consequence that the order of a subgroup divides the order of a group.

If a is in a group G, $< a >$ is the subgroup generated by a and $|a|$ is the order of a.

Cyclic groups such as Z_n. a, an element of Z_n generates Z_n iff $gcd(n, |a|) = 1$. Z_n is also a ring under multiplication mod n. It has units $U(n)$.
$U(n) = \{x \in Z_n | \gcd(x, n) = 1\}$ $U(st) \cong U(s)U(t)$ if $\gcd(s, t) = 1$

Permutation groups and cycles in them. Disjoint cycles commute. Odd and even permutions. Alternating groups

Normal subgroups and factor (or quotient) groups

Homomorphisms and their kernels, Isomorphisms, Cayley's theorem that every group is isomorphic to a subgroup of some group of permutations, Automorphisms and inner automorphisms.

Fundamental Theorem of Finite Abelian Groups (without proof)

Let p be a prime greater than 2. Show that all groups of order p are cyclic, Show that all groups of order $2p$ are cyclic or dihedral.

Internal and external direct products of groups. Show $Z_n \times Z_m$ is cyclic iff $\gcd(m, n) = 1$. Prove that $U(st) \cong U(s) \times U(t)$ if s and t are relatively prime.

Rings, Integral Domains and Fields

Zero Divisors. Prove that a finite integral domain must be a field.x

Ideals and Principal Ideals, Polynomial rings, and the degree of a polynomial.

Factor or quotient rings: R/I where I is an ideal in the ring R

Maximal Ideals, Prime ideals, R/I in each case

Quotient field of an integral domain, construction of rationals from integers

Characteristic of a ring and of a field

Division algorithm in Z and in $F[x]$, where F is any field

Content of a polynomial in $Z[x]$, primitive polynomial, Gauss lemma

Reducibility in $Z[x]$ and in $Q[x]$.

Mod p irreducibility tests and Eisenstein's criterion.

Let $p(x)$ be a polynomial in $F[x]$. Prove that $< p(x) >$ is a maximal ideal iff $p(x)$ is irreducible.

Define principle ideal domains and unique factorization domains. Show that a PID is a UFD, but not conversely.

Unique factorization in $Z[x]$.

Prove that a principle ideal domain satisfies the ascending chain condition.