Homework 12 (due 5/9)

MAT 342: Applied Complex Analysis

Read Sections 82–84 from Chapter 6 and 85–94 from Chapter 7.

Problem 1. Find the isolated singular points of the function $f(z) = \frac{1}{\sin z}$. What is the type of singularities and what is the residue at each singular point?

Problem 2. Compute the integral

$$\int_C \frac{dz}{(z^2-1)^2+3},$$

where C is the positively oriented boundary of the rectangle whose sides lie along the lines $x = \pm 2$, $y = 0$, and $y = 1$. *(Hint: Find the singular points of the function $f(z) = \frac{1}{(z^2-1)^2+3}$ that are enclosed by C. Then compute the corresponding residues and use the Residue Theorem.)*

Problem 3. Compute the integral

$$\int_C \tan z dz,$$

where C is the positively oriented circle $|z| = 2$.

Problem 4. Use residues to show that

$$\int_0^\infty \frac{1}{(x^2+1)^2} dx = \frac{\pi}{4}.$$

Problem 5. Use residues to show that

$$\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx = \frac{\pi}{6}.$$

Problem 6. Use residues to show that

$$\int_0^\infty \frac{1}{x^3+1}\,dx = \frac{2\pi}{3\sqrt{3}}.$$

Do this by integrating over the positively oriented closed contour C consisting of the line segment from 0 to $R > 1$, the circular arc $Re^{i\theta}$, $0 \le \theta \le 2\pi/3$, and the line segment from $Re^{i2\pi/3}$ to 0 (draw the contour).

Problem 7. Use residues to show that

$$\int_0^\infty \frac{\cos 3x}{x^2+1}\,dx = \frac{\pi}{2}e^{-3}.$$

Problem 8. Use residues to evaluate

$$\int_0^\infty \frac{x \sin x}{(x^2+1)(x^2+4)}\,dx.$$