Centre de Recherches Mathématiques

CRM Summer School Course

on

REPRESENTATIONS OF REAL REDUCTIVE GROUPS

Edited by:

A.W. KNAPP

Department of Mathematics

State University of New York

Stony Brook, NY 11794, USA

Les publications **CRM**
Directeur: Francis Clarke

ISBN 2-921120-09-7
Dépôt légal, 4^e trimestre 1990

Université de Montréal
C.P. 6128-A
Montréal, QC H3C 3J7
Canada

CRM Summer School Course

on

REPRESENTATIONS OF REAL REDUCTIVE GROUPS

Montréal, July 2 to August 10, 1990

A. W. Knapp
Department of Mathematics
State University of New York
Stony Brook, NY 11794, USA

Contents

Primary References

Yellow Book: Lie Groups, Lie Algebras, and Cohomology, Princeton Mathematical Notes, 1988.

Brown Book: Representation Theory of Semisimple Groups: An Overview Based on Examples, Princeton Mathematical Series, 1986.

Preface

Robert Langlands organized a summer school "Représentations des Groupes et des Algèbres de Lie" at the Centre de Recherches Mathématiques of the Université de Montréal for the period July 2 to August 10, 1990. Although there have been in the past many summer schools on advanced topics in mathematics, this one was distinctive in that it was truly for graduate students; anyone with a Ph.D. was discouraged from enrolling. About 75 students attended, roughly in equal numbers from Canada, the United States, and Western Europe. Four courses were offered, and students generally attended one or two of the four:

1. R. Bédard, Groupes linéaires algébriques,
2. A. Knapp, Representations of real reductive groups,
3. P. Kutzko, Local classfield theory and the representation theory of GL(N) of a p-adic field,
4. Y. Saint-Aubin, Algèbre de Lie de dimension infinie et leurs représentations.

The course on real reductive groups was a 20-hour exposition built around overhead transparencies. Students had paper copies of the transparencies, so that they could listen more and take notes less. Class discussion consisted of examples and other amplification of the transparencies.

This book is largely of a copy of the transparencies from the course, with minor corrections made. As such, it is a record of about two thirds of the course material.

The purpose of the course was to orient students about the field. Most of the students were writing theses in adjacent fields, not in the representation theory of Lie groups. The need was for general understanding, not details. In fact, the course played the role of easing students into reading what is called the "Brown Book" on the transparencies: A.W. Knapp, *Representation Theory of Semisimple Groups: An Overview Based on Examples*, Princeton Mathematical Series, 1986.

Announced prerequisites were topological groups, fundamental groups, covering spaces, and basic measure theory (including Haar measure), as well as closed linear groups as in Chapter I of what is called the "Yellow Book" on the transparencies: A. W. Knapp, *Lie Groups, Lie Algebras, and Cohomology*, Princeton Mathematical Notes, 1988. In other words, the only things initially assumed about Lie groups were the passage from closed subgroups of matrices to their Lie algebras and the passage from homomorphisms between such groups to homomorphisms between their Lie algebras. Because of the uneven background of the students in the course, it seemed advisable to provide a supplementary lecture on this material; notes from this lecture appear as the first section of Appendix B.

Except for one aberration on page 16, no other prior knowledge of Lie theory was needed. Beginning with page 65, however, some deeper knowledge of basic Lie theory came into play. Notes from a lecture on this material appear as the second section of Appendix B. Finally, beginning with page 152, one needed to know that basic Lie theory can be redone with real analytic manifolds and functions replacing smooth manifolds and functions. The classic book by Chevalley, *Theory of Lie Groups*, operates in this context.

Students were provided with exercises for the first two thirds of the course. Doing the exercises was important for maintaining understanding. The exercises are reproduced in Appendix A.

Preparation of these notes was supported in part by National Science Foundation grant DMS 87-23046. I am grateful to the students in the course for spotting a number of misprints on the original transparencies and for bringing them to my attention. Some material in the notes, including some of the exercises, is taken directly from the Brown Book and the Yellow Book and is reproduced by permission of Princeton University Press.

A. W. Knapp

August, 1990