MATH 132 Second Midterm

Wednesday November 2, 2011

Name: ________________________ **ID:** ____________________ **Rec:** _____

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	15	10	15	15	15	15	20	15	15	135
Score:										

There are 9 problems in this exam, printed on 7 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate **clearly** what is where if you expect someone to look at it. **Books, calculators, electronic devices, extra papers, and discussions with friends are not permitted.** Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.) After receiving the exam but before the end of exam period, you may use a time machine to go back in time and tell yourself what topics you should have studied. However, you must also allow me to use the time machine to go back in time and change the exam accordingly.

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

You must give a correct justification of all answers to receive credit.

You have 90 minutes to complete this exam.

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1. For each of the following sequences, determine whether it converges or diverges. If the sequence converges, give its limit. Justify your answer.

5 pts (a) $\left\{ \dfrac{2n^4 + \cos(n\pi)}{(1+3n)(2+n^3)} \right\}_{n=0}^{\infty} = -\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{10}, \ldots$

5 pts (b) $\left\{ \dfrac{\ln(n)}{2n} \right\}_{n=1}^{\infty} = 0, \dfrac{\ln 2}{4}, \dfrac{\ln 3}{6}, \ldots$

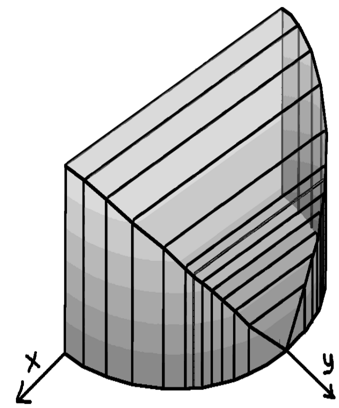
5 pts (c) $\{e + (-1)^n\}_{n=0}^{\infty} = e+1, e-1, e+1, \ldots$

10 pts 2. What value of N do we need to ensure that the sum $\displaystyle\sum_{n=0}^{N} \frac{(-1)^n}{n^2+2}$ is within $1/100$ of the limit? Keep in mind that N must be a whole number.

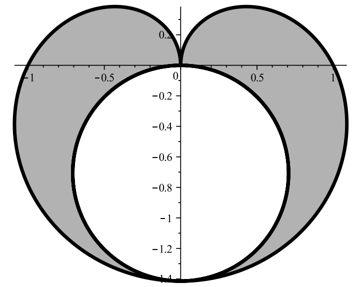
15 pts 3. Find the volume of the solid whose base is the half-circle

$$x^2 + y^2 = 9 \qquad \text{with } y \geq 0$$

and whose cross-sections perpendicular to the y-axis are squares.



15 pts 4. Find the area that lies inside the polar curve $r = \sqrt{1 - \sin\theta}$ but outside the circle $r = -\sqrt{2}\sin\theta$.



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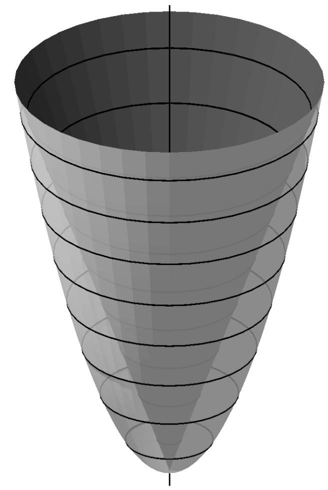
$\boxed{\text{15 pts}}$ 5. Find the volume of the solid obtained by rotating the region between the two curves

$$y = 2x \qquad \text{and} \qquad y = x^2$$



about the y-axis.

(a) Write an integral which represents the volume.

(b) Evaluate the integral in (a).

$\boxed{\text{15 pts}}$ 6. Determine the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(3x-1)^n}{\sqrt{n+5}}$. Don't forget to establish convergence or divergence at the endpoints!

7. For each of the infinite sums below, state whether it converges or diverges. Justify your answer completely.

$\boxed{\text{5 pts}}$ (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{2n} = 0 + \frac{\ln 2}{4} + \frac{\ln 3}{6} + \dots$

$\boxed{\text{5 pts}}$ (b) $\displaystyle\sum_{n=3}^{\infty} \frac{n^2+5}{(n^2-1)(n^2-4)} = \frac{7}{20} + \frac{7}{60} + \frac{1}{16} + \dots$

$\boxed{\text{5 pts}}$ (c) $\displaystyle\sum_{n=5}^{\infty} \frac{5^n-3^n}{7^{n+2}} = \frac{2882}{82543} + \frac{304}{117649} + \frac{75938}{40353607} + \dots$

$\boxed{\text{5 pts}}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{\sqrt{n^2+1}} = \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{5}} + \frac{3}{\sqrt{10}} - \dots$

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15 pts 8. Jack has a goose that lays golden eggs, one each day. Unfortunately each egg is only $7/10$ the mass of the previous one. Jack needs to obtain 120 grams of gold to ransom his sister from the evil monkey-king. If the first egg weighed 30 grams, does Jack ever get enough gold? If so, how long must he wait? Justify your answer.

15 pts 9. Recall that Hooke's law says that the amount of force required to stretch a spring x units beyond its natural length is kx, where k is a constant depending on the spring.

A giant spring designed to hold the gates of Mordor closed has a natural length of 10 meters. If it takes 1800 Joules[1] to stretch the spring from 10 meters to a length of 13 meters, how much work will it take to stretch the spring from 13 meters to a length of 16 meters?

[1] A Joule is the amount of work needed to apply a force of 1 Newton over the distance of 1 meter.