## Nomination for David A. Vogan Jr.

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In recent years the tradition has been that the AMS president is someone of high stature in mathematics research whose presidential duties include representing American mathematics to the nonmathematical world and guiding the AMS committee system in its formulation and carrying out of policies. For this position, as one says in the sports world, David Vogan is the

complete package. He has done stunning research over a long period of time, he is the Head of one of the very best mathematics departments in the world, he has been a quiet but forceful advocate for women in the profession, he is known for his extraordinary mentoring of graduate students, he is the author of four research-level books, and he has served the AMS long and well in several capacities.

David's field is the representation theory of Lie groups. A group representation is a group action by linear transformations, typically on a complex Hilbert space. Behind a representation is often an action of the group on a manifold, transitive or not, and one studies the manifold in part by studying the complex-valued functions on it. Group representations of nonabelian infinite groups were studied first by I. Schur and H. Weyl in the 1920s, and it was not long before this approach made the subject blend with quantum mechanics, as one examined the effect of symmetry and the breaking of symmetry on systems of differential equations. The decomposition of representations into sums or integrals of other representations and the identification of the ultimate irreducible pieces have remained as fundamental problems in the theory since the 1920s.

David has concentrated his research on reductive Lie groups, which one may view as closed subgroups of real or complex matrices that are stable under conjugate transpose. These are the groups whose normal subgroups offer few clues to their structure. Some early names associated with the representation theory of these groups are Bargmann, I. M. Gelfand, Naimark, Godement, and Mackey. But from the early 1950s until 1976, the year of David's thesis, the direction of the field was set by Harish-Chandra and R. P. Langlands.

Harish-Chandra's approach for such a *G* was ultimately analytic, using differential equations and asymptotic properties of the functions  $g \mapsto (R(g)u, v)$  associated to a representation *R* to get a handle on *R*. The fundamental irreducible representations for Harish-Chandra were those in the "discrete series"—the ones that occur as subrepresentations of  $L^2(G)$ . Other representations of interest could be constructed by "parabolic induction" from the discrete series. Harish-Chandra classified the discrete series and then, in part using ideas that Langlands had developed for studying  $L^2(G/\Gamma)$  for arithmetic subgroups  $\Gamma$ , completed the analysis of  $L^2(G)$ . Langlands, for his own part, went on to use asymptotic expansions to classify the irreducible representations. He used his classification as substantive evidence for a body of conjectures and questions that have come to be known as the Langlands program; these relate the solutions of Diophantine problems to infinite-dimensional representation theory, and later progress by Langlands on these conjectures was indispensable to the proof of Fermat's Last Theorem.

That much history brings us to David's thesis in 1976, which was written under the direction of B. Kostant and revolutionized the field. David introduced a completely algebraic theory for studying irreducible representations of reductive groups. The fundamental representations were not discrete series but representations behaving quite differently, and the tools were not differential equations and asymptotic expansions but cohomology theories. The final theorem of the thesis was a classification completely different from the one by Langlands. Building on ideas that G. J. Zuckerman introduced in 1978, David developed a construction now called "cohomological induction" that made his classification easier to formulate and to work with. His completed classification was published in 1981 in the first of his four research books. The Vogan-Zuckerman classification, as it is called, does not replace the Langlands classification; it gives a completely new way of looking at the field, and the passage back and forth between the two approaches is a powerful tool.

Left unaddressed by all this work was the question of which irreducible representations are unitary. Parabolic induction carries unitary representations to unitary representations, but cohomological induction does not necessarily. In a 1984 paper David proved, by a remarkably intricate algebraic construction, that cohomological induction does preserve unitarity when a certain positivity condition holds for the parameters. With this theorem he was able to classify the irreducible unitary representations for the general linear groups over the reals, the complexes, and the quaternions.

David's Hermann Weyl Lectures at the Institute for Advanced Study in 1986, published as an Annals of Mathematics Study in 1987, showed David's thinking about the classification of irreducible unitary representations for general *G*. The book gives great insight into the mind of a first-rate mathematician at work.

This classification problem for irreducible unitary representations remains unsolved in general, but it is now known that cohomological induction is an indispensable tool for the problem. A 1998 *Annals of Mathematics* paper by David with S. Salamanca-Riba reports on some recent progress.

In the 1980s J. Arthur made some conjectures related to the Langlands program. Like the program in general, Arthur's conjectures are first of all about automorphic forms, but they have consequences and analogs in

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representation theory for real and p-adic reductive groups. A 1992 book by David with two coauthors proves most of Arthur's conjectures for real groups. The results in the book provide evidence for the full Arthur conjectures about automorphic forms, as well as tools to approach those conjectures.

David received his Ph.D. from the Massachusetts Institute of Technology at age twenty-one, spent another year as an instructor at MIT, visited the Institute for Advanced Study for two years, and then returned to the MIT faculty. He rose through the ranks and is now Professor and Head of Mathematics. Over the years he served multiple terms as undergraduate director and graduate director.

He became Head of Mathematics in 1999. The MIT mathematics department has been especially successful at having pure and applied mathematics thrive together in a single department. The department has two subdepartments, one for pure mathematics and one for applied mathematics, and each has a select committee to deal with hiring and some other matters. Before becoming Head, David served on the select committee in pure mathematics. Now, as Head, he is responsible for representing the combined views of the two subdepartments to the dean and others. His appointment as Head indicates a level of trust in his ability to carry out this responsibility.

David takes seriously the status of women in the profession. He is a member of the AWM. As department Head, he has extended to mathematics instructors a good MIT faculty-leave policy for those who assume responsibility to care for a newborn child or a child newly placed for adoption or foster care. This extension of the policy is a serious step, as instructors are more likely to benefit from such a policy than are senior faculty.

David has supervised twenty-one Ph.D. theses. In addition, he has organized a weekly Lie Groups Seminar for twenty years whose speakers have kept the greater Boston mathematical research community abreast of current developments in many areas related to Lie groups.

David is admired as a teacher. At the time of his appointment as Head of Mathematics, the MIT News Office said, "Among these students, he is known for his loyalty and generosity with his time and his ideas."

David is married to his childhood sweetheart, and they have two children. He and his wife are pillars of one of the downtown Boston churches. Also, David is a director of The Giving Back Fund, a public charity that provides expertise to athletes, entertainers, and others to help them get the greatest possible impact from their philanthropy.

For the AMS David has been a member of the Council, has served on the Science Policy Committee, has coorganized three special sections at meetings, has been a member of the editorial staff of the *Bulletin* since 1987, and has served as founding editor of the electronic journal *Representation Theory*.

He has jointly organized three non-AMS conferences: a one-week conference at Oberwolfach, a special year at MSRI in representation theory, and the graduate component of one summer's Park City Mathematics Institute.