Corrections since 16 Feb 2015 to
***Lie Groups Beyond an Introduction*, *Second Edition*:**
Short Corrections and Remarks

Short Corrections

Page 150, table (2.43). With A_n, change the condition "$\sum a_i e_i = 0$" to "$\sum a_i = 0$".

Page 463, line 8 of "Proof of Existence in Theorem 7.40." Change " $\mathfrak{a}_0 \oplus \mathfrak{m}_0$ " to " $\mathfrak{a}_0 \oplus \mathfrak{n}_0$ ".

Page 573, equation (9.21). Change " $\displaystyle\sum_{\beta\in\Sigma}$ " to " $\displaystyle\prod_{\beta\in\Sigma}$ ".

Page 573, equation (9.23). Change " $\displaystyle\sum_{\beta\in\Sigma}$ " to " $\displaystyle\prod_{\beta\in\Sigma}$ ".

Remarks

Page 50, Proposition 1.43. Whether or not C is nondegenerate, it is still true that

$$\dim U + \dim U^\perp \geq \dim V.$$

In fact, going over the proof of Proposition 1.43 shows that the equality $\ker\psi = U^\perp$ is still valid. Hence

$$\begin{aligned}\dim V = \dim(\mathrm{domain}(\psi)) &= \dim(\ker(\psi)) + \dim(\mathrm{image}(\psi)) \\ &\leq \dim U^\perp + \dim U^* = \dim U^\perp + \dim U,\end{aligned}$$

and the inequality follows.

Page 50, Corollary 1.44. Whether or not C is nondegenerate, it is still true that $V = U \oplus U^\perp$ if and only if $C|_{U\times U}$ is nondegenerate. In fact, if $V = U \oplus U^\perp$, then $U \cap U^\perp = 0$ and the equality $U \cap U^\perp = \mathrm{rad}(C|_{U\times U})$ of (1.42) shows that $C|_{U\times U}$ is nondegenerate. Conversely if $C|_{U\times U}$ is nondegenerate, then $U \cap U^\perp = 0$ by (1.42). From the previous remark we see that

$$\dim(U + U^\perp) = \dim U + \dim U^\perp - \dim(U^\perp \cap U) \geq \dim V - 0 = \dim V,$$

and thus $U + U^\perp = V$. Hence $V = U \oplus U^\perp$.

Pages 108, line -2, to page 110, statement of Corollary 1.134. This material proves that the exponential map is everywhere regular when the Lie algebra is nilpotent. An alternative approach to this question is to establish the following general formula for the differential of the exponential map:

$$(d\exp)_X = d(L_{\exp X})_1 \circ \frac{1 - e^{-\mathrm{ad}\,X}}{\mathrm{ad}\,X}.$$

When the Lie algebra is nilpotent, each $\mathrm{ad}\,X$ is nilpotent. Consequently $\frac{1-e^{-\mathrm{ad}\,X}}{\mathrm{ad}\,X}$ is everywhere nonsingular, and the differential is everywhere one-one onto.

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