

① For each of the following integers  $n$ , find all the primitive roots mod  $n$

a)  $n = 11$

b)  $n = 20$

c)  $n = 8$ .

② Find the number of primitive roots of 47.

③ Show that 20 has no primitive roots

④ Prove let  $\pi$  be a primitive root mod  $n$ , and let  $s$  be a positive integer

④ Then  $\pi^s$  is a primitive root mod  $n$  iff  $(s, \phi(n)) = 1$

⑤ Decide whether the following statement is true:

If  $d \mid \phi(n)$  then there exists  $a$  with order  $d$  mod  $n$ .

⑥ Prove that  $\sigma_{F_n}(2) \leq 2^{h_n}$  where  $F_n = 2^{2^n} + 1$  is the  $n$ th Fermat number.

⑦ Find the continued fraction expansion of  $22/7$  and all the convergents. Plot the convergents in the real line

⑧ Find the continued fraction expansion of  $f_{k+1}/f_k$ , where  $f_k$  is the  $k$ -th Fibonacci number.

⑨ Show that if  $a_0 > 0$  then

$$\frac{P_k}{P_{k-1}} = \langle a_k, a_{k-1}, a_{k-2}, \dots, a_1, a_0 \rangle$$

$$\frac{q_k}{q_{k-1}} = \langle a_k, a_{k-1}, \dots, a_2, a_1 \rangle$$

where  $C_i = P_i/q_i$   $i \geq 1$  are the successive convergents of  $\langle a_0, a_1, \dots, a_n \rangle$

⑩ Show that  $q_k \geq f_k$ , for  $k=1, 2, \dots$  where  $C_k = \frac{P_k}{q_k}$  is the  $k$ -th convergent of the continued fraction  $\langle a_0, a_1, \dots, a_n \rangle$  and  $f_k$  denotes the  $k$ -th Fibonacci number.