Problem set 7

1. Determine the number of elements of order 4 mod 2^s. What is the smallest positive k such that $n^k \equiv 1 \pmod{32}$, for all a odd. Hint: Start with $s = 5$ and then study the general case.)

2. Determine for which positive integers n, $2^n + 3^n$ is divisible by 17.

3. Show that if m has a primitive root and $m > 2$ then $x^2 \equiv 1 \pmod{m}$ has exactly two solutions.

4. Suppose that p is prime and that the order of a mod p is 3. Prove that:

   (a) 3 divides $p - 1$.

   (b) $a^2 + a + 1 \equiv 0 \pmod{p}$.

   (c) the order of $a + 1$ mod p is 6.