

Problem set 9

1. Solve the Diophantine equations

(a) $-718x + 123y = 1$.

(b) $417x - 172y = 1$.

(c) $1163x - 815y = 1$.

2. Show that for any real number $x > 0$, $\langle a_0, a_1, \dots, a_{k-1}, x \rangle = \frac{xp_{k-1} + p_{k-2}}{xq_{k-1} + q_{k-2}}$

3. Let C_k be the k -th convergent of $\frac{a}{b} = \langle a_0, a_1, \dots, a_n \rangle$. Prove that each convergent is nearer to $\frac{a}{b}$ than the preceding one.

4. Let $\langle a_0, a_1, \dots, a_n \rangle$ be the continued fraction expansion of a rational number r . Find the continued fraction expansion of $1/r$ in terms of a_0, a_1, \dots, a_n .