Math 122 - Overview of Calculus (Fall 2016)

Instructor: Ben McMillan
Email: bmcmillan@math.stonybrook.edu
Locataion: TuTh 10:00am-11:20am in Javits lecture hall 102

Announcements

- The old quizzes are now posted at the bottom of the page.

Course Information:

The course syllabus is here. Some critical points:

- The midterms will be in class on Thursday 9/29 and Thursday 11/3. The final is on Wednesday 12/14 from 11:15am--1:45pm.
- My office hours are Monday and Tuesday from 11:30am--12:30pm in Simons Center 510 and Thursday 1:30--2:30pm in the MLC.
- TA office hours:
  - Apratim Chakraborty---Friday 10-11am in MAT-3105 and Friday 11am-1pm in the MLC.
  - Nathan Chen---Monday 1-2pm in S204A and Monday 2-3pm/Wednesday 12-1pm in the MLC.
  - Qianyu Chen---Monday 4-6pm and Thursday 8:30-9:30am in the MLC.

Schedule and Homework:

The following is a tentative schedule for the course. As homework is assigned it will be posted here. Homework is due to your TA at the beginning of section the following week. You are responsible for your own homework---mathematics takes practice, and if you find ways to avoid it you will have a hard time on the tests.

Unless stated otherwise, the homework is from the corresponding section of the book. E.g., your first homework is problems 1,4, 11, etc. from section 1.1. All of the problems assigned in week n are due to your TA (at the start of recitation) in week n+1.

For each homework you turn in, please write an estimate of how long it took you at the top of the page.

Week	Date	Topic(s) Covered	Reading	Homework
1	8/30	Functions	Section 1.1	1.1: 1, 4, 11, 13, 28, 35, 36
	9/1	Linear functions, Applications to economics	Section 1.2, Section 1.4	1.2: 4, 5, 9, 12 1.4: read pages 28-30, do problem 19
2	9/6	Labor Day, no class!	.	Have a good break!
	9/8	Average rate of change	Section 1.3	1.3: 1, 2, 3, 4, 5, 6, 9, 12, 24, 34, 38, 40
3	9/13	Exponential functions	Section 1.5	1.5: 2, 3, 4, 11, 27, 29, 33
	9/15	The natural logarithm Applications of exponentials	Section 1.6, Section 1.7	1.6: 8, 15, 22, 23, 25, 40, 48 1.7: 2, 3, 10, 11, 15, 21
4	9/20	Function composition, scaling, shifting	Section 1.8	1.8: 4, 3, 6, 11, 17, 23, 31, 37, 38, 39, 41
	9/22	Polynomials, 1/x	Section 1.9	1.9: 1, 2, 4, 5, 8, 13, 17, 19, 27
5	9/27	Review!	Chapter 1	Study
	9/29	Midterm 1	Chapter 1	None
6	10/4	Instantaneous rate of change	Section 2.1	2.1: 1, 4, 6, 11, 14, 15
	10/6	Limits	"Focus on theory" (Pages 131-135)	Page 135: 1, 2, 3, 5, 7, 19, 22
7	10/11	More aspects of the derivative	Section 2.1	2.1: 17, 18, 22
	10/13	The derivative of a function is a function	Section 2.2 Section 2.3	2.2: 1, 7, 8 2.3: 1, 3, 11, 17 3.1: 1, 2, 4, 7, 11, 25, 51, 53
8	10/18	Graphing derivatives	Section 2.2 Section 2.4	2.2: 2, 5, 6, 7, 9, 18, 19, 20, 21, 25, 26 2.4: 1, 4, 7, 10, 11, 26
	10/20	More on calculating derivatives	Section 3.1 Section 3.2	3.1: 6, 11, 13, 15, 23, 32, 27, 47 3.2: 2, 5, 11, 23, 37
9	10/25	The product rule	Section 3.4	3.4: 2, 3, 7, 13, 14, 17, 22, 25, 42
	10/27	The chain rule	Section 3.3	3.3: 1, 2, 4, 8, 11, 13, 14, 22, 34
10	11/1	Review	Chapters 2 & 3	.
	11/3	Midterm 2		
11	11/8	Local maxima and minima	Section 4.1	4.1: 1, 3, 4, 7, 14, 15, 17, 21
	11/10	Antiderivatives	Section 6.2	6.2: 15, 19, 23, 40, 43, 51, 55, 59
12	11/15	More antiderivatives, properties	Section 6.2	6.2: 1, 3, 8, 9, 12, 26, 31, 36, 45, 49, 53, 59, 65, 71
	11/17	The Fundamental Theorem of Calculus, Graphs of antiderivatives	Section 6.3 Section 6.1	6.3: 1, 3, 7, 13, 21, 29 6.1: 5, 7, 11, 17, 18
13	11/22	The integral is an antiderivative	Section	.
	11/24	Thanksgiving break, no class!		

14	11/29	Integration by parts	Section 6.7	1, 4, 7, 10, 11, 22, 23, 24
	12/1	Integration by substitution	Section 6.6	7, 11, 14, 19, 21, 29, 51, 53
15	12/6	Integrals as areas	Section 5.3	.

Previous Quizzes:

Quiz 1

Quiz 2

Quiz 3

Quiz 4

Quiz 5

Quiz 6

Quiz 7

Quiz 8

Solutions!

Math 122: Overview of Calculus

Fall 2016 Stony Brook

Instructor:	Ben McMillan	**Time:**	TuTh 10:00am–11:20am
Email:	bmcmillan@math.stonybrook.edu	**Place:**	102 Javits

Course Page: You will find up to date information, homework, and announcements at

math.stonybrook.edu/~bmcmillan/math122fall16/

Office Hours: Office hours are an invaluable resource, one that you really should use! You are welcome to attend any of the office hours held by either me or the TAs. In particular, if you can't make your TA's office hours, you should go to another TA's.

My office hours this term are Monday and Tuesday 11:30am–12:30pm in 510 Simons Center and Thursday 1:30–2:30pm in the MLC. Your TAs for the course are Apratim Chakraborty, Nathan Chen, and Qianyu Chen. You can find their office hours at math.stonybrook.edu/office-hours

Textbook: The textbook for the course is *Applied Calculus, 5th Edition* by Hughes-Hallett et al.

Exams: The midterms will be held in class on 9/29 and 11/3. The final is Wednesday, Dec 14, 11:15am–1:45pm. Unfortunately, there will be no makeup exams, so you *must* be able to make those dates to take this course. Please let me know if you will need DSS accomodations well in advance of the first midterm.

Homework: Each week I will post homework questions on the course webpage and you will turn it in to your TA the following week at the beginning of section. The homework is graded for completion. You should think of it as practice for the quizzes.

Quizzes: There will be a quiz at the end of section each week. These are based heavily on the homework. If you complete (and understand) the homework from the previous week, you should do well on them. There will be no make-up quizzes. Instead, we will cancel your lowest 2 quiz scores at the end of term.

Grading Policy: Homework/Quizzes/Participation: 20%, Midterm 1: 25%, Midterm 2: 25%, Final: 30%.

Americans with Disabilities Act: If you have a physical, psychological, medical or learning disability that may impact your course work, please contact Disability Support Services, ECC (Educational Communications Center) Building, Room 128, (631)632–6748. They will determine with you what accommodations, if any, are necessary and appropriate. All information and documentation is confidential. http://studentaffairs.stonybrook.edu/dss/index.html.

Academic Integrity: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty is required to report any suspected instances of academic dishonesty to the Academic Judiciary. Faculty in the Health Sciences Center (School of Health Technology Management, Nursing, Social Welfare, Dental Medicine) and School of Medicine are required to follow their school-specific procedures. For more comprehensive information on academic integrity, including categories of academic dishonesty please refer to the academic judiciary website at http://www.stonybrook.edu/commcms/academic_integrity/

Critical Incident Management: Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of University Community Standards any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn. Faculty in the HSC Schools and the School of Medicine are required to follow their school-specific procedures. Further information about most academic matters can be found in the Undergraduate Bulletin, the Undergraduate Class Schedule, and the Faculty-Employee Handbook.

Name: ______________________

Nathan Chen

Math 122 - Quiz 1

0. Please write your name and your TA's name at the top. Label which problems you are working on in the space below. Please box your answers.
1. What is the slope of the line that passes through the points (-2, 1) and (4, -1)? Use this to find the equation of the line.
2. The cost function of a company is given by $C(q) = 10q + 500$ and the revenue function is $R(q) = 30q$. Find the smallest quantity that must be sold to make a profit.
3. The number of US households with typewriters was 60,000,000 in 1970 and 12,000,000 in 1986. Estimate the average rate of change in the number of US households with typewriters during this 16-year period. Give units and interpret your answer.

Solutions:

Name: ______________________

Apratim Chakraborty

Math 122 - Quiz 2

0. Please write your name at the top. Label which problems you are working on in the space below. Please box your answers.
1. In 2010, there were 100 tigers in a sanctuary. The population increased by 15% each year.Write a formula for number of tigers in that sanctuary after t years.
2. Find t such that $4e^{2t} = 10$

Solutions:

Name: ______________________________

Qianyu Chen

Math 122 - Quiz 3

0. Please write your name at the top. Label which problems you are working on in the space below. Please box your answers.
1. Let $f(x) = x^2 - 2x - 8$ and $g(x) = x + 3$. Find and simplify $f(g(x))$.
2. The exponential function $y(x) = Ce^{\alpha x}$ satisfies the conditions $y(0) = 5$ and $y(1) = 1$. Find the constant C and α. What is $y(-1)$?

Solutions:

Name: ____________________

Nathan Chen

Math 122 - Quiz 4

0. Please write your name at the top. Label which problems you are working on in the space below. Please **box** your answers.

1. In a time of t seconds, a particle moves a distance of s meters from its starting point, where $s = 3t^2 + 5$. Find the average velocity between $t = 2$ and $t = 2 + h$ (where h is some positive constant) and give units.

2. Find the following limit:

$$\lim_{h \to 0} \frac{2 \cdot (3+h)^2 - 2 \cdot 3^2}{h}.$$

<u>Solutions</u>:

Name: ____________________

Apratim Chakraborty

Math 122 - Quiz 5

0. Please write your name at the top. Label which problems you are working on in the space below. Please box your answers.
1. Use the limit definition of the derivative to find the derivative of $f(x) = x^2 + 1$

(Hint : Remember that $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$)

2. Find the derivative of the function $y = 3x^3 - 2x - 8$

<u>Solutions</u>:

Name: ____________________________

Qianyu Chen

Math 122 - Quiz 6

0. Please write your name at the top. Label which problems you are working on in the space below. Please box your answers.
1. Find the equation of the tangent line to $y = e^{3t} + 3$ at $t = 0$. Check by sketching a graph of $y = e^{3t} + 3$ and the tangent line on the same axes.
2. If $y(t) = 5 \ln t + 7e^t - 4t^2 + 12$, find $f'(t)$ and $f''(t)$.

Solutions:

Name: ____________________

Nathan Chen

Math 122 - Quiz 7

0. Please write your name at the top. Label which problems you are working on in the space below. Please **box** your answers.

1. Find the derivative of the following functions:
   a. $g(t) = e^t + 3t$
   b. $f(x) = \ln(e^x + 3x)$.
2. Let $h(x) = x^{-2}e^{3x}$.
   a. Find the derivative of $h(x)$.
   b. Find $h'(1)$.

<u>Solutions</u>:

Name: ______________________

Apratim Chakraborty

Math 122 - Quiz 8

0. Please write your name at the top. Label which problems you are working on in the space below. Please box your answers.
1. Find and classify its local maxima/minima of the function - $f(x) = x^3 - 2x^2 + 1$.

2. Find the anti-derivative of the function $f(x) = x^5 + x - 1$.

<u>Solutions</u>:

Quiz 1

Problem 1. The slope is given by the slope formula, so

$$m = \frac{-1-1}{4-(-2)} = -1/3.$$

Using the point-slope formula for a line, we get the equation

$$y = -\tfrac{1}{3}(x-4) - 1.$$

Problem 2. We need to find when the revenue function is equal to the cost function. This gives the equation

$$10q + 500 = 30q.$$

Solving, we find that

$$q = 25.$$

Problem 3. The slope formula gives

$$\frac{12,000,000 - 60,000,000}{1986 - 1970} = -3,000,000.$$

This is in units of typewriters per year. The number means that, on average, US households had 3 million fewer typewriters every year.

Quiz 2

Problem 1. The initial value is 100 and the annual rate of increase was 1.15. Thus there are

$$P(t) = 100(1.15)^t$$

tigers after t years.

Problem 2. Taking the logarithm of both sides, and then applying the rules of logarithms,

$$\begin{aligned}
\ln(4e^{2t}) &= \ln(10) \\
\ln(4) + \ln(e^{2t}) &= \ln(10) \\
\ln(4) + 2\ln(e^{t}) &= \ln(10) \\
\ln(4) + 2t &= \ln(10) \\
2t &= \ln(10) - \ln(4) \\
t &= \frac{\ln(10/4)}{2} \\
t &= \frac{\ln(5/2)}{2}
\end{aligned}$$

Note that

$$\ln(4e^{2t}) \neq 2\ln(4e^{t}).$$

Try plugging in random values for t on a calculator if you don't believe this.

Quiz 3

Problem 1.

$$\begin{aligned} f(g(x)) &= (g(x))^2 - 2g(x) - 8 \\ &= (x+3)^2 - 2(x+3) - 8 \\ &= (x^2 + 6x + 9) - 2x - 6 - 8 \\ &= x^2 + 4x - 5 \end{aligned}$$

Problem 2. Since $y(0)$ is the initial value, $C = 5$. This means that $y(x) = 5e^{\alpha x}$. Using the other given piece of information,

$$1 = y(1) = 5e^{\alpha}.$$

Solving the equation, one finds

$$\alpha = \ln(1/5).$$

So

$$y(x) = 5e^{\ln(1/5)x}$$

and

$$y(-1) = 5e^{-\ln(1/5)} = 5e^{\ln((1/5)^{-1})} = 5e^{\ln(5)} = 25.$$

Quiz 4

Problem 1. The formula for average velocity gives

$$\frac{s(2+h)-s(2)}{2+h-2} = \frac{3(2+h)^2+5-3\cdot 2^2-5}{h} = \frac{12+12h+3h^2-12}{h} = 12+3h$$

Problem 2.

$$\begin{aligned}
\lim_{h\to 0} \frac{2\cdot(3+h)^2-2\cdot 3^2}{h} &= \lim_{h\to 0} \frac{18+12h+2h^2-18}{h} \\
&= \lim_{h\to 0} \frac{12h+2h^2}{h} \\
&= \lim_{h\to 0} \frac{h(12+2h)}{h} \\
&= \lim_{h\to 0} 12+2h \\
&= 12
\end{aligned}$$

Quiz 5

Problem 1.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{(x+h)^2+1-x^2-1}{h} \\
&= \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} \\
&= \lim_{h\to 0} \frac{2xh+h^2}{h} \\
&= \lim_{h\to 0} \frac{h(2x+h)}{h} \\
&= \lim_{h\to 0} 2x+h \\
&= 2x
\end{aligned}$$

Problem 2. We could use the limit definition, but it is quicker and less likely that we will make calculation errors if we use the rules that we have learned for calculating deratives. Thus

$$\begin{aligned}
\frac{d}{dx}y &= \frac{d}{dx}(3x^3-2x-8) \\
&= \frac{d}{dx}(3x^3) + \frac{d}{dx}(-2x) + \frac{d}{dx}(-8) && \text{by sum law} \\
&= 3\frac{d}{dx}(x^3) - 2\frac{d}{dx}(x) && \text{because constants cross } \tfrac{d}{dx} \\
&= 9x^2-2 && \text{by derivatives of power laws}
\end{aligned}$$

Quiz 6

Problem 1. Recall that the formula for the tangent line to $f(t)$ at $t = a$ is given by the formula

$$y = f'(a)(t - a) + f(a).$$

In this case, $f(t) = y(t)$, and we may calculate that

$$y'(t) = 3e^{3t}$$

so that

$$y'(0) = 3.$$

Thus the equation for the tangent line is given by

$$y = 3t + 4.$$

Problem 2. There is a slight notational error in the question. We should compute $y'(t)$ and $y''(t)$. This results in

$$y'(t) = 5/x + 7e^t - 8t.$$

Taking the derivative again,

$$y''(t) = -5/x^2 + 7e^t - 8.$$

Quiz 7

Problem 1.

(1) We use the rules of differentiation:

$$\begin{aligned}\frac{d}{dt}(e^t + 3t) &= \frac{d}{dt}(e^t) + \frac{d}{dt}(3t) \\ &= e^t + 3.\end{aligned}$$

(2) Now we must use the chain rule. The inner function is $e^x + 3x$ and the outer function is $\ln(x)$.

$$\frac{d}{dx}(\ln(e^x + 3x)) = \frac{1}{e^x + 3x} \cdot (e^x + 3)$$

Problem 2.

(1) Now we use the product rule.

$$\frac{d}{dx}(x^{-2}e^{3x}) = (-2x^{-3})(e^{3x}) + (x^{-2})(3e^{3x})$$

(2) Plugging $x = 1$ into the previous answer, we get

$$h'(1) = (-2(1)^{-3})(e^{3(1)}) + ((1)^{-2})(3e^{3(1)}) = -2e^3 + 3e^3 = e^3.$$

Quiz 8

Problem 1. First we need to find the critical points of $f(x)$—those values of $x = a$ for which $f'(a) = 0$. So we must solve the equation

$$f'(x) = 3x^2 - 4x = 0.$$

We can solve this by factoring:

$$0 = 3x^2 - 4x = x(3x - 4).$$

The solutions are thus $x = 0$ and $x = 4/3$.

We use the second derivative test to determine if these are local minima or maxima. First note that the second derivative of $f(x)$ is

$$f''(x) = 6x - 4.$$

Thus $f''(0) = -4$ and so 0 is a maximum.

On the other hand, $f''(4/3) = 4$, so $4/3$ is a minumum.

Problem 2. By the rules of integrals,

$$\begin{aligned}\int (x^5 + x - 1)\ dx &= \int x^5\ dx + \int x\ dx - \int 1\ dx \\ &= x^6/6 \qquad + x^2/2 \qquad - x \qquad\quad + c.\end{aligned}$$