

HOMWORK 4

Due: Thursday September 27th at 10:00am in Physics P-124

Please write your solutions legibly; the TA may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first) and HW number in the upper-right corner of the first page.

Problem 1: Let \mathcal{F} be a σ -field on a set Ω and let $f : \Omega' \rightarrow \Omega$ be any function. Show that

$$f^{-1}(\mathcal{F}) := \{f^{-1}(E) : E \in \mathcal{F}\}$$

is a σ -field.

Problem 2: Let $(\Omega, \mathcal{F}, \mu)$ be a measure space. For measurable functions $f, g : \Omega \rightarrow \mathbb{R}$ on a measure space $(\Omega, \mathcal{F}, \mu)$ show that

- (1) $\text{ess sup}(f + g) \leq \text{ess sup}(f) + \text{ess sup}(g)$ and
- (2) $\text{ess inf}(f + g) \geq \text{ess inf}(f) + \text{ess inf}(g)$.

Problem 3: Let (Ω, \mathcal{F}, P) be a probability space and let X be a random variable. Define the *cumulative distribution function*

$$F_X : \mathbb{R} \rightarrow \mathbb{R}, \quad F_X(x) := P(X^{-1}((-\infty, x])).$$

- (1) Show that there is a countable set $Q \subset \mathbb{R}$ so that $F_X|_{\mathbb{R}-Q}$ is continuous.
- (2) Give an example of a random variable whose cumulative distribution function has infinitely many points where it is discontinuous.