ERRATUM: SYMPLECTIC INVOLUTIONS OF $K3^{[n]}$ TYPE AND KUMMER N TYPE MANIFOLDS

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Abstract. In this note we present a corrected formula for the enumeration of connected components of the locus fixed by a symplectic involution inside hyperkähler manifolds of types $K3^{[n]}$ and generalized Kummer. We also provide further precisions concerning the involutions considered in the Kummer case.

In the [3] the enumerative part of Theorems 1.2 and 1.3 is incorrect. The source of error is an erroneous interpretation of the results of [1] in the proof of Lemma 3.13. The error does not affect the enumeration of the top dimensional components as in Theorem 3.7 and 3.9. Moreover, the result of Theorem 1.3 does not apply to all involutions with a non trivial action on H^3, and we detail here the correct type of involutions considered.

The lower-dimensional component counts are special cases of Theorems 3.0.3 and 3.0.7 from [4]. To state the results let introduce the standard theta function:

$$\vartheta(\eta;q)=\sum_{n\in\mathbb{Z}} q^{n^2}\eta^n.$$

Theorem 1. *Let S be a K3 surface and let $\imath$ be a symplectic involution of S. Let $S^{[n]}$ be the Hilbert scheme of n points on S and let us consider the induced action of $\imath$ on $S^{[n]}$. Then all the irreducible components of the locus stabilized by $\imath$ are deformation equivalent to Hilbert schemes of points on a K3 surface or isolated points, and their number for each dimension $2k$ is*

$$N_k=\Theta(n-2k),$$

where $\Theta(j)$ is the j-th coefficient of the Theta series

$$\Theta(q)=\sum_{i\geqslant 0}\Theta(i)q^i=\vartheta(q;q^2)^8.$$

Remark 1. All involutions on generalized Kummer manifolds act trivially on the second cohomology, and they can be subdivided in three different geometrical categories:

- Involutions obtained by a translation of a two torsion point.
- Involutions obtained by a sign change composed with a translation by a two torsion point.
- Involutions obtained by a composition of a translation by a point of order at least three with a sign change.

The first kind of involutions only appears when n is odd.

In this paper we deal with the second kind of involutions, and we call them *regular* involutions. The fixed locus of the first kind of involutions was analyzed in [5, Lemma 3.5], while the third kind was analyzed only in the case of Kummer sixfolds in [2, Lemma 2.4]. It would be interesting to compute the fixed locus in general for an involution of the third kind.

Theorem 2. *Let $X = K_n(A)$ be a n-Kummer hyperkähler manifold and let $\imath \in Aut(A)$ be a regular symplectic involution of the abelian surface A. Then all irreducible components of $X^\imath$ are of $K3^{[k]}$ type, and their number is*

$$N_k = \Theta(n-2k;1),$$

where $\Theta(j;1)$ is the j-th coefficient of the Theta series

$$\Theta(q) = \sum_{i\geqslant 0,\gamma\in A,\gamma^2=1} \Theta(i;\gamma)q^i\gamma = \prod_{\gamma\in A,\gamma^2=1} \vartheta(\gamma\cdot q;q^2).$$

Remark 2. All three kinds of involutions of Remark 1 can be recognized by their action on higher cohomology: involutions obtained by pure translations have a trivial action on H^3, and involutions of the third kind, those obtained by composing a translation of order at least three with -1, are recognizable by their action on higher cohomology. As an example, if $n \equiv 3$ modulo 4, in H^6 there are 256 distinguished classes of codimension three subvarieties of a generalized Kummer. These subvarieties are in correspondance with points in $A[4]$, each of them is the locus of subschemes having support of multiplicity at least four on a given four torsion point of A. Regular involutions fix 16 of these subvarieties and permute the rest, while involutions of the third kind (obtained with an order four translation) freely permute these subvarieties.

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