MAT126.R01: QUIZ 9

SOLUTIONS

Find the volume of the solid obtained by rotating the region bounded by the curves

\[ y = x^3, \quad x = 0, \quad y = 1 \]

about \( y = 2 \).

(Sketch the region first.)

Intersection points: \( x = 0 \) and \( y = x^3 \) intersect at the origin.
\( y = 1 \) and \( y = x^3 \) intersect when \( x^3 = 1 \), i.e. at \( x = 1 \).

Since the region is “below” the axis of revolution, the inner shell is formed by \( y = 1 \) and the outer shell by \( y = x^3 \).

The volume is

\[
\pi \int_0^1 \pi \left( (x^3 - 2)^2 - (1 - 2)^2 \right) \, dx = \pi \int_0^1 x^6 - 2x^3 + 4 - 1 \, dx = \pi \left( \frac{x^7}{7} - 2 \frac{x^4}{4} + 3x \right)_0^1 = \pi \left( \frac{1}{7} - 2 \frac{1}{4} + 3 - 0 \right) = \frac{37\pi}{17}
\]