Stony Brook University
Mathematics Department
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Advanced Topology, Geometry I
MAT 540
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Homework 1

1. Prove that continuous maps $f, g : \mathbb{R}^n \to X$ are homotopic iff $f(0)$ and $g(0)$ belong to the same path-connected component of X.

2. Let $f, g : X \to S^n$ be continuous maps. Prove that if $f(x) \neq g(x)$ for each $x \in X$, then g is homotopic to the map $X \to S^2 : x \mapsto -f(x)$.

3. Prove that the set of connected components of an open set on the plane R^2 is countable.

4. Prove that the graph of any continuous function $[0, 1] \to \mathbb{R}$ is closed, connected, path-connected, Hausdorff and compact.

5. Let X and Y be topological spaces, and Y be compact and Hausdorff. Prove that $f : X \to Y$ is continuous if and only if its graph $\Gamma_f = \{(x, y) \in X \times Y \mid y = f(x)\}$ is closed.