Theorem. There are are infinitely primes of the form $4n + 3$.

Proof. We try to adapt the proof of the fact that there's infinitely many prime numbers. Suppose there's only finitely primes of the form $4n + 3$, and let $p_1, p_2, \dots p_k$ be all such primes. Consider the number

$$m = 4p_1p_2 \dots p_k - 1.$$

Note that m has the form $4s + 3$, and that none of the primes $p_1, p_2, \dots p_k$ divide m. (Why?) If the number m is prime itself, then we get a contradiction by finding a prime number different from $p_1, p_2, \dots p_k$. If m is not prime, then it must have some prime factors. In general, 2 is the only even prime, so all prime numbers have the form $4n + 1$ or $4n + 3$. By construction, m can have no prime factors of the form $4n + 3$, because we are assuming that all of those are given by $p_1, p_2, \dots, p_k$. Since m is odd, all its prime factors must then have the form $4n + 1$. But any product of factors of this form must itself have the form $4n + 1$ (why?), which is contradiction because m doesn't have this form.