Stony Brook University
Mathematics Department
Julia Viro

Introduction to Advanced Mathematics
MAT 250
Fall 2019

Syllabus

Course description: An introduction to the Advanced Track mathematics program. Provides the core of basic logic, elementary set theory and language of maps. The rigorous language will be applied to define and study some notions of number theory, combinatorics, elementary analysis, Euclidean geometry, topology, etc. No preliminary knowledge of advanced mathematics is required. MAT 250 serves as an alternative to MAT 200 for students in the Advanced Track.

Credits: 3.

Instructor: Julia Viro. e-mail: julia.viro@stonybrook.edu
Office hours: Monday, Wednesday at 4pm-5pm (MLC);
Tuesday, Thursday at 10am-11am (MLC) or by appointment.

Textbook: Peter J. Eccles, *An Introduction to Mathematical Reasoning*, Cambridge University Press.

Meetings: TuTh 1:00pm-2:20pm in Physics P117.

Homework: will be assigned weekly through the Blackboard and collected in class on Tuesdays. The emphasis of the course is on writing proofs, so please try to write legibly and explain your reasoning clearly and fully. You are encouraged to discuss the homework problems with others, but your write-up must be your own work. Suspiciously similar papers won't be graded.

Grading system: your grade for the course will be based on: homework 10%, quizzes 10%, two midterms 20% each, final exam 40%.

Make-up policy: Make-up examinations are given only for work missed due to unforeseen circumstances beyond the student's control. Late home work will not be accepted.

Student Accessibility Support Center (SASC) statement: If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact SASC (631) 632-6748 or http://studentaffairs.stonybrook.edu/dss/. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and SASC. For procedures and information go to the following website:
http://www.stonybrook.edu/ehs/fire/disabilities/asp.

Academic integrity statement: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at http://www.stonybrook.edu/uaa/academicjudiciary

Critical incident management: Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn.

Weekly Plan (tentative)

Tu 8/27, Th 8/29. Introduction to logic. Propositions and predicates. Logical connectives. Truth tables. Compound propositions. Conjunctive and disjunctive normal forms. Conditional and biconditional sentences. Denials. Logical identities.

Reading: 1, 2.

Tu 9/3, Th 9/5. Quantifiers and quantified sentences. Analyzing and constructing propositions involving several quantifiers.

Reading: 7.

Tu 9/10, Th 9/12. Logical structure of definitions and theorems. How to read and understand mathematical texts.

Structure of a mathematical theory: basic objects, axioms, definitions and theorems. The role of proofs. Examples and counterexamples.

Reading: 3; Lecture notes.

Th 9/17, Th 9/19. Proof techniques: direct proof, proof by contraposition, proof by contradiction, proof by exhaustion. Strategies for constructing proofs.

Reading: 3, 4.

Tu 9/24, Th 9/26. Principle of mathematical induction in various forms: induction, strong induction, well-ordering principle.

Reading: 5.

Tu 10/1, Th 10/3. Basic notions of set theory: set and its elements, empty set, subset, intersection, union, difference and complement. Families of sets. Relations between logical and set-theoretical operations. Set-theoretic identities.

Maps: definitions and notations. Basic terminology associated with maps: domain, codomain, image and preimage.

Examples of maps: functions in one variable, numerical sequences, identity map, constant map.

Reading: 6, 8

Tu 10/8, Th 10/10. Review. **Midterm 1 is on Tu 10/8.**

Tu 10/15 (no classes), Th 10/17. Composition of maps: definition and properties. Inclusion map. Restriction of a map to a subset. Submap. Characteristic function of a set. Power set and the set of all maps to a two-element set. The set of all maps $X \to Y$.

Injections, surjections and bijections. Definition and properties of inverse map. Equivalence between invertibility and bijectivity.

Reading: 9.

Tu 10/22, Th 10/24. Cartesian product of sets. Coordinate projections and fibers. Graph of a map.

Relations. Functions in several variables as functions on a Cartesian product. Metric on a set.

Equivalence relations and partitions. Quotient sets. Canonical factorization of a map into composition of surjection, bijection and injection.

Constructions of integers and rational numbers. Construction of complex numbers.

Reading: 13; Lecture notes.

Tu 10/29, Th 10/31. Congruence classes. Modular arithmetic.

Reading: 19, 20, 21.

Tu 11/5, Th 11/7. Definitions of equipotent sets and cardinality of a set. Finite and infinite sets.

Finite arithmetic. Pigeonhole principle.

Reading: 10, 11, 12.

Tu 11/12, Th 11/14. Review. **Midterm 2 is on Tu 11/12.**

Tu 11/19, Th 11/21. Examples of infinite sets of the same and different cardinalities. Hilbert's Grand Hotel. Cantor theorem about non-equipotency of a set and its power set.

Denumerable arithmetic. Countable and uncountable sets.

Reading: 14.

Tu 11/26, Th 11/28 (no classes) Cantor-Shröder-Bernstein theorem. Ordering of cardinal numbers. Cantor's theorem about uncountability of $\mathbb{R}$. Continuum hypotheses.

Reading: 14.

Tu 12/3, Th 12/5 Review.

Th 12/20 at 11:15am-1:45pm. Final exam.