Homework 6 (due 3/14)

MAT 342: Applied Complex Analysis

Read Sections 41–47 from Chapter 4.

Problem 1. Evaluate the following integrals:

(i) $\int_0^1 (2-3it)^2 dt$ (ii) $\int_0^{\sqrt{\pi/3}} te^{it^2} dt$

Show all steps in your calculations.

Problem 2. For each $k \in \mathbb{Z}$ consider the function $f_k(z) = z^k$. Let C be the unit circle, parametrized by $z(t) = e^{it}$, $0 \leq t \leq 2\pi$. Evaluate

$$\int_C f_k(z) dz.$$

Note that for different values of $k \in \mathbb{Z}$ the answers will differ.

Problem 3. Evaluate each of the following expressions (that is, write them in (x, y)-coordinates) by selecting an appropriate branch of the logarithm each time and using that branch for your evaluation:

(i) $(-5)^{1-i}$ (ii) i^{e^i}

Explain carefully which branch of the logarithm you are using each time, by writing the domain of definition and the formula of the logarithm in polar coordinates.

For the functions f and the contours C given in Problems 4–6 draw the contour C and evaluate

$$\int_C f(z) dz.$$

Problem 4. $f(z) = z^2+1$ and C consists of the segment $z_1(t) = t$, $0 \leq t \leq 1$, and the semicircle $z_2(t) = \frac{1}{2} + \frac{1}{2}e^{it}$, $0 \leq t \leq \pi$.

Problem 5. $f(z) = 2e^{i\bar{z}}$ and C is the boundary of the rectangle with vertices at the points -1, 1, $1+2i$, $-1+2i$, oriented in the counterclockwise direction.

Problem 6. $f(z) = z^{2i}$ (defined using the principal branch of the logarithm) and C is the arc $z(t) = e^{it}$, $0 \leq t \leq \pi/2$.

Problem 7. Let C be the arc $z(t) = 4e^{it}$, $0 \leq t \leq \pi/2$. Without evaluating the integral show that

$$\left| \int_C \frac{iz - 10}{2z^2 - 1 - i} dz \right| \leq \frac{28\pi}{32 - \sqrt{2}}.$$