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Denumerable Markov Chains

John G. Kemeny *J. Laurie Snell*
Anthony W. Knapp

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John G. Kemeny
President
Dartmouth College
Hanover, New Hampshire 03755

J. Laurie Snell
Dartmouth College
Department of Mathematics
Hanover, New Hampshire 03755

Anthony W. Knapp
Cornell University
Department of Mathematics
Ithaca, New York 14850

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PREFACE TO THE SECOND EDITION

With the first edition out of print, we decided to arrange for republication of *Denumerable Markov Chains* with additional bibliographic material. The new edition contains a section Additional Notes that indicates some of the developments in Markov chain theory over the last ten years. As in the first edition and for the same reasons, we have resisted the temptation to follow the theory in directions that deal with uncountable state spaces or continuous time. A section entitled Additional References complements the Additional Notes.

J. W. Pitman pointed out an error in Theorem 9-53 of the first edition, which we have corrected. More detail about the correction appears in the Additional Notes. Aside from this change, we have left intact the text of the first eleven chapters.

The second edition contains a twelfth chapter, written by David Griffeath, on Markov random fields. We are grateful to Ted Cox for his help in preparing this material. Notes for the chapter appear in the section Additional Notes.

J.G.K., J.L.S., A.W.K.
March, 1976

PREFACE TO THE FIRST EDITION

Our purpose in writing this monograph has been to provide a systematic treatment of denumerable Markov chains, covering both the foundations of the subject and some topics in potential theory and boundary theory. Much of the material included is now available only in recent research papers. The book's theme is a discussion of relations among what might be called the descriptive quantities associated with Markov chains—probabilities of events and means of random variables that give insight into the behavior of the chains.

We make no pretense of being complete. Indeed, we have omitted many results which we feel are not directly related to the main theme, especially when they are available in easily accessible sources. Thus, for example, we have only touched on independent trials processes, sums of independent random variables, and limit theorems. On the other hand, we have made an attempt to see that the book is self-contained, in order that a mathematician can read it without continually referring to outside sources. It may therefore prove useful in graduate seminars.

Denumerable Markov chains are in a peculiar position in that the methods of functional analysis which are used in handling more general chains apply only to a relatively small class of denumerable chains. Instead, another approach has been necessary, and we have chosen to use infinite matrices. They simplify the notation, shorten statements and proofs of theorems, and often suggest new results. They also enable one to exploit the duality between measures and functions to the fullest.

The monograph divides naturally into four parts, the first three consisting of three chapters each and the fourth containing the last two chapters.

Part I provides background material for the theory of Markov chains. It is included to help make the book self-contained and should facilitate the use of the book in advanced seminars. Part II contains basic results on denumerable Markov chains, and Part III deals with discrete potential theory. Part IV treats boundary theory for both transient and recurrent chains. The analytical prerequisites for the two chapters in this last part exceed those for the earlier parts of the book and are not all included in Part I. Primarily, Part IV presumes that the reader is familiar with the topology and measure theory of compact metric spaces, in addition to the contents of Part I.

Two chapters—Chapters 1 and 7—require special comments. Chapter 1 contains prerequisites from the theory of infinite matrices and some other topics in analysis. In it Sections 1 and 5 are the most important for an understanding of the later chapters. Chapter 7, entitled "Introduction to Potential Theory," is a chapter of motivation and should be read as such. Its intent is to point out why classical potential theory and Markov chains should be at all related.

The book contains 239 problems, some at the end of each chapter except Chapters 1 and 7.

For the most part, historical references do not appear in the text but are collected in one segment at the end of the book.

Some remarks about notation may be helpful. We use sparingly the word "Theorem" to indicate the most significant results of the monograph; other results are labeled "Lemma," "Proposition," and "Corollary" in accordance with common usage. The end of each proof is indicated by a blank line. Several examples of Markov chains are worked out in detail and recur at intervals; although there is normally little interdependence between distinct examples, different instances of the same example may be expected to build on one another.

A complete list of symbols used in the book appears in a list separate from the index.

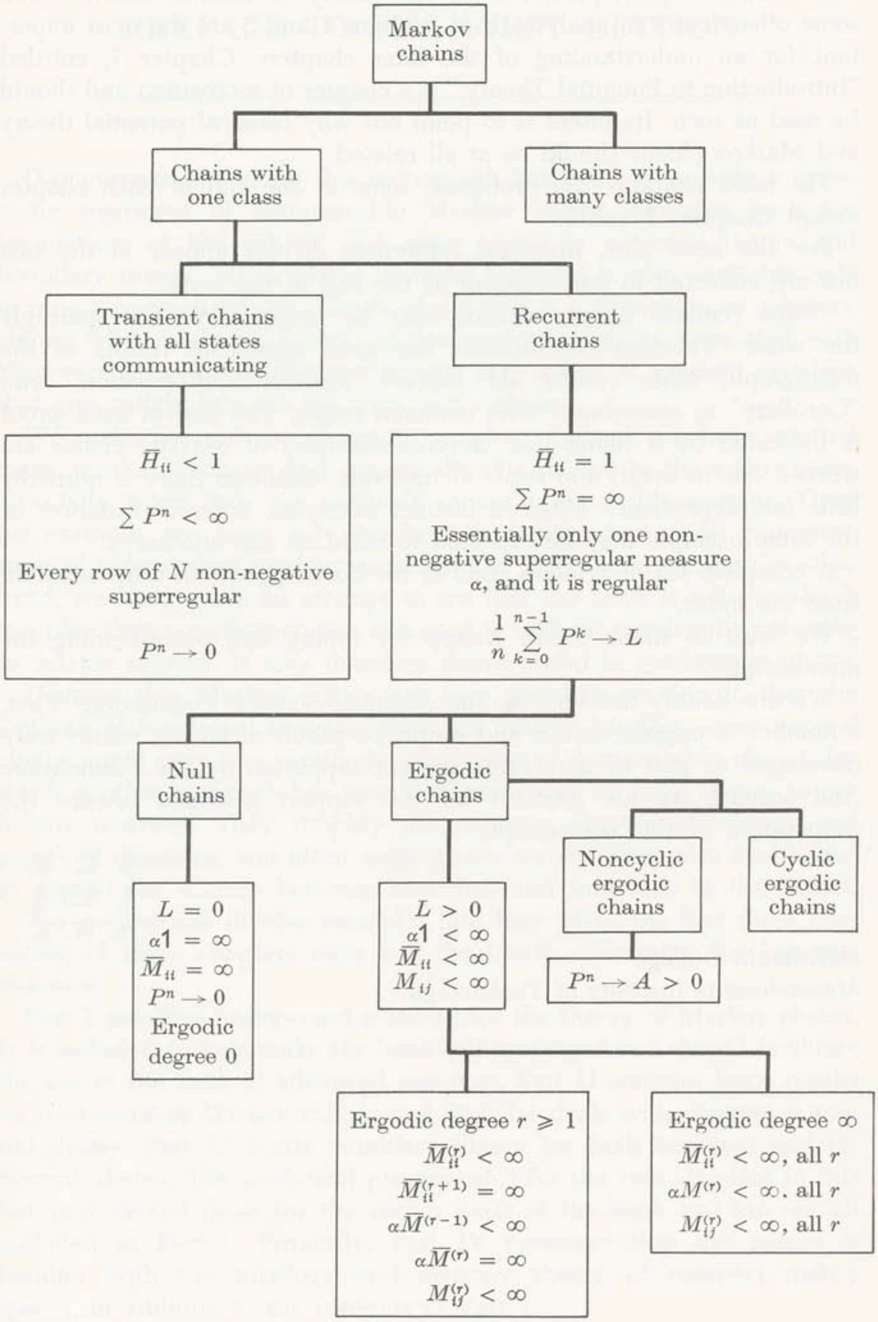
We wish to thank Susan Knapp for typing and proof-reading the manuscript.

We are doubly indebted to the National Science Foundation: First, a number of original results and simplified proofs of known results were developed as part of a research project supported by the Foundation. And second, we are grateful for the support provided toward the preparation of this manuscript.

J. G. K.
J. L. S.
A. W. K.

Dartmouth College
Massachusetts Institute of Technology

RELATIONSHIPS AMONG MARKOV CHAINS



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