MAT 305 - Calculus IV, Spring 2007

Monday, Wednesday 5:20-6:40 p.m. in Physics 116

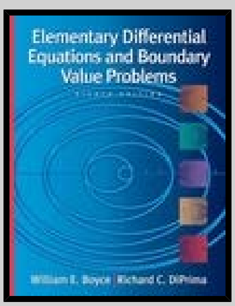
FINAL EXAM INFO
TIME: Wednesday, May 9th, 5:00pm-7:30pm
PLACE: JAVITS 105 (This is NOT our usual classroom)
REVIEW I: Friday, May 4th, 1:00pm-2:30pm (in Math tower P-131)
REVIEW II: Monday, May 7th, 4:00pm-6:00pm (in Math tower P-131)
Here is a practice final exam from 2003. Suggested problems are posted now (follow the link to the syllabus).

Instructor: Hrant Hakobyan
hhakob@math.sunysb.edu
Office: Math Tower 3-104

Office Hours: Tuesday 11:30 a.m.-12:30 p.m. in MLC (**Note: MLC is in the basement of the Math Tower, S-240A**),
Tuesday 12:30 p.m.-02:30 p.m. in Math Tower 3-104

Recitation: Thursday 5:20-6:15 p.m. in Physics P117
T.A.: Robert Findley, rfindley@math.sunysb.edu
Office Hours:

Textbook: Boyce, DiPrima : Elementary Differential Equations and Boundary Value Problems, Wiley, 8th Edition



What is this course about?
Observables in natural and physical sciences are subject to change. The rate of change (with respect to time/location/etc.) may depend on many parameters, quite often including the observable itself. In a mathematical model, this leads to *differential equations*. We will study different kinds of differential equations and the methods used to solve them. We will cover : first order equations, second order linear equations (both homogeneous and non-homogeneous), power series solutions, systems of first order linear equations, some basic partial differential equations and methods used to solve them including Fourier series. (This will roughly be Chapters 2, 3, 5, 7, and 10 of Boyce and DiPrima.) A tentative syllabus will be updated according to the progress of the class.

Prerequisites: This is an upper division's course. Knowledge of basic calculus (Fundamental Theorem, Separable Differential Equations) is expected, knowledge of material from MAT 203/205 is beneficial, especially for the last part of the course on partial differential equations.

Grading Policy: Your grade will be determined by your scores on

- Midterm 1 : 2%
- Midterm 2 : 20%
- Final : 40%
- Recitations : 20%

Homework: Homework problems will be assigned weekly (see syllabus). They are due the next week in recitation. Two or three problems will be graded each week. You get 5 points if you (reasonably) attempt to solve at least 50% of the problems and 5 points if you solve the graded problems correctly. You are expected to participate in recitations. The T.A. will assign a recitation grade at the end of the course based on your graded homework problems and your performance in recitations classes.

Exam Schedule:

Midterm 1 : Monday, February 26th, 5:30-6:50 p.m. in Physics P117
Midterm 2 : Wednesday, April 11th, 5:30-6:50 p.m. in Physics P117
Final Exam : Wednesday, May 9th, 5:00-7:30 p.m. in Physics P117

Students with Disabilities: If you have a physical, psychological, medical, or learning disability that may impact on your ability to carry out assigned course work, you are strongly urged to contact the staff in the Disabled Student Services (DSS) office: Room 133 in the Humanities Building; 632-6748v/TDD. The DSS office will review your concerns and determine, with you, what accommodations are necessary and appropriate. A written DSS recommendation should be brought to your lecturer who will make a decision on what special arrangements will be made. All information and documentation of disability is confidential. Arrangements should be made early in the semester (before the first exam) so that your needs can be accommodated.

Back to the Department of Mathematics.

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Topics and Assignments

Week of	Topics	Assignments
Jan 22	Introduction and Review : Chapter 1	
Jan 29	**First Order Differential Equations** Methods of Solving Differential Equations : 2.1, 2.2, 2.6	Section 2.1: 15, 16, 17, 28 Section 2.2: 4, 6, 10, 25 Section 2.6: 4, 6, 18, 19, 24, 27 due February 8th
Feb 5	**Second Order Differential Equations** **Homogeneous Equations** Fundamental Solutions : 3.1, 3.2	Section 3.1: 5, 6, 11, 12, 22, 28 Section 3.2: 4, 9, 14, 19, 24, 31 due February 15th
Feb 12	**Constant Coefficient Equations** Complex Roots : 3.4,	Section 3.3: 4, 8, 10, 17, 21, 25 Section 3.4: 10, 12, 18, 19 Section 3.5: 6, 14, 20, 25 Section 3.6: 3, 7 due February 22nd
Feb 19	Repeated Roots 3.5 Non-homogeneous Equations : 3.6, 3.7 Practice problems	Section 3.4: 34*, 37 Section 3.5: 28, 31 Section 3.6: 13, 15, 17, 32*, 33 Section 3.7: 2, 9, 14, 17, 28*, 30 due March 1st
Feb 26	**MIDTERM 1** (Monday 26th February) **Power Series Solutions**	

	Review of Series : 5.1	
Mar 5	**Series Solutions** Ordinary Points : 5.2, 5.3	Section 5.2: 9, 12, 17 Section 5.3: 3, 7 due March 15th
Mar 11	**Series Solutions** Regular Singular Points and Euler's Equation: 5.4, 5.5 **The Laplace Transform** Definition and solution of IVPs : 6.1, 6.2	Section 5.4: 8, 9, 18, 19, 21, 24 Section 5.5: 4, 6, 15, 19, 21, 23 due March 22nd
Mar 18	**The Laplace Transform** Discontinuous functions : 6.3, 6.4	Section 6.1: 5, 7, 17, 19 Section 6.2: 5, 8, 14, 16, 22 Section 6.3: 2, 9, 14, 18 due March 29th
Mar 25	Spring Break	
April 8	**MIDTERM 2** (Wednesday 11th April) Practice midterm, Solutions to the practice midterm (Note: There won't be problems about Bessel's equation. Instead there will be more problems about the Laplace transform.)	Section 6.4: 3, 7, 9, 15 Section 6.5: 4, 6, 16 (part (d) is optional) Section 6.6: 6, 7, 9, 10, 13, 16 due April 19th
Apr 16	**The Laplace Transform** Impulse functions and convolution : 6.5, 6.6	Section 10.1: 3, 8, 13 Section 10.2: 11, 14, 16, 17 due April 24th
Apr 23	**Fourier Series** Boundary value problems and Fourier series : 10.1, 10.2 Fourier series, even and odd functions : 10.3, 10.4 **Partial Differential Equations** The heat equation and separation of variables : 10.5	Sec. 10.3: 4, 5, 8, 9, 12 Sec. 10.4: 8, 9, 17, 18, 24, 35 Sec. 10.5: 2, 5, 8, 10, 12 due May 3rd
Apr 30	Other Heat Conduction problems : 10.6 The Wave Equation: Vibration of an elastic string : 10.7	Section 10.6: 2, 4, 9, 12 Section 10.7: 2, 6, 9 (ignore part (d) in 2, 6)
May	**Final from 2003** **Practice topics and problems.**	

9	Practice set : 2.6-27, 3.5-30, 3.6-17, 3.7-17, 5.2-6, 5.5-20*, 6.4-9, 6.5-6, 10.4-25(a),(b), 10.5.10* (* =extra credit)	

Time allowed: 2 hours 30 minutes. Name:

MAT 305 Calculus IV : Final Exam

Calculators not permitted. All problems are of equal value; attempt all eight.

1. (i) Show that the first order ODE

$$(2y - e^x) + xy' = 0$$

is *not* exact.
(ii) Find the equation that an integrating factor $\mu(x)$, which depends only on x, must satisfy to make the equation exact. Then solve for $\mu(x)$ and find an implicit solution y of the ODE.

2. (i) Find the value of r for which $y_1(x) = e^{rx}$ is a solution to the second order ODE

$$y'' - 6y' + 9y = 0.$$

(ii) Using $y_1(x)$ and reduction of order, find the general solution to the non-homogeneous ODE

$$y'' - 6y' + 9y = 2e^{3x}.$$

In other words, substitute a solution of the form $y(x) = v(x)y_1(x)$ and solve for $v(x)$.

3. Find the general solution to the second order non-homogeneous ODE

$$y'' + 4y = \frac{2}{\cos 2t}, \qquad 0 < t < \pi/4$$

by first solving the corresponding homogeneous ODE and then using variation of parameters.

4. Solve the second order ODE

$$y'' - xy' - 2y = 0$$

by substituting a series $y = \sum_{n=0}^{\infty} a_n x^n$, and finding two linearly independent series solutions (write the first four terms and the general term of each series). Use the ratio test to determine the radius of convergence of the two series.

NOTE: This will be a question about Euler's equation.

5. (i) Show that the second order ODE

$$4x^2y'' + (1 - 4x)y = 0$$

has a regular singular point at $x_0 = 0$.
(ii) By substituting a series $y = \sum_{n=0}^{\infty} a_n x^{n+r}$, or otherwise, find the indicial equation for r and the exponents r_1 and r_2.
(iii) Find the series solution which corresponds to the larger exponent r_1 (write the first four terms and the general term).

6. Using the Laplace transform, solve the IVP

$$y'' + 2y' + 5y = \delta(t-4), \qquad y(0) = 0, \qquad y'(0)$$

where δ is the Dirac delta function.

7. (i) Consider the function $f(x) = L - x$, $0 < x < L$. Describe how to extend f to *both* an even and an odd function with period $2L$. Sketch three periods of each function.
(ii) Find the Fourier sine series of the odd extension of $f(x)$.

8. (i) Consider the heat equation

$$u_t = u_{xx}, \qquad 0 < x < 10, \qquad t > 0$$

with initial and boundary conditions

$$u(x,0) = f(x) = 30, \qquad 0 < x < 10,$$

$$u(0,t) = u(10,t) = 0, \qquad t > 0.$$

Show that separation of variables, $u(x,t) = X(x)T(t)$, leads to a second order ODE for $X(x)$ and a first order ODE for $T(t)$, and write explicitly these ODEs.
(ii) Given that the fundamental solutions are

$$u_n(x,t) = \exp(-n^2\pi^2 t/100)\sin(n\pi x/10), \qquad n = 1, 2, \ldots$$

find the solution $u(x,t)$ to the heat equation, as a series.