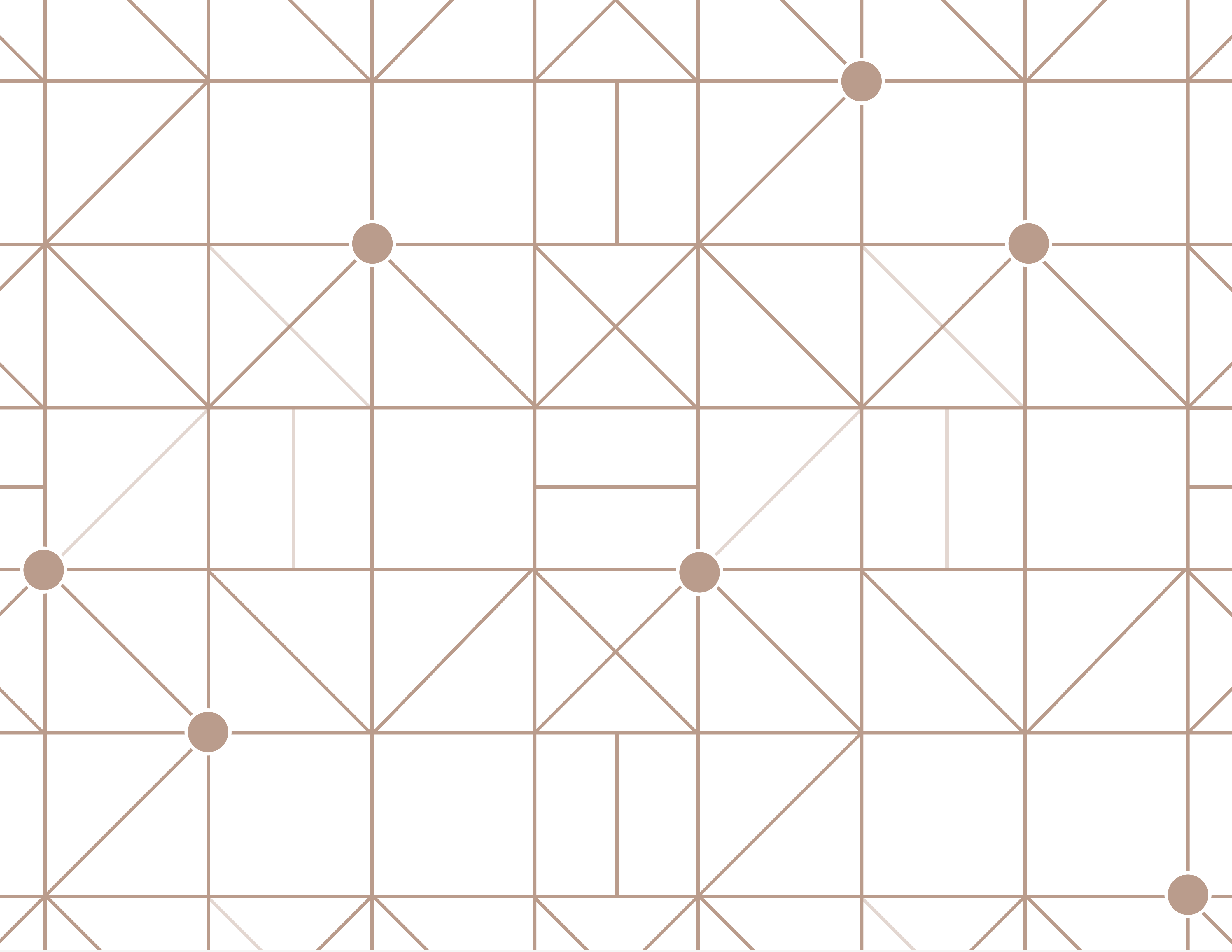


During T'ang dynasty (618-906), a body of mathematical books was assembled for official use in the imperial examinations.

If 1115, a printed edition of the Nine Chapters of the Mathematical Art appeared.

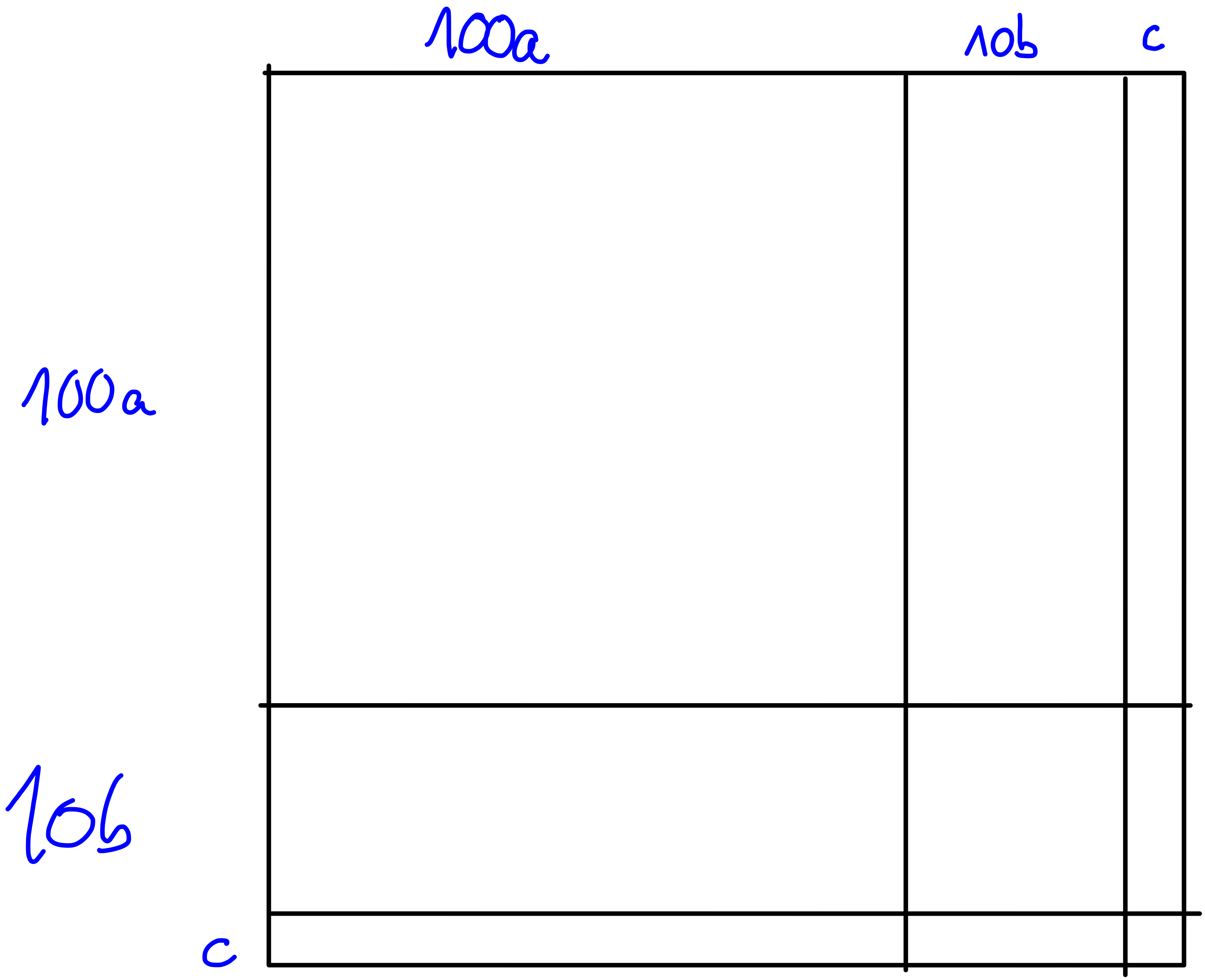
The Song (Sung) dynasty (960-1279) and the early years of the Mongol dynasty of the Yuan were a period of greatest flowering of ancient Chinese mathematics.

- Ch'in Chiu-Shao 1247 book of indeterminate equations
  - Chinese reminder theorem
  - solution of higher degree equations.
  - extraction of square and cubic roots.

we will study this work

- Yan Hui - 1261 "A detailed Analysis of the Mathematical Methods in the Nine Chapters"
  - 1274-5 "The Method of Computation of Yang Hui"
  - worked with decimal fractions, wrote them in a way reminescent of our present method.
  - earliest representation of the Pascal triangle.

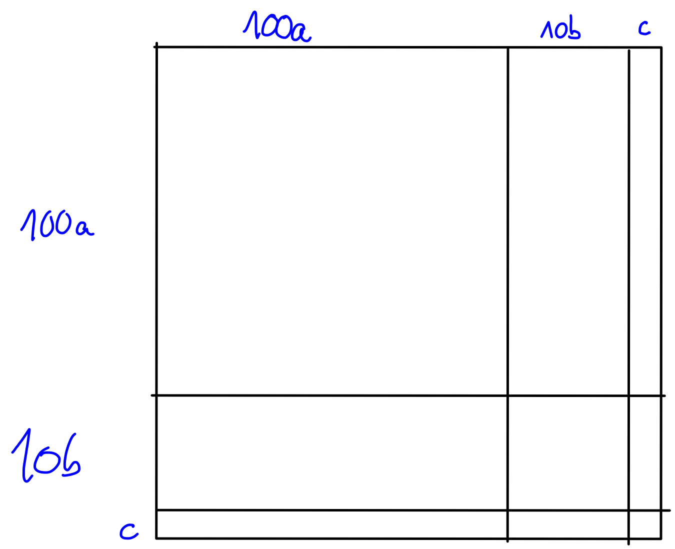
- Zhu Shijie (Chu Shihchieh)
  - chinese arithmetic algebraic-computational method.
  - "matrix" sol'n of systems of linear equation extended to equations of higher degree.


100a
10b
c
100a
10b
c

Algorithm to compute the square root of 55225

· Since $100^2 < 55225 < (1000)^2$, we know that 3 fig

· $x \,/\, x^2 = 55225$



$$x = 100a + 10b + c$$

$$x^2 = \underbrace{100^2 a^2} + 10^2 b^2 + c^2 +$$

1) Find largest a / $100^2 a^2 < 55225$ $\boxed{a=2}$

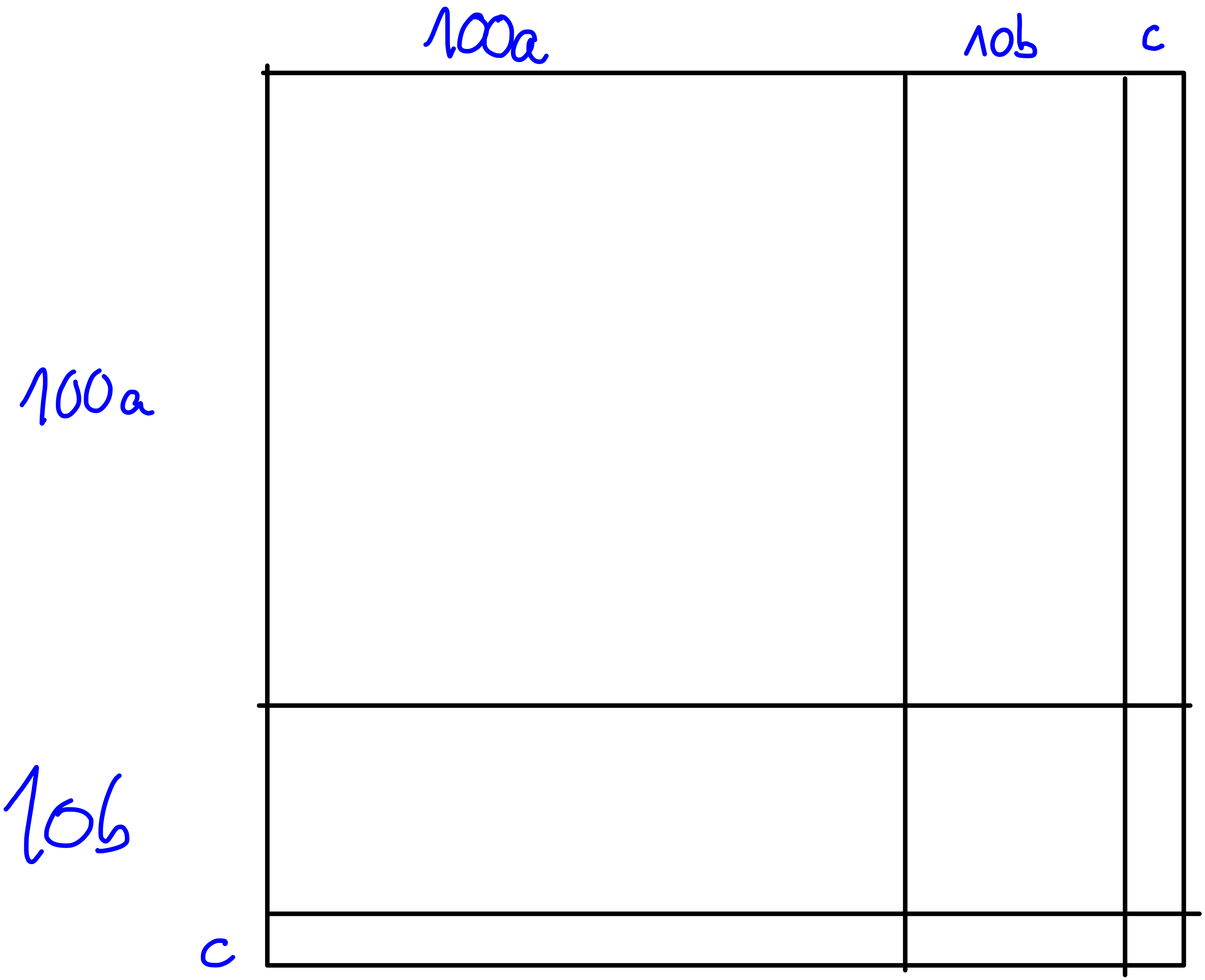
$$55225 - (2 \cdot 100)^2 = 10^2 b^2 + 100 \cdot 10 \cdot 2 \cdot 2 \cdot b + c^2 + 2 \cdot 10bc + 2 \cdot 100 \cdot 2c$$

$$15225 = 12000 + 900 + 2 \cdot 10 \cdot 3 \cdot c + c^2 + 100 \cdot 4c + 400c$$

$b=3$

$$2325 = 60c + c^2 + 400c = 460c + c^2$$

$\boxed{c=5}$



$0 \leq a, b, c \leq 9$

Find a, b and c if the area of the square is 55225 step 1 置積[1]爲實[2].

$x^2 = 55225$. Find x

Put the (known) square (of a certain unknown number) (in the second row from the top of the counting-board) to be the *Shih* 實, dividend.

55225	dividend

step 2 借[3]一算

Make use of one counting-rod (and put it in the bottom row of the counting-board in the furthest right-hand digit column) [This one counting-rod is to be called the preliminary *Chieh-suan* 借算].

55225	dividend
1	preliminary chieh-suan

step 3 步[1] 之，超一等.

This one counting-rod is moved forward (from right to left) by steps of two places each (as far as it can go without transgressing the furthest left digit of the dividend) [This one counting-rod, with its new place-value, is to be called the *Chieh-suan* 借算].

55225	dividend
10000	chieh-suan

1 → 10000

Move this counting-rod to the left by 2 steps of 2 places step 4 議[2] 所得.

(The first figure of the root is selected through trial, taking 1, 2, 3, one after another). Discuss the *So-tê*, 所得[3]). (The *So-tê* is the product of the first root figure under trial multiplied by the *Chieh-suan*). (What is meant by 'discussion' is that when the selected number has multiplied the *So-tê* once, the product must not be greater than the dividend; and at the same time the largest possible root figure must be selected)[4]).

[The selected figure is placed in the top row of the countingboard. This is the *Fang* 方 row which will ultimately contain the answer.]

Too small

1	
55225 1x1x10000	dividend
1x10000	
10000	chieh-suan

OK

2	
55225 2x2x10000	dividend
2x10.000	
10000	chieh-suan

Too big

3	
55225 3x3x10000	dividend
3x10000	
10000	chieh-suan

step 5 以一[2] 乘所借一算爲法[1].

The *Chieh-suan* is multiplied by the (selected) first figure of the root [2]. The product is the divisor, *Fa* 法 (which is put in the third row from the top). [It should be noted that in this square root series, but not in the cube root series, the values of *So-tê* and *Fa* are identical.]

2	
	dividend
20000	divisor
10000	chieh-suan

step 6 而以除.

This divisor, *Fa*, is used to divide the dividend (and the remainder is put in the second row from the top of the counting-board). [This is to be called the first remainder].

$$55225 = 2 \times 20000 + 15225$$

2	
15225	first remainder
20000	divisor
10000	chieh-suan

step 7 除已，倍法爲定法，其復除，折法. NOTE!

a) After the division has been made, the divisor, *Fa*, is doubled to form the *Ting-fa*,
b) The *Ting-fa*[1]) is cut short (i.e. moved back by one digit) [and this is the (first) fixed divisor, *Ting-fa*$_1$] in preparation for the next division operation.

2	
15225	first remainder
2x20 000	
10000	chieh-sua

2	
15225	first remainder
4000	divisor
10000	chieh-sua

step 8 而下復置借算步之如初.

Again the counting-rod (which took up its position in step 3) in the bottom row is moved (backward from left to right by one step of two places) as before [1]). [This counting-rod, with its new place-value, is to be called the *Chieh-suan*$_1$.]

2	
15225	first remainder
4000	divisor
10000	chieh-sua

2	
15225	first remainder
4000	divisor
100	chieh-suan$_1$

Second Phase:

step 9 [2])

(Again, the second figure of the root is selected through trial and discussion. The discussion aims to find the *Ting-fa*$_2$ by the process given in step 10. The product of the *Ting-fa*$_2$ multiplied by the second figure of the root under trial must not be greater than the first remainder. The largest figure which does not violate this condition is selected).

21	
15225 1× 4100	first remainder
4000 + 1×100	divisor
100.	chieh-sua

22	
15225 2 × 4200	first remainder
4000 + 2×100	divisor
100	chieh-sua

OK

23	
15225 3 × 4300	first remainder
4000 + 3×100	divisor
100	chieh-sua

24	
15225 4 × 4400	first remainder
4000 + 4×100	divisor
100	chieh-sua

step 10 以復議一乘之，所得副以加定法.

The *Chieh-suan*$_1$ is multiplied by the second figure of the root [1]. (The product is the *So-tê*$_2$). The *so-tê*$_2$ is added to the *Ting-fa*$_1$. (The result is called *Ting-fa*$_2$, which is put in the third row from the top.)

23	
15225 3 x 4300	first remainder
4000+3x100	divisor
100	chieh-sua

23	
15225	second remainder
4300	divisor
100	chieh-sua

Step 11. 以除.

(The first remainder) is divided by (*Ting-fa*$_2$) (and the remainder is put in the second row from the top). [This remainder is the second remainder].

$$15225 = 3 \times 4300 + 2325$$

23	
2325	second remainder
4300	divisor
100	chieh-sua

Step 12 以所得副從定法, 復除, 折.

So-tê is added [2]) to *Ting-fa*$_2$, and the sum [to be called *Ts'ung Ting Fa*] is cut short (i.e. moved back by one place), [and this is the *Ts'ung-ting-fa*$_1$';] in preparation for the next division operation.

23	
2325	second remainder
4300	divisor
100	chieh-sua

→

23	
2325	second remainder
$\frac{4300+3\times100}{10}$	divisor
100	chieh-sua

Step 13 下如前.

Proceed similarly to the previous operations (step 8). [The *Chieh-suan*$_1$, cut short, i.e. moved back, by two places, becomes the *Chieh-suan*$_1'$].

23	
2325	second remainder
460	divisor
100	chieh-sua

23	
2325	second remainder
460	divisor
1	chieh-sua

Third Phase:

Steps 14, 15, and 16.

(will be necessary only if the root comes to three figures; in which case they will follow steps 9, 10, and 11 precisely).

231	
2325 461 × 1	second remainder
460+1	divisor
1	chieh-sua

232	
2325 462 × 2	second remainder
460+2	divisor
1	chieh-sua

233	
2325 463 × 3	second remainder
460+3	divisor
1	chieh-sua

234	
2325 464 × 4	second remainder
460+4	divisor
1	chieh-sua

235	
2325 465 × 5	second remainder
460+5	divisor
1	chieh-sua

236	
2325 466 × 6	second remainder
460+6	divisor
1	chieh-su

Step 17 開之不盡者爲不可開，以面命之.

If the (last remainder) is not equal to zero (when the *Chieh-suan*$_1^{n'}$ has been moved back to the unit digit position) this means that the operation cannot be completed (within the bounds of an integral root) [1]), but the operation is continued as before [2]).