Notations. $N(\mu, \sigma)$ is the normal distribution with the mean $\mu$ and standard deviation $\sigma$.

The percentage of data lying between values $a$ and $b$ is denoted $P(a < x < b)$ (“$P$” stands for percentage or proportion).

The percentage of data lying above $a$ is denoted $P(x > a)$; below $b$, $P(x < b)$.

$z$-score. Under the normal $N(\mu, \sigma)$, the $z$-score of the value $x$ is $z = \frac{x - \mu}{\sigma}$. The process of computing the $z$-score is called standardization.

Example. Consider the distribution $N(100, 10)$. The $z$-score of 100 is $\frac{100 - 100}{10} = 0$. The $z$-score of 105.3 is $\frac{105.3 - 100}{10} = 0.53$.

Finding the percentage. In order to compute percentages under a normal distribution, you need to standardize every given value. For example, to find $P(x < b)$ under the normal distribution $N(\mu, \sigma)$, you first standardize $b$ to $\frac{b - \mu}{\sigma}$. Then you need to find $P(z < \frac{b - \mu}{\sigma})$. Look up the value of $\frac{b - \mu}{\sigma}$ in table A (“Standard normal probabilities”). The corresponding number in the table is the required proportion. To convert to percentages, multiply by 100%.

Example, continued. Consider the normal distribution $N(100, 10)$. To find the percentage of data below 105.3, that is $P(x < 105.3)$, standartize first:

$$P(x < 105.3) = P \left( z < \frac{105.3 - 100}{10} \right) = P(z < 0.53).$$

Then find the proportion corresponding to 0.53 in Table A: look for the intersection of the row labeled 0.5 and the column labeled .03. The number is .7019. Thus $P(x < 105.3) = 0.7019$ or 70.19%.

Table A gives only proportions of the kind $P(z < b)$. To find other proportions, we use geometric facts that $P(a < z < b) = P(z < b) - P(z < a)$ (see the picture) and $P(z > a) = 1 - P(z < a)$.

Example, continued. Consider the normal distribution $N(100, 10)$. To find $P(97.1 < x < 105.3)$, standartize first:

$$P(97.1 < x < 105.3) = P \left( \frac{97.1 - 100}{10} < z < \frac{105.3 - 100}{10} \right) = P(-0.29 < z < 0.53).$$
Then
\[ P(-0.29 < z < 0.53) = P(z < 0.53) - P(z < -0.29). \]
The last two proportions can be found in Table A: \( P(z < 0.53) = 0.7019 \) and \( P(z < -0.29) = 0.3859 \) (row \(-0.2\), column 0.09). Thus
\[ P(97.1 < x < 105.3) = 0.7019 - 0.3859 = 0.3160 \text{ or } 31.6\% . \]

**From percentages to values.** There is another kind of problems: given a percentage, find the corresponding boundary value. For example, given the percentage \( P(x < b) = P \), what is \( b \)? Here to find \( b \), we look up \( P \) or the value closest to \( P \) in the table and find the corresponding \( z \)-score. Then, we need to solve \( z = \frac{b-\mu}{\sigma} \) for \( b \). Algebra shows that \( b = z\sigma + \mu \).

**Example, continued.** Consider the normal distribution \( N(100, 10) \). What values lie in the lower 80% of the data?

We need to find \( b \) such \( P(x < b) = 80\% \). First we find the \( z \)-score \( Z \) such that \( P(z < Z) = 80\% \). The table does not contain 0.8; the closest number is 0.7995. It lies in the row 0.8 and column 0.04. Thus the \( z \)-score of \( b \) is approximately 0.84:
\[ 0.84 = \frac{b - 100}{10} \]
Hence \( b - 100 = 0.84 \times 10 = 8.4 \) and \( b = 100 + 8.4 = 108.4 \). We conclude that the lower 80% of this distribution is formed by values below 108.4